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Brief paper

Optimization-free robust MPC around the terminal region[☆]

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ABSTRACT

We present a novel dual-mode MPC scheme that significantly reduces the computational effort of robust MPC (RMPC). Specifically, we propose a method for the computation of a large set $\mathcal{C}$ on which no optimal control problem (OCP) needs to be solved online. The method is motivated by the trivial observation that, for classical MPC, no optimization is required for the states in the terminal set $\mathcal{T}$, because the unconstrained linear–quadratic regulator is optimal there. While this observation cannot be directly transferred to RMPC, we show that suitable sets $\mathcal{C}$ exist in the neighborhood of $\mathcal{T}$ and state an algorithm for their computation. We stress that the resulting sets $\mathcal{C}$ are significantly larger than robust positively invariant sets that are typically exploited in RMPC and on which it is well-known that no OCP needs to be solved online. The approach is illustrated with three examples for which we observe a reduction of the numerical effort between 22.36% and 95.60%.

1. Introduction and problem statement

Model predictive control (MPC, Camacho & Bordons, 1999; Kouvaritakis & Cannon, 2015; Maciejowski, 2001; Rawlings & Mayne, 2009) has become a standard tool for the regulation of dynamical systems with state and input constraints. MPC is well-established in theory and practice particularly for processes that can accurately be modeled by linear systems. Here, we study the predictive control of linear discrete-time systems of the form

$$x(k+1) = A\,x(k) + B\,u(k) + w(k) \tag{1}$$

with state, input, and disturbance constraints

$$x(k) \in \mathcal{X}, \quad u(k) \in \mathcal{U}, \quad \text{and} \quad w(k) \in \mathcal{W} \tag{2}$$

for every $k \in \mathbb{N}$. Conceptually, MPC is based on periodically solving an optimal control problem (OCP) that allows to explicitly account for the constraints (2) and some performance criterion. In classical linear MPC, disturbances are ignored and the OCP is formulated under the assumption that $w(k) = 0$. This simplification is tolerable, to a certain extent, since nominal MPC is intrinsically robust (see, e.g., Grimm, Messina, Tuna, & Teel, 2004; Kerrigan, 2000; Limón, Alamo, & Camacho, 2002; Scokaert, Rawlings, & Meadows, 1997). However, there exist many robust MPC (RMPC) schemes that allow to explicitly include disturbances in the OCP (see, e.g., Alamo, Muñoz de la Peña, Limon, & Camacho, 2005; Bemporad & Morari, 1999; Campo & Morari, 1987; Langson, Chryssochoos, Raković, & Mayne, 2004; Mayne, Seron, & Raković, 2005; Raković, Kouvaritakis, Cannon, & Panos, 2012). For disturbed systems, these methods outperform deterministic MPC since they guarantee robust asymptotic stability (with convergence to a neighborhood of the origin) for every feasible initial state. RMPC schemes can be distinguished based on their performance criterion and disturbance handling. In tube-based RMPC (as in Langson et al. (2004) and Mayne et al. (2005)), the performance of the nominal system (which, again, results for $w(k) = 0$) is considered and a tube around the nominal trajectory is constructed to account for disturbances. In contrast, in min–max RMPC (Alamo et al., 2005; Campo & Morari, 1987), the worst-case scenario is optimized. Tube-based RMPC and classical MPC are structurally very similar and it is the purpose of this paper to further investigate those similarities. To this end, we note that the underlying OCP can be formulated as

$$V(x_0) := \min_{\substack{\hat{x}(0),\ldots,\hat{x}(N)\\ \hat{u}(0),\ldots,\hat{u}(N-1)}} \; \|\hat{x}(N)\|_P^2 + \sum_{k=0}^{N-1} \|\hat{x}(k)\|_Q^2 + \|\hat{u}(k)\|_R^2 \tag{3}$$

$$\text{s.t. } x_0 - \hat{x}(0) \in \mathcal{S},$$
$$\hat{x}(k+1) = A\,\hat{x}(k) + B\,\hat{u}(k), \qquad \forall k \in \{0,\ldots,N-1\}$$

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$$\hat{x}(k) \in \mathcal{X}_{\rm MPC}, \qquad \forall k \in \{0, \dots, N-1\}$$
$$\hat{u}(k) \in \mathcal{U}_{\rm MPC}, \qquad \forall k \in \{0, \dots, N-1\}$$
$$\hat{x}(N) \in \mathcal{T}$$

