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Brief paper

Trajectory tracking control of thrust-vectoring UAVs☆

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ABSTRACT

In this paper a geometric approach to the trajectory tracking control of Unmanned Aerial Vehicles (UAVs) with thrust vectoring capabilities is proposed. The control problem is developed within the framework of geometric control theory, yielding a control law that is independent of any parametrization of the configuration space. The proposed design works seamlessly when the thrust vectoring capability is limited, by prioritizing position over attitude tracking. The control law guarantees almost-global asymptotic tracking of a desired full-pose (attitude and position) trajectory that is compatible with the platform underactuation according to a specific trackability condition. Finally, a numerical example is presented to test the proposed control law on a tilt-rotor quadcopter UAV. The generality of the control strategy can be exploited for a broad class of UAVs with thrust vectoring capabilities.

1. Introduction

The development of Unmanned Aerial Vehicles (UAVs) with thrust vectoring capabilities has grown significantly in recent years. These aerial vehicles are endowed with a propulsion system that can deliver both a net torque and a force with respect to the aircraft frame, which makes them end-effector-like devices. Among the different technological solutions, several multirotor configurations have shown great potentiality in terms of fast disturbance rejection and maneuverability (Crowther, Lanzon, Maya-Gonzalez, & Langkamp, 2011; Hua, Hamel, Morin, & Samson, 2015; Jiang & Voyles, 2014; Rajappa, Ryll, Bülthoff, & Franchi, 2015; Ryll, Bülthoff, & Robuffo Giordano, 2015). Indeed, while the standard coplanar multirotor architecture (Abdessameud & Tayebi, 2010; Lee, Leok, & McClamroch, 2010; Naldi, Furci, Sanfelice, & Marconi, 2017) combines good performance and a simple mechanical design, it is inherently underactuated as the control force can be applied only in a fixed direction of the aircraft frame. On the contrary, thrust vectoring vehicles overcome this intrinsic maneuverability limitation and widen the operational range of the conventional system. Among the different architectures that have been developed, it is worth mentioning the fixed-tilted hexacopter (Rajappa et al., 2015) and the tilt-rotor quadcopter (Ryll et al., 2015).

The trajectory tracking control problem for these aerial vehicles is challenging for two main reasons: the maneuver may involve large rotational motions and there may be limitations in the thrust vectoring capability, thus reducing the actual maneuverability. In particular, propulsion systems of thrust-vectoring UAVs cannot usually deliver thrust in any direction of the aircraft frame (Rajappa et al., 2015), which makes the platform underactuated. These issues have been addressed explicitly in Hua et al. (2015), in which the control strategy is based on prioritizing position over attitude tracking to handle the actuation limitation. The resulting control law guarantees almost-global tracking (in the sense of Koditschek (1989)) but requires a sufficiently fast loop for the stabilization of the angular velocity. Following similar ideas, Franchi, Carli, Bicego and Ryll, (2018) tackled the tracking problem for a more general class of UAVs with laterally bounded input force. The approach presented therein includes an optimization step to handle the actuation limitation but it guarantees only local exponential convergence of the tracking errors. Thrust vectoring control techniques have also been exploited to solve the position tracking problem of ducted-fan vehicles (Abdessameud & Tayebi, 2010; Pflimlin, Souères, & Hamel, 2007; Roberts & Tayebi, 2011).

In this work, the trajectory tracking problem for UAVs with thrust vectoring capabilities is solved directly in $SO(3) \times \mathbb{R}^3$, with thrust and torque as inputs. We start by showing that tracking of a desired full-pose trajectory (position and attitude) is not feasible if the thrust can be produced only in a cone region around the vertical body axis of the vehicle. However, by relaxing attitude tracking requirements, position tracking can always be achieved. In particular, a reference attitude, different from the desired one, is computed by means of a dynamic controller such that it is always possible to deliver the control force required to guarantee position tracking. Then, the modified attitude trajectory is used as the actual reference for the attitude control subsystem. By exploiting a

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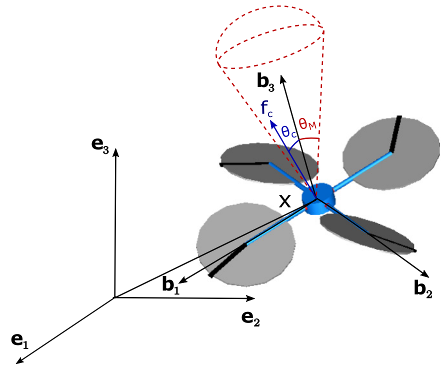

