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Brief paper

Finite-time and asymptotic left inversion of nonlinear time-delay systems ☆

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ABSTRACT

In this paper we investigate the left invertibility problem for a class of nonlinear time-delay systems. In both cases of time delay systems with and without internal dynamics the invertibility conditions are given. A new approach based on the use of higher order sliding mode observer is developed for finite-time left invertibility and for asymptotic left inversion. Causal and non-causal estimations of the unknown inputs are respectively discussed. The results are illustrated by numerical examples in order to show the efficiency of the method and its limits.

1. Introduction

Time delay systems represent one of the most studied class of systems in control theory. Since the 60s, many different problems are studied such as stability and stabilization of time delay systems, observation and observer design, parameter identification, etc. In the present work we are interested in the left invertibility problem of time delay systems with internal dynamics. The problem of unknown inputs recovering from the outputs is crucial. Such a problem has attracted the interest of the control community since it has direct applications in many domains, such as data secure transmission where the unknown input is the message, and fault detection and isolation where the fault is the unknown input. In fact, left invertibility problem has been studied since at least forty five years ago in linear control theory (Sain & Massey, 1969; Silverman, 1969) and thirty five year ago in nonlinear control theory (Hirschorn, 1979; Singh, 1981). Most of those works are in the context of nonlinear systems without time delays, or linear systems with commensurate delays (Bejarano & Zheng, 2014; Zheng, Bejarano, Perruquetti, & Richard, 2015).

In the literature, an important tool based on non-commutative ring, proposed in Xia, Márquez, Zagalak, and Moog (2002), is used to analyze nonlinear time delay systems in algebraic framework. Using this framework, many notions are extended to the case of nonlinear time delay systems and many results have already been obtained and published (Zheng, Barbot, & Boutat, 2013; Zheng, Barbot, Boutat, Floquet, & Richard, 2011). In the context of constant time delay, the notions of Lie derivatives and relative degree are defined, and the differences between causal and non-causal invertibility are clarified in Zheng, Barbot, Boutat, Floquet, and Richard (2010a). The canonical form of invertibility is also given in Zheng, Barbot, Boutat, Floquet, and Richard (2010b), and in Zheng and Richard (2015) a method for estimating the unknown inputs is proposed. However, the algorithm for left invertibility proposed in Zheng et al. (2010b) was only for system without internal dynamics (see also Califano, Marquez-Martinez, & Moog, 2013, Germani, Manes, & Pepe, 2003 and their references). For system with or without time-delay, the main difficulty when the internal dynamics appears is to estimate the state of such dynamics. One interesting solution in order to overcome such a difficulty is to allow the derivative of the unknown inputs (Respondek, 1990) with a geometrical approach and with an algebraic one. If, however, the input derivatives are not possible, then it is necessary to compute and analyze the internal dynamics.

For nonlinear systems without delay, if the vector fields associated to the inputs verify involutivity property, then the internal dynamics does not depend on the unknown input. However, this rule is not valid for nonlinear systems with time delay. In order to analyze the internal dynamics and estimate its state, one solution is to rewrite this internal dynamics such that its dynamics become

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independent of the unknown input. Thus in the paper we will adapt a new way to determine the internal dynamics for the nonlinear systems with time-delay (Barbot, Boutat, & Busawon, 2015). This method is based on the finite-time convergence by using the existing observer for time-delay system in the literature (Bhat & Koivo, 1976; Germani, Manes, & Pepe, 2001; Ghanes, De Leon, & Barbot, 2013; Hou, Zítek, & Patton, 2002; Ibrir, 2009; Kharitonov & Hinrichsen, 2004) and the high order sliding mode proposed in Levant (1998, 2003) (see also Fridman, Shtessel, Edwards, & Yan, 2008 for unknown input observer) is applied in this paper.

As a continuity of our preliminary work in Kader, Zheng, and Barbot (2014), this paper provides a method for finite-time and asymptotic left invertibility for nonlinear time delay systems with and without internal dynamics. This method is based on a finite-time estimation of the state variables using a higher order sliding mode observer and on an asymptotic estimation of the internal dynamics. The causality of the unknown input estimation is given. Finally, in order to illustrate the result, numerical examples are provided.

