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Vehicular platoons in cyclic interconnections☆

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ABSTRACT

Vehicle platooning has received considerable attention as a means to achieve more efficient transportation networks and vehicle autonomy. It is well known that certain systems experience undesired behaviours (instability and string instability) when the length of the platoon grows. In this context, we study homogeneous platoons of vehicles with linear dynamic models, having bi-directional communication and maintaining a constant (velocity independent) target inter vehicle spacing. We investigate both stability and string instability if the string length increases, where the vehicles utilise weighted information from multiple vehicles ahead and behind. We find conditions for the weighting factors and the linear systems that are necessary for stability of long platoons. We then show that by selecting a communication range that increases linearly with the platoon size, it is possible to achieve string stability in some cases. Further, we show that careful selection of the weighting factors can reduce the disturbance amplification considerably.

1. Introduction

Vehicle platooning is commonly studied, see for example Barooah and Hespanha (2005), Cook (2007), Herman, Martinec, Hurák, and Śebek (2015), Lestas and Vinnicombe (2007), Levine and Athans (1966), Martinec, Herman, and Sebek (2016), Melzer and Kuo (1971), Middleton and Braslavsky (2010), Peppard (1974), Rogge and Aeyels (2008), Seiler, Pant, and Hedrick (2004), Swaroop and Hedrick (1996), Swaroop and Hedrick (1999), Verginis, Bechlioulis, Dimarogonas, and Kyriakopoulos (2017), Zheng, Li, Li, and Wang (2016), Zheng, Li, Wang, Cao, and Li (2016). In this application multiple vehicles are controlled to maintain a certain formation.

While early works focused on a centralised control approach (Levine & Athans, 1966; Melzer & Kuo, 1971), the large number of vehicles demands a distributed or decentralised solution. Here, the vehicle controller uses local information from other neighbouring vehicles (Barooah & Hespanha, 2005; Cook, 2007; Herman, Martinec, Hurák et al., 2015; Martinec et al., 2016; Peppard, 1974; Verginis, Bechlioulis, Dimarogonas, & Kyriakopoulos, 2015; Verginis et al., 2017) and in some cases from the leading vehicle (Cook, 2007; Fax & Murray, 2004; Middleton & Braslavsky, 2010; Seiler et al., 2004; Swaroop & Hedrick, 1999; Zheng, Li, Li et al., 2016; Zheng, Li, Wang et al., 2016). Some of these distributed systems can exhibit two issues with increasing platoon size. The first issue is instability, while the second is an unbounded amplification of disturbances (commonly known as string instability (Peppard, 1974)).

The methods used to analyse these systems range from classical control theory to spatial–temporal system techniques (Bamieh, Paganini, & Dahleh, 2002; Knorn, 2012). Recent works combine control theoretic approaches with graph theory. In that context the vehicle's behaviour is governed by an individual dynamic system, while the information exchange among the vehicles is represented as a graph. In this paper we follow this approach. There are three design choices that have a profound impact on stability and string stability: (1) the formation and spacing policy; (2) the dynamic system of the individual vehicles; and (3) the information flow.

Two formations are common: string and cyclic formations. In the former the vehicles form a line with a leader. This is the most common interconnection and can represent highway traffic. In the cyclic formation, the vehicles form a real or virtual circle. This is present for example in light rail, subway circuits, or densely used road systems (e.g. ring roads) in cities. Additionally, cyclic formations, simplify the analysis due to structural properties. Hence, these systems can be useful to establish results for the string formation, as argued in Herman, Martinec, and Veerman (2016) and Herman, Martinec, Veerman, and Sebek (2015), however they experience different issues. For example, while stability is usually

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achieved in uni-directional string formations, this no longer holds for cyclic formations.

Within the given formation the vehicles aim to maintain a predefined inter vehicle distance, defined via the spacing policy. Two spacing policies are common: constant spacing and a velocity dependent spacing. While a constant spacing is known to cause (string) stability issues in some cases, it has the desirable property of achieving a more efficient traffic flow compared to a velocity dependent distance, which experiences less of the known stability issues. Sometimes the use of a velocity dependent spacing policy is suggested to avoid (string) instability (Peters, Middleton, & Mason, 2016; Rogge & Aeyels, 2008).

In this paper we utilise the structural properties of cyclic formations to obtain conditions for stability and string stability, when a constant spacing policy is utilised.

The second design choice is the dynamic system model used. Even though a vehicle is a non-linear system, a common approach is to linearise the dynamics. For example, feedback linearisation is used in Zheng, Li, Li et al. (2016) and Zheng, Li, Wang et al. (2016). In this paper, we assume that such a technique has been used and base our analysis on a general linear time invariant system containing two integrators. Note that such a general representation includes commonly used models (Zheng, Li, Li et al., 2016; Zheng, Li, Wang et al., 2016).

We acknowledge that the use of non-linear controllers can improve the exhibited issues Huang, Huang, Deng, and Chen (2017) and Verginis et al. (2017).