Fig. 1. Reference frames - tilt-rotor quadcopter UAV.

geometric PID controller, the modified attitude motion is tracked even in the presence of a constant disturbance torque. We prove that under a specific trackability condition, also the desired attitude motion can be exactly tracked almost globally. By means of established cascade arguments (Naldi et al., 2017; Raptis & Valavanis, 2010), we demonstrate that our solution ensures almost-global asymptotic tracking (AGAT), which is the strongest result one can obtain on $\mathrm{SO}(3) \times \mathbb{R}^3$ with continuous time-invariant control laws (Koditschek, 1989). Furthermore, the proposed control law improves the transient performance, in terms of position overshoot, with respect to the most common solution in the literature, as it is shown in a numerical example.

Notations. For $A \in \mathbb{R}^{n\times n}$, the minimum and maximum eigenvalues are denoted as $\lambda_m(A)$ and $\lambda_M(A)$, respectively, and $\mathrm{skew}(A) := \frac{A-A^T}{2}$ is the skew-symmetric part of A. The ith canonical base in $\mathbb{R}^n$ is $e_i := [0\cdots 1 \cdots 0]^T$, while the identity element in $\mathbb{R}^{n\times n}$ is $I_n := [e_1 \cdots e_n]$. Given the vectors a, b we often denote $(a, b) := [a^T, b^T]^T$. Given a bounded function $f : \mathbb{R} \to \mathbb{R}^n$, we denote the positive constants f_m and f_M as lower and upper bounds of f, respectively, such that $f_m \le \|f(t)\| \le f_M \ \forall t \in \mathbb{R}$. The *hat* map $\hat{\cdot} : \mathbb{R}^3 \to \mathfrak{so}(3)$ is an isomorphism between $\mathbb{R}^3$ and $\mathfrak{so}(3)$, the space of third order skew-symmetric matrices, such that $\hat{\omega} y = \omega \times y, \forall y \in \mathbb{R}^3$, where $\times$ is the cross product. The corresponding inverse is the *vee* map $(\cdot)^\vee : \mathfrak{so}(3) \to \mathbb{R}^3$. We will often employ the modified trace function $\Psi_K(R) := \frac{1}{2}\mathrm{tr}(K(I_3 - R))$, where $K = K^T \in \mathbb{R}^{3\times 3}$ is such that $\mathrm{tr}(K)I_3 - K \in \mathbb{R}^{3\times 3}_{>0}$, to measure attitude errors in SO(3). The time derivative of Ψ_K along the flows of $\dot{R} = R\hat{\omega}$ is $\dot{\Psi}_K(R) = e_R^T \omega$, where $e_R := \frac{\mathrm{skew}(KR-R^TK)^\vee}{2}$ is the left-trivialized derivative of Ψ_K. When K has distinct positive eigenvalues, Ψ_K is an example of polar Morse function (Koditschek, 1989).

2. Mathematical modeling

The class of aerial vehicles considered in this work can be described as rigid bodies subjected to external actions and with an actuation mechanism that can produce torque in any direction and thrust in a spherical sector around the vertical axis of the body frame.

2.1. Dynamical model

The motion of a rigid body is described by the motion of a body-fixed frame $\mathcal{F}_B = (O_B, \{b_1, b_2, b_3\})$ with respect to an inertial reference frame $\mathcal{F}_I = (O_I, \{e_1, e_2, e_3\})$, as shown in Fig. 1 (for the sake of simplicity, we assume that the inertial frame axes coincide with the canonical basis of $\mathbb{R}^3$). The configuration of a rigid body is uniquely and globally defined by $G := (R, x) \in \mathrm{SO}(3) \times \mathbb{R}^3 =: \mathcal{M}$, where $R := [b_1\, b_2\, b_3] \in \mathrm{SO}(3)$ is the rotation matrix describing the orientation of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$ and $x \in \mathbb{R}^3$ is the position vector of the origin O_B with respect to O_I, resolved in the reference frame $\mathcal{F}_I$. The tangent vector to a curve (velocity) at a given configuration $G \in \mathcal{M}$ is the pair $(\omega, v) \in T_R\mathrm{SO}(3) \times \mathbb{R}^3 \simeq T_G\mathcal{M}$, by direct identification of $\mathbb{R}^3$ with its tangent space $T_x\mathbb{R}^3$. The equations of motion of a rigid body moving in a constant gravity field $-ge_3$, actuated by a control wrench $(f_c, \tau_c) \in \mathbb{R}^3 \times \mathbb{R}^3$ and subjected to external disturbances $(f_d, \tau_d) \in \mathbb{R}^3 \times \mathbb{R}^3$, which include unmodeled dynamics and aerodynamic effects, are described by the following system (Naldi et al., 2017):