2. Algebraic framework and notations

Consider the following class of multi-input multi-output nonlinear time-delay systems:

$$\begin{cases} \dot{x} = f(x(t-j\tau)) + \sum_{i=1}^{m} g_i(x(t-j\tau))u_i(t), \\ y = h(x(t-j\tau)),\ x = \psi(t), u(t) = \varphi(t),\ t \in [-s\tau, 0], \end{cases} \quad (1)$$

where $x \in W \subset \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the vector of the unknown inputs and $y \in \mathbb{R}^p$ is the output, with $p \geq m$. τ represents the basic commensurate time delay, and it is assumed in this paper that f, g_i and h are all meromorphic functions, where $f(x(t-j\tau))$, $g_i(x(t-j\tau))$, and $h(x(t-j\tau))$ denote functions depending on $x(t)$, $x(t-\tau)$, $x(t-2\tau)$ until $x(t-s\tau)$. This class of nonlinear time-delay systems is widely studied in the literature, and a lot of tools are used to study their properties (see for instance Califano et al., 2013, Germani et al., 2003, Richard, 2003 and references herein), among which we are interested in the use of algebraic tools to develop the invertibility conditions of system (1) when its internal dynamics are not vanished. Since the delay is assumed to be commensurate, this paper utilizes the algebraic framework, developed in Conte, for Information Processing, Moog, and Perdon (1999), Márquez-Martínez, Moog, and Velasco-Villa (2000) and Moog, Castro-Linares, Velasco-Villa, Márquez-Martínez, et al. (2000), to study the structure of nonlinear time-delay systems. Under this framework, defining $\mathscr{K}$ as a field of meromorphic functions of finite number of variables of the form $\{x_i(t-j\tau), i \in [1, n], j \in [0, s]\}$ such that $\mathscr{E}$ represents the vector space over $\mathscr{K}$: $\mathscr{E} = span_{\mathscr{K}}\{d\varsigma : \varsigma \in \mathscr{K}\}$ and τ represents the basic commensurate time delay. After, in Xia et al. (2002) this algebraic framework is generalized and the algebraic properties of the field $\mathscr{K}$ are studied.

Denote the operator δ as a backward shift operator, which means $\delta^j\varsigma(t) = \varsigma(t-j\tau)$ and $\delta^j(a(t)d\varsigma(t)) = a(t-j\tau)d\varsigma(t-j\tau)$. With this operator, we can then define the following set of polynomials $\mathscr{K}(\delta]$: $a(\delta] = \sum_{j=0}^{r_a} a_j\delta^j$, $a_j \in \mathscr{K}$. The addition for the entries in $\mathscr{K}(\delta]$ is defined as usual, but its multiplication is given by the following criteria:

$$a(\delta]b(\delta] = \sum_{k=0}^{r_a+r_b} \sum_{i+j=k}^{i\leq r_a, j\leq r_b} a_j(t)b_i(t-j\tau)\delta^k. \quad (2)$$

With the standard differential operator d, denote by $\mathscr{M}$ the left module over $\mathscr{K}(\delta]$:

$$\mathscr{M} = span_{\mathscr{K}(\delta]}\{d\varsigma, \varsigma \in \mathscr{K}\}. \quad (3)$$

Hence, $\mathscr{K}(\delta]$ is a non-commutative ring satisfying the associative law, and it has been proved in Jan (1996) and Xia et al. (2002) that it is a left Ore ring, which enables us to define the rank conception. In order to take into account the non-causal case, following the same idea, we can introduce as well the forward shift operator ∇ such that $\nabla^j x(t) = x(t+j\tau)$ (see also Fliess, 1987), which means as well $\nabla x(t) = \delta^{-1}x(t)$. In the following, for the sake of simplicity, a function $f(x, \delta, \nabla)$ simply means it is a function of x, and the backward and forward value of x.

Thanks to the above algebraic framework the nonlinear time-delay system (1) can be represented in a compact algebraic form as follows:

$$\begin{cases} \dot{x} = f(x, \delta) + \sum_{i=1}^{m} G_i(x, \delta)u_i(t), \\ y = h(x, \delta),\ x = \psi(t), u(t) = \varphi(t),\ t \in [-s\tau, 0], \end{cases} \quad (4)$$

where the notation $f(x, \delta)$ means $f(x, \delta) = f(x, x(t-\tau), \ldots, x(t-s\tau))$ and the same is considered for $G(x, \delta) = (G_1(x, \delta), \ldots, G_m(x, \delta))$ and $h(x(t), \delta)$, $G_i = \sum_{j=0}^{s} g_i^j\delta^j$ with entries belonging to $\mathscr{K}(\delta]$. Let us now give some definitions, which will be used in the sequel to develop our main results.