Finally, the third main impact factor is the information flow, which can be captured using a graph. Hence, the Laplacian of the graph is very important (Barooah & Hespanha, 2005; Herman, Martinec, Hurák et al., 2015; Yadlapalli, Darbha, & Rajagopal, 2006).

It is shown in Peters et al. (2016) that instability is unavoidable with increasing platoon lengths using only the information of the predecessor. So without other measures, the extension of the communication range is an option to avoid string stability issues. Hence, we allow an extended bi-directional communication range r, where the information of r vehicles in front and back is considered. The information of the first vehicle is not passed on. This set up is similar to the one in Herbrych, Hazirakis, Christakis, and Veerman (2017) and Middleton and Braslavsky (2010) except that (1) we allow weighting between the information received by different vehicles[1] ; and (2) we consider a communication length r that can grow with the platoon size n.

This paper expands on the results presented in Stüdli, Seron, and Middleton (2017b). In detail, additional insight is given for positional symmetric weighting factors in regard to stability and the results on string stability are extended for non-equal weighting factors. For completeness some parts are repeated. The paper is structured as follows. In Section 2, the system is described in detail. We investigate stability in Section 3 and string stability in Section 4. In Section 5 we conclude the paper and point out future work.

2. System definition

We use the general system description as in Stüdli et al. (2017b), which is commonly used, see for example Barooah and Hespanha (2005), Herman, Martinec, Hurák et al. (2015), Li, Duan, and Chen (2011), Tonetti and Murray (2010), Tonetti and Murray (2011), Yadlapalli et al. (2006), You and Xie (2013).

[1] The inclusion of the weighting factors is an important difference to other works including some on non-cyclic structures as the ones in Zheng, Li, Li et al. (2016) and Zheng, Li, Wang et al. (2016).

2.1. Plant

We consider a platoon of n homogeneous vehicles which aim to keep a predefined distance to their predecessors. Such a set up was considered in several studies for example Barooah, Mehta, and Hespanha (2009), Middleton and Braslavsky (2010) and Seiler et al. (2004). The transfer function $\bar{P}(s)$ of each vehicle maps the control input $u_i(s)$ to the vehicle x-coordinate, such that

$$x_i(s) = \bar{P}(s)u_i(s). \tag{1}$$

We assume that $\bar{P}(s)$ is strictly proper, does not have any unstable hidden modes, and has no zero at $s = 0$.

Further, let the desired target separation of the ith vehicle to its predecessor be denoted by $\delta_i(t)$. Then, the inter-vehicle distance error is defined as

$$e_i(t) = \delta_i(t) + x_{i-1}(t) - x_i(t) \tag{2}$$

for vehicles $i = 1, \ldots, n$. Since we look at a cyclic formation the first vehicle is following the last vehicle, i.e. $x_k = x_{k+n}$ for all $k \in \mathbb{N}$.

The vectors $x \triangleq (x_1, \ldots, x_n)$, $u \triangleq (u_1, \ldots, u_n)$, $e \triangleq (e_1, \ldots, e_n)$ and $\delta \triangleq (\delta_1, \ldots, \delta_n)$, are related by

$$e(s) = \delta(s) - Mx(s) \text{ and} \tag{3}$$

$$x(s) = \bar{P}(s)Iu(s), \tag{4}$$

where I is the $n \times n$ identity matrix and M is a *system coupling matrix*, defined as

$$M \triangleq \begin{bmatrix} 1 & 0 & \ldots & 0 & -1 \\ -1 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 \end{bmatrix}. \tag{5}$$

2.2. Controller

For the system described in the previous section we will consider a multi-variable controller which is built upon a scalar controller $\bar{K}(s)$ that stabilises $\bar{P}(s)$ and a *control coupling matrix* Γ. Thus, the input of the plant can be computed by

$$u(s) = \bar{K}(s)\Gamma e(s), \tag{6}$$

where $e(s)$ is defined in (3). We will use $\tilde{e}_i(s)$ to denote the ith entry of $\Gamma e(s)$.

The control coupling matrix Γ defines what measurements each vehicle can utilise to implement the control action. In this paper, we consider a cyclic control structure where each vehicle can use the relative distance to its r predecessors and followers. Unlike in other literature, for example Middleton and Braslavsky (2010), we allow the measurements to be weighted. This is a significant variation, as it not only allows the implementation of asymmetric control, but also gives the controller the freedom to choose the importance of the information. We will explain the used weighting in detail in Section 2.4 and see in the remainder of the paper its impact on the platoon. We will next inspect the interconnected system and representation thereof.

2.3. Interconnected system

We allow an output disturbance w_o to act on the system as shown in Fig. 1. Then, from (3) to (6) we find that the inputs of the networked system relate to the output by

$$x(s) = (I + L(s)\Gamma M)^{-1}w_o(s) \\ +(I + L(s)\Gamma M)^{-1}L(s)\Gamma\delta(s) \tag{7}$$