$$\dot{x} = v \tag{1}$$

$$\dot{R} = R\hat{\omega} \tag{2}$$

$$m\dot{v} = -mge_3 + Rf_c + f_d \tag{3}$$

$$J\dot{\omega} = -\omega \times J\omega + \tau_c + \tau_d, \tag{4}$$

where $m \in \mathbb{R}_{>0}$ and $J = J^T \in \mathbb{R}^{3\times 3}_{>0}$ are the mass and inertia matrix of the rigid body, respectively. Note that the control wrench $(f_c,\ \tau_c)$ is defined with components in the body frame $\mathcal{F}_B$.

3. Control problem: trajectory tracking in SO(3) × $\mathbb{R}^3$ under thrust vectoring constraints

The thrust-vectoring limitation of the actuation mechanism are now formally defined. In the following, the control torque τ_c is assumed to span $\mathbb{R}^3$, *i.e.*, the rotational dynamics is fully actuated. However, the control force f_c spans only the spherical sector, around the third body axis b_3, defined as:

$$\begin{aligned} &0 < \cos(\theta_M) \le \frac{f_c^T(t)e_3}{\|f_c(t)\|} := \cos(\theta_c(t)) \\ &\|f_c(t)\| \le f_M \quad \forall t \ge 0. \end{aligned} \tag{5}$$

These assumptions may be reasonable approximations for UAVs like the tilt-rotor quadcopter in Fig. 1 (Franchi et al., 2018). Let us now consider a smooth desired trajectory $t \mapsto (R_d(t), x_d(t)) =: G_d(t) \in \mathcal{M}$ that is assigned as a function of time and the corresponding tangent vector that is given by $t \mapsto \xi_d(t) := (v_d(t),\ \omega_d(t)) \in T_{G_d}\mathcal{M}$, where $\omega_d(t) = (R_d^T(t)\dot{R}_d(t))^\vee \in \mathbb{R}^3$ is the desired (*body*) angular velocity and $v_d(t) = \dot{x}_d(t) \in \mathbb{R}^3$ is the desired (*inertial*) translational velocity. Due to the thrust vectoring limitation in (5), an arbitrary full-pose trajectory in $\mathcal{M}$ cannot be followed. This can be understood by inspecting the inputs at steady state obtained by inverting the system dynamics:

$$f_c^n(t) := mR_d^T(t)(\dot{v}_d(t) + ge_3) \tag{6}$$

$$\tau_c^n(t) := J\dot{\omega}_d(t) + \omega_d(t) \times J\omega_d(t), \tag{7}$$

in which, for simplicity, we assumed nominal conditions, *i.e.*, $(f_d, \tau_d) = (0, 0)$. Clearly, whereas Eq. (7) is always fulfilled for any sufficiently smooth trajectory, the control force (6) may not be compatible with constraint (5) for a given desired attitude motion $R_d(t)$ and a given vector $m(\dot{v}_d(t) + ge_3)$. As a consequence, the control objective must be relaxed to deal with the platform underactuation. In particular, since position tracking is of utmost importance in applications involving aerial vehicles, we will devise a strategy that always ensures position tracking and that tries to achieve the attitude tracking objective at best. In this regard, Eq. (6) provides a useful hint: it is always possible to find a rotation matrix such that the resulting control force is inside the cone region defined by (5). Therefore, it is assumed that the actual attitude reference is at least a twice differentiable curve defined as:

$$t \mapsto R_a(t) \in \mathrm{SO}(3) \cap \mathcal{C}_2, \tag{8}$$

which will be computed dynamically in order to satisfy the cone region constraint (5) and to be as close as possible to the desired attitude $t \mapsto R_d(t)$.