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Issues in separable identification of continuous-time models with time-delay☆

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ABSTRACT

This paper discusses several issues related to the identification of time-delayed continuous-time systems using the refined instrumental variable method. The proposed estimation procedure is iterative where, at each iteration, the rational system parameters and time-delay are estimated separately. The main contribution of this paper covers three aspects. Firstly, conditions for persistent excitation are established, which should be satisfied to guarantee the identifiability of the system parameters and time-delay. Secondly, existence of multiple minima in the loss function is investigated. Due to the nonlinear nature of the loss function to be optimized with respect to the time-delay, initialization is a particularly important issue for correct estimation of the time-delay. Lastly, to reliably initialize the identification algorithm, based on the previous analysis, some guidelines are proposed to facilitate the choice of reliable initial parameters. The main results derived in this paper are verified by means of both theoretical analyses and numerical simulations.

1. Introduction

Time-delays are a common feature of many industrial processes and occur for many different reasons, such as large scale heat or energy exchange, long distance material transportation, or communication delay. In the presence of a time-delay, the output response of a system is delayed for some time after the input command is applied, which inevitably weakens the achievable feedback performance. Therefore, obtaining an accurate estimate of the time-delay is of crucial importance for controller design, in order to achieve good control system performance. With the fast development of digital computers, system identification has mainly been based on discrete-time (DT) models to facilitate the digital implementation. In such cases, the time-delay is typically constrained to be an integer number of the sampling interval, and can be conveniently handled by shifting the data. However in recent years, there has been an increased interest in modeling industrial processes in the continuous-time (CT) domain. This is motivated by the advantages provided by CT models, for example the ability to handle arbitrarily time-delayed systems and irregularly sampled data, and the physical insight provided by the CT model parameters (Garnier, 2015; Garnier & Wang, 2008; Garnier & Young, 2014; Wang, Zheng, & Chen, 2009; Young & Garnier, 2006; Young, Garnier, & Gilson, 2008; Yuz, Alfaro, Agüero, & Goodwin, 2011). One of the most popular experiment tests for process modeling is the step test, see Aström and Hägglund (2005). However, as it was pointed out in Ahmed, Huang, and Shah (2006), the identified model captures only the basic behavior of the system dynamics since a step test may not provide sufficient excitation of the system dynamics. Therefore, in this paper, we consider excitation signals which are sufficiently rich in harmonics – for example a pseudo-random binary sequence (PRBS) or white-noise signal – to generate accurate model estimates.

In the signal processing literature, estimation of the pure time-delay between signals received at two separated sensors has been studied. This is normally completed by tuning the time-delay to maximize the correlation function of the two signals (Knapp & Carter, 1976). However, the problem is more tricky in control-related applications, since we are interested not only in estimating the pure time-delay but also in modeling the unknown process dynamics. Various methods have been proposed to address this model-dependent time-delay estimation problem, see Björklund

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(2003) for a survey. A simple method is to approximate the time-delay by a rational model (Gawthrop & Nihtilä, 1985), but this leads to an increase in the number of parameters to be estimated, and it also presents a problem when the time-delay is long. Another method is to first estimate a model with an expanded numerator polynomial that covers the maximum time-delay range. Then, by analyzing the estimated numerator coefficients, the time-delay is extracted (Kurz & Goedecke, 1981). A combined correlation and recursive least-squares method was reported in Rad, Lo, and Tsang (2003), where the time-delay is estimated by maximizing the input–output correlation function, whereas the rational model parameters are updated by a recursive least-squares method. The main drawback of this method is that the estimated time-delay is generally biased, since it not only picks up the pure time-delay but also picks up the system phase-lag. A remedy to overcome this deficiency was suggested in Ni, Xiao, and Shah (2010), with the main idea being to construct the correlation function based on the wavelet transforms of the input and output data rather than based on the raw data. Since the system phase-lag converges to zero as the frequency tends to zero, it is possible to factor out the pure time-delay at that limit. If the input is a stationary random process, time-delay estimation can be much easier. It was pointed out in Zheng and Feng (1990) that the accurate time-delay corresponds to the instant when the input–output correlation becomes nonzero, not when it attains the maximum. Other methods include adaptive estimation methods (Gomez, Orlov, & Kolmanovsky, 2007; Na, Ren, & Xia, 2014), where the rational model parameters and time-delay are updated simultaneously using a recursive least-squares approach; relay feedback methods (Hang, Aström, & Wang, 2002), in which the unknown parameters are retrieved based on the measured shape factors of the relay-produced limit cycle.