Definition 1 (*Left Invertibility*). System (4) is said to be asymptotically left invertible (or equivalently, u can be asymptotically estimated) if $\|u(t) - \hat{u}(t)\| = 0$ when $t \to \infty$, with $\hat{u}(t) = \varphi(\hat{\xi}, \delta, \nabla) \in \mathbb{R}^m$ and $\dot{\hat{\xi}} = \bar{f}(\hat{\xi}, y, \delta) \in \mathbb{R}^{\bar{n}}$ with some user chosen functions φ and $\bar{f}$ with proper dimension $\bar{n}$. It is finite-time left invertible (or equivalently, u can be finite-time estimated) if there exist $T > 0$ such that $\|u(t) - \hat{u}(t)\| = 0, \quad \forall t > T$.

Definition 2 (*Causality*). System (4) is said to be causally asymptotically (or finite-time) left invertible if it is asymptotically (or finite-time) left invertible with $\hat{u}(t) = \varphi(\hat{\xi}, \delta)$. Otherwise, it is said to be non-causally asymptotically (or non-causally finite-time) left invertible .

Let $f(x(t-j\tau))$ and $h(x(t-j\tau))$ for $0 \leq j \leq s$ be an n and p dimensional vector fields respectively, with f_r the entries of f belonging to $\mathscr{K}$ for $1 \leq r \leq n$ and $h_i \in \mathscr{K}$ for $1 \leq i \leq p$, with $p \geq m$, and then $\frac{\partial h_i}{\partial x} := \left[\frac{\partial h_i}{\partial x_1}, \ldots, \frac{\partial h_i}{\partial x_n}\right] \in \mathscr{K}^{1\times n}(\delta]$ where for $1 \leq r \leq n$, $\frac{\partial h_i}{\partial x_r} := \sum_{j=0}^{s} \frac{\partial h_i}{\partial x_r(t-j\tau)}\delta^j \in \mathscr{K}(\delta]$, we can define the Lie derivative in the algebraic framework as follows: $L_f h_i := \sum_{r=1}^{n}\sum_{j=0}^{s} \frac{\partial h_i}{\partial x_r(t-j\tau)}\delta^j f_r$ and $L_G h_i := \sum_{r=1}^{m}\sum_{j=0}^{s} \frac{\partial h_i}{\partial x_r(t-j\tau)}\delta^j G_r \delta^j$.

The conventional concepts like Lie derivative and relative degree have been generalized to nonlinear time-delay systems by Califano et al. (2013), Germani, Manes, and Pepe (1996), Oguchi (2007), Oguchi, Watanabe, and Nakamizo (2002) and the following adapted definitions using the above algebraic framework are recalled (for more details see Márquez-Martínez et al., 2000).

Definition 3 (*Relative Degree*). System (4) has the relative degree $(\nu_1, \ldots, \nu_p)$ in an open set $W \subseteq \mathbb{R}^n$ if the following conditions are satisfied for $1 \leq i \leq p$:

(1) for all $x \in W$, $L_{G_j}L_f^r h_i = 0$ for all $1 \leq j \leq m$ and $0 \leq r < \nu_i - 1$;
(2) there exists $x \in W$ such that $\exists j \in \{1, \ldots, m\}$, $L_{G_j}L_f^{\nu_i-1}h_i \neq 0$.

If the first condition is satisfied for all $r \geq 0$ and some $i \in \{1, \ldots, p\}$, we set $\nu_i = \infty$.

Definition 4 (*Observability Indices*). System (4) has the observability indices $(k_1, \ldots, k_p)$, where for $1 \leq k \leq p$ $k_i = card\{d_k \geq i, 1 \leq k \leq n\}$ with $d_1 = rank_{\mathscr{K}(\delta]}\mathscr{F}_1$, and $d_k = rank_{\mathscr{K}(\delta]}\mathscr{F}_k - rank_{\mathscr{K}(\delta]}\mathscr{F}_{k-1}$ for $2 \leq k \leq n$, where $\mathscr{F}_k := span_{\mathscr{K}(\delta]}\{dh, dL_f h, \ldots, dL_f^{k-1}h\}$.