in both cases (Mayne et al., 2005). Thereby, N is the prediction horizon, P, Q and R are weighting matrices, $\mathcal{S}$ bounds variations between the current system state x_0 and the initial state $\hat{x}(0)$ of the predicted trajectory, $\mathcal{X}_{\rm MPC} \subseteq \mathcal{X}$ and $\mathcal{U}_{\rm MPC} \subseteq \mathcal{U}$ describe (potentially tightened) state and input constraints, and $\mathcal{T}$ is a terminal set. Clearly, for $\mathcal{S} = \{0\}$ (i.e., $\hat{x}(0) = x_0$), $\mathcal{X}_{\rm MPC} = \mathcal{X}$, and $\mathcal{U}_{\rm MPC} = \mathcal{U}$, we obtain a classical MPC scheme. In contrast, choosing a robust positively invariant (RPI) set $\mathcal{S} \supset \{0\}$ and tightened constraints $\mathcal{X}_{\rm MPC} \subset \mathcal{X}$ and $\mathcal{U}_{\rm MPC} \subset \mathcal{U}$, results in tube-based RMPC (see Section 2.3 for details). For both classical MPC and tube-based RMPC, (3) is solved in every time-step for the current state x_0 and only the first elements of the optimal state and input trajectories are used to control the system. More precisely, the control law

$$\varrho(x_0) := \hat{u}^*(0) + K\,(x_0 - \hat{x}^*(0)), \tag{4}$$

is applied (Mayne et al., 2005, Eq. (23)), where the role of K will be specified in Section 2.3. Here, we stress that K is only relevant for RMPC, since we have $x_0 - \hat{x}^*(0) = 0$ for classical MPC.

It is obviously desirable to reduce the numerical effort required for solving (3) whenever possible. A radical approach in this direction is explicit MPC (EMPC, see Bemporad, Morari, Dua, & Pistikopoulos, 2002; Seron, DeDona, & Goodwin, 2000). In EMPC, the OCP (3) is solved offline (i.e., before runtime of the controller) for every feasible x_0 and the resulting piecewise affine control law is cataloged. In principle, EMPC allows to completely avoid online optimization. However, the computation of the explicit control law is numerically hard for systems with many constraints or long prediction horizons. Moreover, the number of affine segments is often too high to implement or store the explicit controller on the target hardware. However, in classical MPC, online optimization can be avoided for some x_0 even if EMPC cannot be applied. In fact, for the standard choice of P and $\mathcal{T}$ (see Section 2.1), the optimal input $\hat{u}^*(0)$ equals $K_{\rm LQR} x_0$ for every $x_0 \in \mathcal{T}$, where $K_{\rm LQR}$ is the feedback gain of the (unconstrained) linear–quadratic regulator (LQR). This observation is used in dual-mode MPC (see, e.g., Mayne, Rawlings, Rao, & Scokaert, 2000, Sects. 2.4.2.3 and 3.7.3), where numerically solving (3) is avoided in the whole terminal set $\mathcal{T}$.

The role of the terminal set $\mathcal{T}$ in RMPC is different than in classical MPC. In fact, for tube-based and min–max RMPC, trivial solutions to the OCPs are only known for the RPI set $\mathcal{S}$ (see Kouvaritakis & Cannon, 2015, Alg. 4.5; Mayne et al., 2005, Prop. 2; Scokaert & Rawlings, 1998, Alg. 1), or Section 3.1), which usually is significantly smaller than $\mathcal{T}$. In this paper, we show that a simple solution is also available on some set $\mathcal{C} \supset \mathcal{S}$. Surprisingly, $\mathcal{C}$ can even be larger than the terminal set $\mathcal{T}$ (see the first and third examples in Section 4.1). The set $\mathcal{C}$ can be used to design a dual-mode RMPC scheme that avoids the numerical solution of (3) whenever the current system state lies in $\mathcal{C}$. Similar to event-triggered control (see Heemels, Johansson, & Tabuada, 2012 and references therein), dual-mode (R)MPC is useful in situations, where energy-aware control is needed, where optimization tasks are solved centralized, or where communication is costly (see, e.g., Jost, Schulze Darup, & Mönnigmann, 2015). In this context, we show that our dual-mode controller allows to significantly reduce the numerical effort during runtime compared to standard RMPC.

The paper is organized as follows. We state notation and assumptions in the remainder of this section. In Section 2, we recall stabilizing MPC and RMPC parametrizations. Our main result, i.e., an efficient dual-mode RMPC scheme, is presented in Section 3. The approach is illustrated with three numerical examples in Section 4 before we state conclusions in Section 5.