In this paper, we focus on time-delay estimation by minimizing a loss function defined in the least-squares sense. Extensive methods have been proposed to solve this problem, prediction- and output-error methods (Chen, Gilson, Garnier, & Liu, 2017; Ljung, 2002; Sung & Lee, 2001), instrumental variable (IV) methods (Baysse, Carrillo, & Habbadi, 2011; Yang, Iemura, Kanae, & Wada, 2007; Young & Garnier, 2006) and ordinary least-squares methods (Ahmed et al., 2006; Na et al., 2014), to name but a few. It should be noted that ordinary least-squares estimators are consistent if and only if the residual of the regression model is white. To cope with measurement noise, we restrict our attention to the application of IV methods, which have the advantage of yielding consistent parameter estimates even if the noise model is unspecified or poorly estimated, while using only linear regression (Gilson, Welsh, & Garnier, 2018; Söderström & Stoica, 1983). One of the most reliable implementation of IV estimators is the refined IV method, which was first developed by Young and Jakeman in Young and Jakeman (1979). The relation to other methods, such as prediction-error and maximum-likelihood methods, was demonstrated in Young (2015). Recently, the simplified refined instrumental variable method for CT models (SRIVC) was extended to handle arbitrarily time-delayed systems (Chen, Garnier, & Gilson, 2015). Here *simplified* means that no noise modeling is considered (Young, 2011). The developed method, renamed TFSRIVC (SRIVC for transfer function models) in this paper, is based on the principle of variable projection (Golub & Pereyra, 2003), combining an IV method for the rational model parameters and an adaptive search for the time-delay. The TFSRIVC method has shown to be effective in terms of numerical simulations (Chen et al., 2015). However, a theoretical analysis has not yet been presented, which is an issue that will be addressed in this paper. Note that the convergence of a gradient-based method to the same problem has been analyzed in our previous work (Chen et al., 2017). Compared with this existing work, the new contribution of the current paper are basically that (1) the refined IV method is considered, and (2) conditions for persistent excitation are established.

Identification of time-delayed systems is challenging due to the presence of a nonlinear parameter, *i.e.*, the time-delay, in a rational model. Here we call a *nonlinear parameter* if differentiation with respect to this parameter cannot yield a constant. This nonlinear nature may result in multiple minima being created in the loss function, making the global minimum hard to approach (Ferretti, Maffezzoni, & Scattolini, 1996; Zheng & Feng, 1991). Therefore, the question of how to improve the convergence performance in time-delayed system identification has been an interesting and practically important problem in the community of process control. The aim of this paper is to provide some effective solutions to the above problem. The main contribution lies in the answers to the following questions.

1. What kinds of systems potentially have multiple minima in the loss function? This question was partially answered some time ago in terms of numerical examples, see *e.g.*, Ferretti et al. (1996) and Zheng and Feng (1991), in which the loss function was shown to be multimodal if the system had complex poles. In this paper, we will stick to this issue, and further analyze not only the impact of complex poles but also the impact of real poles as well as zeros on the loss function shape.
2. How can a reliable initialization of the TFSRIVC method be achieved? On this issue, several guidelines and techniques will be suggested to facilitate the choice of reliable starting values, especially for models which have zeros in the numerator.

The TFSRIVC method belongs to the category of separable identification methods, where the rational model parameters and time-delay are estimated in two separate steps. Similar methods include (Baysse et al., 2011; Yang et al., 2007; Zheng & Feng, 1991). A common feature of these methods is that they all employ a numerical search to solve for the optimal time-delay. The difference lies in the method of estimating the rational model parameters, as well as in the technique to improve the convergence performance of the time-delay. In Zheng and Feng (1991), identification of time-delayed exogenous autoregressive moving average (ARMA) models was considered. An ordinary least-squares method was employed to handle the rational model parameters. To help the convergence of the time-delay, prefilters were designed such that the multimodal error function was transformed into a unimodal function with respect to the time-delay. In Yang et al. (2007), identification of CT output-error models was considered. Least-squares and IV methods were utilized to estimate the rational model parameters. To compute time-derivatives that are required in direct CT modeling, a linear filter was used. In order to improve the convergence performance in the estimation of the time-delay, a stochastic perturbation was added to the gradient vector, which greatly reduced the probability of being trapped in a local minima. However, it was latter recognized in Baysse et al. (2011) that the IV parameter estimates generated by Yang et al. (2007) were unbiased but did not have minimum variance — as the prefilter used was suboptimal/not optimal. Motivated by the SRIVC method, an modification was made by employing the model denominator to build up the prefilter. As such, the variance of the estimated parameters was minimized. In this paper, we aim to provide a thorough discussion on several important issues related to the identification problem under consideration, including conditions for persistent excitation, impacts of poles and zeros on the loss function shape, and methods to generate reliable starting values.

The remainder of this paper is arranged as follows: the identification problem is formulated in Section 2; the TFSRIVC method is derived in Section 3; the conditions for persistent excitation are presented in Section 4; then, the existence of multiple minima in the loss function is investigated in Section 5; based on the