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Stabilization of linear systems with distributed infinite input delays: A low gain approach[☆]

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ABSTRACT

This paper investigates the stabilization problem of linear systems with distributed infinite input delays. By introducing a stability result on systems with infinite delays, it is shown that a stabilizable linear system with distributed infinite input delays can be globally asymptotically stabilized with low gain controllers as long as the open-loop system is not exponentially unstable. Our result includes constant delay and bounded distributed delay as its special cases. It should be noted that the stabilization problem of systems with distributed infinite input delays, to our best knowledge, has not been considered in existing literatures. Simulation examples are provided to illustrate the effectiveness of our theoretical results.

1. Introduction

In this paper, we consider the stabilization problem of the following linear systems with distributed infinite delay in the input:

$$\dot{x}(t) = Ax(t) + \int_0^{+\infty} B(\eta)u(t-\eta)d\eta,$$
$$x_0(\theta) = \phi(\theta), \quad \theta \in (-\infty, 0], \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $B(\eta)$ is a matrix function whose elements are all Lebesgue integrable on $[0, +\infty)$. In this study, the distributed infinite delay is formulated by the integral over all past time of the history of the input. The matrix function $B(\eta)$ is the coupling of the system matrix and the delay kernel.

Time-delay systems have been extensively studied for the past decades, see, for example Cao, Lin, and Hu (2002), Chen and Latchman (1995), Gu, Chen, and Kharitonov (2003), Hale and Lunel (2013), Niculescu (2001), Richard (2003), Wu and Zheng (2009a, b) and Xu, Feng, Zou, and Huang (2012) and the references therein. In practical systems, time delays are inherent features and are often the main cause for instability and poor performance. Time-delay systems are usually very challenging to deal with mainly because they belong to infinite dimensional FDEs (functional differential equations). Distributed delays, as one of the most common types, have received considerable attention in control community. The stability or stabilization problems of systems with distributed delays have been widely considered, see, for example Chen and Zheng (2006, 2007), Olbrot (1978), Xu and Chen (2004), Wu, Shi, Gao, and Wang (2009) and Xie, Fridman, and Shaked (2001).

However, all aforementioned works on time delayed systems consider bounded time delays. Infinite delay, also known as unbounded delay, is much more general but also much more difficult to deal with. There are two typical types of infinite delays, that is, time-varying infinite delays and distributed infinite delays. In this study, we focus on distributed infinite delays, which have been considered in many practical systems, such as the HIV-spread model (Culshaw, Ruan, & Webb, 2003), the modelling of oscillators (Atay, 2003), the modelling of traffic flow (Sipahi, Atay, & Niculescu, 2007), the analysis of neural networks (Liao, Li, & Chen, 2004). It is however worth pointing out that the stabilization problem for systems with distributed infinite delays, to our best knowledge, has not been considered so far. Systems with distributed infinite delays, compared with bounded ones, present challenges in three aspects. First, tools for addressing infinite delays are quite limited. Many stability or stabilization results have been developed for systems with bounded delays via using methods of the Lyapunov–Razumikhin functions, the Lyapunov–Krasovskii functionals and the frequency-domain criteria. However, the conditions of those existing results cannot be satisfied when time delays become infinite. In other words, those results cannot be applied to the case of infinite delays directly. Therefore, extended versions of those methods or new methods need to be developed in order to analyse infinite delayed systems. Second, the

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solutions in the case of infinite delays are sensitive to the initial conditions as their dynamics always contain a part of the initial functions. Thus the proper definition of initial conditions is critical, which is different from the bounded delay cases. Last but not least, since there is no bound for scaling the integrals relevant to infinite time delays, the proofs for stability or stabilization of the concerned systems are much more complex and difficult in general.

In this paper, we concentrate on the stabilization problem for systems with distributed infinite input delays. Input delays, which are often caused by transmission of the control signal, have been studied widely (Bekiaris-Liberis & Krstic, 2011; Fang & Lin, 2006; Krstic, 2010; Lin & Fang, 2007; Zhou, Gao, Lin, & Duan, 2012; Zhou, Lin, & Duan, 2012). Low gain feedback has proved to be effective to solve the stabilization problem of linear systems with bounded input delays, including constant delays (Lin & Fang, 2007), bounded distributed delays (Zhou, Gao et al., 2012) and bounded time-varying delays (Zhou, Lin et al., 2012). Details of low gain feedback can be found in Lin (1999, 2009). However, the low gain approaches used in the aforementioned literatures cannot be utilized to deal with the case of infinite input delays as the design of low gain parameters in those papers depends on the bounds of delays, which do not exist in the case of infinite delays. By utilizing a different low gain approach, two low gain feedback controllers are proposed in this paper for two different classes of linear systems with distributed infinite input delays respectively, one class with their unstable or marginally stable eigenvalues on the imaginary axis and the other class with their unstable or marginally stable eigenvalues at the origin. It is shown that both classes of linear systems can be globally asymptotically stabilized with the proposed low gain feedback controllers respectively, while the controller for the second class is of a simpler form.

Our main contributions can be summarized into the following four aspects. First, our work, to our best knowledge, is the first attempt at solving the stabilization problem of systems with infinite input delays. Second, it is shown that the low gain approach is also effective to the case of distributed infinite input delays. It should be noted that the techniques used in this work are very different from those in Lin and Fang (2007), Zhou, Gao et al. (2012) and Zhou, Lin, and Duan (2010) though the similar low gain feedback controllers are adopted. In contrary to the aforementioned papers which use the time domain low gain approach, the frequency domain low gain approach is used in our paper. Third, noting that most of the existing stability result cannot be applied to infinite delay cases directly, a new stability result, which can be seen as the extension of the frequency domain method, is utilized to establish our main results. In addition, an important property of low gain feedback is revealed and proved with which our main results can thus be established. Last but not least, our results include some of the existing results on constant delays and bounded distributed delays as their special cases, see for example Lin and Fang (2007), Zhou, Gao et al. (2012) and Zhou et al. (2010).