Notation and assumptions. For sets $\mathcal{C}, \mathcal{D} \subset \mathbb{R}^n$, $\alpha \in \mathbb{R}$, and $A \in \mathbb{R}^{n\times n}$, we define $\alpha\,\mathcal{C} := \{\alpha x \mid x \in \mathcal{C}\}$, $A\,\mathcal{C} := \{Ax \mid x \in \mathcal{C}\}$, $\mathcal{C} \oplus \mathcal{D} := \{x + \xi \mid x \in \mathcal{C},\ \xi \in \mathcal{D}\}$ (Minkowski sum), and $\mathcal{C} \ominus \mathcal{D} := \{x \mid \forall \xi \in \mathcal{D} : x + \xi \in \mathcal{C}\}$ (Pontryagin difference). The boundary and the interior of $\mathcal{C}$ are denoted by $\partial\mathcal{C}$ and $\mathrm{int}(\mathcal{C})$. The Minkowski function of $\mathcal{C}$ is defined as $\Psi_{\mathcal{C}}(x) := \inf\{\alpha \geq 0 \mid x \in \alpha\,\mathcal{C}\}$. We further define $\mathcal{B}(\delta) := \{x \in \mathbb{R}^n \mid \|x\|_2 \leq \delta\}$. The smallest and largest singular values of a matrix $\Omega \in \mathbb{R}^{q\times n}$ are denoted $\sigma_{\min}(\Omega)$ and $\sigma_{\max}(\Omega)$. The matrix I_q refers to the identity matrix in $\mathbb{R}^{q\times q}$, e_i is the ith canonical unit vector in $\mathbb{R}^q$, a q-dimensional vector with all entries equal to 1 is written as $\mathbf{1}_q$, and, for $P \in \mathbb{R}^{n\times n}$, $\|x\|_P^2$ is understood as $x^\top P x$. Finally, throughout the paper, we assume that (A, B) is stabilizable, that $\mathcal{X} \subset \mathbb{R}^n$, $\mathcal{U} \subset \mathbb{R}^m$, and $\mathcal{W} \subset \mathbb{R}^n$ are convex and compact polytopes containing the origin as an interior point, that $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are positive definite matrices, and that $N \in \mathbb{N}$ is positive.

2. Stabilizing (R)MPC parametrizations

Conditions guaranteeing stability in (R)MPC are well-known (see, e.g., Mayne et al., 2000, 2005). We recall some results required here.

2.1. Choice of P and $\mathcal{T}$

We briefly summarize the standard choice of P and $\mathcal{T}$ for stabilizing MPC (see, e.g., Scokaert & Rawlings, 1998; Sznaier & Damborg, 1987). First, P is chosen as the solution of the Riccati equation

$$A^\top (P - P\,B(R + B^\top P\,B)^{-1} B^\top P) A - P + Q = 0.$$

Next, the LQR gain $K_{\rm LQR} := -(R + B^\top P\,B)^{-1} B^\top P\,A$ is computed. Finally, $\mathcal{T}$ is chosen as

$$\mathcal{T} := \{x \in \mathbb{R}^n \mid \forall k \in \mathbb{N} : (A + BK_{\rm LQR})^k x \in \mathcal{L}\} \tag{5}$$

where $\mathcal{L} := \{x \in \mathcal{X}_{\rm MPC} \mid K_{\rm LQR}\, x \in \mathcal{U}_{\rm MPC}\}$ (Gilbert & Tan, 1991). Note that $\mathcal{L}$ depends on the constraints $\mathcal{X}_{\rm MPC}$ and $\mathcal{U}_{\rm MPC}$ considered in (3). The choice of $\mathcal{X}_{\rm MPC}$ and $\mathcal{U}_{\rm MPC}$ is detailed in Sections 2.2 and 2.3.

2.2. Choice of $\mathcal{S}$, $\mathcal{X}_{\rm MPC}$, and $\mathcal{U}_{\rm MPC}$ for classical MPC

Setting $\mathcal{S} = \{0\}$, $\mathcal{X}_{\rm MPC} = \mathcal{X}$, and $\mathcal{U}_{\rm MPC} = \mathcal{U}$ results in classical MPC. Due to $\hat{x}(0) = x_0$, we find $V(x_0) = \|x_0\|_P^2$ and

$$u^*(0) = K_{\rm LQR}\, x_0 \quad \text{for every} \quad x_0 \in \mathcal{T} \tag{6}$$

with P, $K_{\rm LQR}$, and $\mathcal{T}$ as in Section 2.1 (Sznaier & Damborg, 1987). Hence, classical MPC can be implemented as the dual-mode control law

$$\varrho(x_0) = \begin{cases} K_{\rm LQR}\, x_0 & \text{if} \quad x_0 \in \mathcal{T}, \\ u^*(0) & \text{otherwise}, \end{cases} \tag{7}$$

which requires a numerical optimization only if $x_0 \notin \mathcal{T}$.

2.3. Choice of $\mathcal{S}$, $\mathcal{X}_{\rm MPC}$, and $\mathcal{U}_{\rm MPC}$ for tube-based RMPC

In tube-based RMPC, the original constraints (2) are tightened to compensate disturbances. To this end, an RPI set for the system $x(k+1) = (A + BK)x(k) + w(k)$ is computed, where K is such that $A + BK$ is Schur. The defining property of an RPI set $\mathcal{R}$ is

$$(A + BK)\,\mathcal{R} \oplus \mathcal{W} \subseteq \mathcal{R}. \tag{8}$$