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Optimization in curbing risk contagion among financial institutes[☆]

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ABSTRACT

Financial institutions are interconnected by holding debt claims against each other. A default bank may cause its creditors to default, and the risk may be further propagated to up-stream institutes (risk contagion). Such interconnection is a key contributing factor to the past worldwide financial crisis. We show that a good mechanism of default liquidation may improve the total wealth of the financial system and therefore may curb the risk contagion. We formulate this problem as a nonlinear optimization problem with constraints and propose an optimal liquidation policy to minimize the system's loss. We show that the problem resembles a Markov decision problem (MDP) and therefore we can apply the direct-comparison based optimization approach to solve this problem. Higher order directional derivatives and some optimality properties are obtained. Furthermore, we derive an iterative algorithm which combines both the policy iteration and the gradient based approach to find a local optimal policy, and under some conditions, a global optimal policy. Our work provides a new direction in curbing the risk contagion in financial networks; and it illustrates the advantages of the direct-comparison based approach, originated in the field of discrete event dynamic system, in nonlinear optimization problems.

1. Introduction

This paper is motivated by two recent developments in financial engineering and performance optimization. First, financial institutions are interconnected by borrowing-and-lending activities among themselves or holding marketable securities against each other (Chen, Liu, & Yao, 2014). Such interconnection is a critical influencing factor to the past worldwide financial crisis and the European sovereignty debt crisis, and could potentially threaten the stability of financial networks. For example, a default bank may cause its creditors to default, and the risk may be further propagated to up-stream institutes. During default liquidation, it is extremely significant to curb such risk contagion among financial networks. The goal of this paper is to propose a new liquidation scheme based on performance optimization and to provide computation algorithms that solve the problem.

The problem formulated above has a large dimension and many highly nonlinear constraints, and we need to develop an efficient algorithm for an optimal solution. On this side, a direct-comparison based approach has been developed in the past years to the optimization of nonlinear problems and has been successfully applied to many problems, such as optimization of singular controlled diffusion processes (Ni & Fang, 2013), MDP with long-run average criterion (Cao, 2007, 2015), and variance criterion (Xia, 2016), and nonlinear performance with probability distortion (Cao & Wan, 2017). In this paper, we show that the special features of the financial risk contagion problem make it possible to be solved by the direct-comparison based approach, leading to some new insights to the problem.

While network connections have a positive effect by diversifying risk (Haldane, 2009; Summer, 2013), it can have a negative effect by adding spreading channels for risks. When a global shortage for liquidity happens, the systemic risk will be transmitted through the risk-sharing mechanism (Allen & Gale, 2000; Leitner, 2005; Rochet & Tirole, 1996). Such risk contagions may result in consecutive consequences for the financial system, such as declines in asset prices, higher price volatility, more bank defaults, and market inefficiency (Allen & Gale, 2004; Brunnermeier & Pedersen, 2009; Holmstrom & Tirole, 2000).

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In response to the financial crisis and its consequences, it is natural to expect the central bank (CB) to take an active role in controlling systemic risk. Yellen's speech appeals governments to improve financial stability and reduce the risks posed by the interconnected financial system (Yellen, 2013). Dasgupta (2004) claims that a CB is indispensable no matter the structure of a financial system is complete or not. Castiglionesi (2007) presents a model in which a CB can prevent financial contagion by imposing reserve requirements.

In this paper, we propose another possible role that the CB may take in curbing contagion: arbitrating the liquidation among banks in the system during financial crisis, and providing required compensation to achieve fairness. In the literature, it is often assumed that all financial institutes are paid off in proportion to the size of their claims on bank assets (Chen et al., 2014; Eisenberg & Noe, 2001). We show that by allowing different liquidation schemes we may reduce the system's total debts and save banks from defaulting. However, this may violate the fairness of the prorated scheme, and CB arbitration and compensation are needed; this is consistent with both Yellen's speech (Yellen, 2013) and others' work on the roles of CBs (Castiglionesi, 2007; Dasgupta, 2004).

The above problem can be formulated as a performance optimization problem. However, it has a large dimension and many highly nonlinear constraints and is therefore difficult to analyze. Fortunately, we solve this problem through an approach originally developed for the optimization of discrete event dynamic systems (Cao, 2007), called the direct-comparison based approach. The approach is intuitively clear, and it can provide new insights, leading to new results to many problems (Cao, 2007, 2015; Cao & Wan, 2017; Ni & Fang, 2013; Xia, 2016).

More precisely, a liquidation scheme determines how the financial institutes pay their debts among themselves, and is also called a policy. After implementing a liquidation scheme, some banks are default and some others non-default, leading to a partition of default and non-default banks. All the policies leading to the same partition are called a region in the policy space. The policy space may have multiple regions corresponding to different partitions. The optimization problem for policies in the same region was reported in the conference paper (Ye, Xue, Gao, & Cao, 2017). In this paper, we study both the one-region and multi-region optimization problems; we show that by choosing the right liquidation scheme, we may significantly reduce system's total debts and save banks from defaulting.