The rest of this paper is organized as follows. In Section 2, some preliminaries are provided, including the definition of stability and a stability result on systems with infinite delays introduced in Hale (1974). Our main results are presented in Section 3. Simulation examples are presented in Section 4 and conclusions are drawn in Section 5. Some technical details can be found in the Appendix.

Notations: Throughout this paper, the following notations are used. $\mathbb{R}$ and $\mathbb{C}$ are the sets of real and complex numbers, respectively. $\mathbb{R}^n$ and $\mathbb{C}^n$ denote the sets of n-dimensional real and complex vectors, respectively. $\mathbb{R}^{n\times m}$ and $\mathbb{C}^{n\times m}$ denote the sets of $n\times m$ real and complex matrices, respectively. $\mathbb{C}^-, \mathbb{C}^+, \overline{\mathbb{C}}^-, \overline{\mathbb{C}}^+$ represent the open left half, the open right half, the closed left half and the closed right half of complex plane, respectively. $j\mathbb{R}$ represents the imaginary axis. $\|\cdot\|$ denotes any vector or matrix norm. The notation $|\cdot|$ can represent the absolute value for real numbers, the module for complex numbers, the Euclidean norm for vectors or the subordinate 2-norm for matrices. $\sigma(\cdot)$ is the spectrum of a matrix. The notation $det\{\cdot\}$ means the determinant of a matrix. I represents the identity matrix with appropriate dimension.

2. Preliminaries

2.1. Stability definition for systems with infinite delays

In this subsection, we extend the concepts of stability so that systems with infinite delays can be included. Suppose $0 \le r \le +\infty$ is given. If $r = +\infty$, then $[-r, 0] = (-\infty, 0]$. For any $t \ge 0$, let $x_t : [-r, 0] \to \mathbb{R}^n$ be defined by $x_t(\theta) = x(t+\theta), \theta \in [-r, 0]$, $\mathcal{B} = \mathcal{B}([-r, 0], \mathbb{R}^n)$ be the Banach space of functions defined on $[-r, 0]$ equipped with norm $\|\cdot\|_{\mathcal{B}}$ and $x_t \in \mathcal{B}$. Consider the following time delayed system

$$\dot{x}(t) = f(x_t), t \ge 0$$
$$x_0(\theta) = \phi(\theta), \theta \in [-r, 0], \quad (2)$$

where the operator $f : \mathcal{B} \to \mathbb{R}^n$ is continuous and bounded with $f(0) = 0$ and $\phi = x_t|_{t=0} \in \mathcal{B}$ is the initial condition.

Remark 1. Let $\mathcal{C}([-r, 0], \mathbb{R}^n)$ denote the Banach space of continuous functions defined on $[-r, 0]$. One first considers the case that $r < +\infty$, which implies that the delay is bounded. Because the solution $x(t)$ is continuous for all $t \ge 0$, after one delay interval r, there will always be $x_t \in \mathcal{C}([-r, 0], \mathbb{R}^n)$ for $t \ge r$. Though the initial condition $\phi = x_t|_{t=0} \in \mathcal{B}$ could be discontinuous, by choosing a new initial time r and a new initial condition $\phi' = x_t|_{t=r}$, there is always $\phi' \in \mathcal{C}([-r, 0], \mathbb{R}^n)$. Then the new system

$$\dot{x}(t) = f(x_t), t \ge r$$
$$x_r(\theta) = \phi'(\theta), \theta \in [-r, 0], \quad (3)$$

where $x_t \in \mathcal{C}([-r, 0], \mathbb{R}^n)$, is equivalent to system (2). Therefore, the space $\mathcal{B}$ is often chosen, without loss of generality, as $\mathcal{C}([-r, 0], \mathbb{R}^n)$ if $r < +\infty$. However, when $r = +\infty$, which implies the delay is infinite, x_t will always contain a part of the initial condition for any $t > 0$. Hence, we cannot assume that $\mathcal{B} = \mathcal{C}((-\infty, 0], \mathbb{R}^n)$. These discussions show that the choice of the initial condition ϕ and the space $\mathcal{B}$ is critical in the case of infinite delays. As a result of this, new definitions and results are required for discussing stability of systems with infinite delays.

The definition of stability of system (2) is formally given as follows.

Definition 1 (*Kato, 1978*)**.** The zero solution of system (2) is said to be

(1) stable if for any $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$\|\phi\|_{\mathcal{B}} < \delta(\varepsilon)$ implies $|x(t)| < \varepsilon$.

(2) asymptotically stable if in addition to stability, there exist $\delta_0 > 0$ and $T = T(\varepsilon) > 0$ of ε such that

$\|\phi\|_{\mathcal{B}} < \delta_0$ and $t \ge T$ implies $|x(t)| < \varepsilon$.

(3) globally asymptotically stable if in addition to stability, for any $\varepsilon > 0$ and any $\phi \in \mathcal{B}$, there exists $T = T(\varepsilon) > 0$ such that

$t \ge T$ implies $|x(t)| < \varepsilon$.

2.2. A stability result on systems with infinite delays

It should be noted that most of the existing stability results are developed to solve the bounded delay case, i.e., $r < +\infty$ in