The difficulty of this optimization problem is mainly caused by the nonconvexity of the regions and nonlinear constraints involved; and there are discontinuities in the performance gradients across the boundaries of the different regions in the policy space. In this paper, we apply the direct comparison based approach to solve this optimization problem in curbing risk contagion among financial institutes. The approach is based on a performance difference formula (PDF), and this PDF explicitly shows the cost difference across different regions. Based on it, we may develop a policy iteration–performance gradients combined algorithm, which leads to a local optimal policy, and under some conditions, a global optimal policy. Numerical examples indicate a significant improvement in the system's loss and the number of default banks. Our work casts new insights to the problem, extends the Eisenberg–Noe model (Eisenberg & Noe, 2001) and obtains the optimal liquidation scheme in curbing risk contagion.

The reminder of the paper is organized as follows. In Section 2, we review the Eisenberg–Noe model (Eisenberg & Noe, 2001) and other related works, and we formulate the optimization problem. In Section 3, we apply the direct-comparison based approach and propose a policy iteration–gradient combined algorithm for the optimal liquidation scheme to minimize the possible system's loss. In Section 4, we provide two numerical examples to demonstrate the efficiency of our approach. Finally, we conclude this paper in Section 5.

2. Problem formulation

2.1. A brief review

Our work is based on the structural framework for contagion in financial network proposed in Eisenberg and Noe (2001). This model illustrates how shocks to individual agents can be propagated through interbank networks, and it was followed by many subsequent works (Chen et al., 2014; Glasserman & Young, 2015; Liu & Staum, 2010).

There are n banks with interconnected balance sheets. The banks in the financial system may have liabilities to each other. The interconnection of the banks is represented via an $n \times n$ liability matrix $L := (L_{i,j})$, where $L_{i,j}$ denotes the nominal obligation of bank i to bank j. Naturally, $L_{i,j} \geq 0$ for $i \neq j$ and $L_{i,i} = 0$. Every bank may also have some liabilities to creditors outside the network, denoted as a row vector $b = (b_i)$, $b_i \geq 0$. We denote the liability vector as $l := (l_i)$, $l_i := b_i + \sum_{j \neq i} L_{i,j}$, and assume $l_i > 0$, for $i = 1, 2, \ldots, n$. We set $r_{i,j} := L_{i,j}/l_i$ to denote the relative liability, and let $R := (r_{i,j})$. We assume that there is only one seniority for the liability. We use a vector $\alpha := (\alpha_i)$, $\alpha_i \geq 0$, to represent the values of exogenous assets of the banks. Then the total asset of bank i is $\alpha_i + \sum_{j \neq i} L_{j,i}$. A bank is defined to be in default if its total liability exceeds its total asset. It is often assumed that bank default will not change the prices outside the network, i.e., α is independent of defaults.

Let $P := (p_{i,j})$ be the liquidation matrix of a liquidation scheme, meaning that in this scheme bank i pays bank j proportionally to $p_{i,j}, j \neq i, j = 1, 2, \ldots, n$. More precisely, let x_i be the total debt that bank i pays to others, then bank i pays bank j with $x_i p_{i,j}$. $x = (x_i)$ is called a clearing vector. In normal situation, $P = R$. It is called a pro rata scheme, i.e., debts are paid proportionally to the relative liabilities.

Next, x and P satisfy the following conditions:
a. *Limited liability*.

$$x_i \leq \alpha_i + \sum_{j=1}^{n} x_j p_{j,i}, \quad \forall i = 1, 2, \ldots, n.$$

b. *Absolute priority*. Either liabilities are paid in full $x_i = l_i$, or all value is paid to creditors, i.e.,

$$x_i = \alpha_i + \sum_{j=1}^{n} x_j p_{j,i}, \quad \forall i = 1, 2, \ldots, n.$$

Putting them into a matrix form as the fixed-point characterization, we have

$$x = \min[l, \alpha + xP]. \tag{1}$$

Based on fixed-point arguments, Eisenberg and Noe (2001) proves that a clearing vector exists for any realization of α, and under some mild regularity conditions, the clearing vector is unique. As an example, if every bank has a positive external liability, i.e., $b_i > 0$ for all i (Glasserman & Young, 2015), then the matrix P is substochastic (all the row sums are strictly less than 1), and thus there exists a unique clearing vector x, because the above fixed-point formulation (1) is a contraction.

Simple and fast algorithms have been developed to calculate the clearing vector x. Eisenberg and Noe (2001) shows that it can be obtained by solving the following linear programming problem, with the performance measure $\eta := \sum_{i=1}^{n} x_i$:

$$\max_{x} \eta, \quad s.t., \quad x(I - P) \leq \alpha, \quad 0 \leq x \leq l. \tag{2}$$