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Brief paper

Attitude estimation by multiplicative exogenous Kalman filter[☆]

Bård Nagy Stovner[a,*], Tor Arne Johansen[b], Thor I. Fossen[b], Ingrid Schjølberg[a]

[a] Department of Marine Technology, Norwegian University of Science and Technology, Trondheim, Norway
[b] Center for Autonomous Marine Operations and Systems (NTNU-AMOS), Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway

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ABSTRACT

This paper presents a novel attitude estimator called the multiplicative exogenous Kalman filter. The estimator inherits the stability properties of a nonlinear observer and the near-optimal steady-state performance of the linearized Kalman filter for estimation in nonlinear systems. The multiplicative exogenous Kalman filter is derived in detail, and its error dynamics is shown to be globally exponentially stable, which provides guarantees on robustness and transient performance. It is shown in simulations and experiments to yield similar steady-state performance as the multiplicative extended Kalman filter, which is the workhorse for attitude estimation today. The filter assumes biased angular rate measurements and two or more time-varying vector measurements, and it estimates the attitude represented by the quaternion and the angular rate sensor bias.

1. Introduction

Estimation of the attitude of a rigid body is an essential part of many navigation systems, whether it is in marine, terrestrial, aerial, or extraterrestrial applications. Solutions typically involve comparing nonparallel vector measurements in the rigid body's body-fixed frame to the corresponding known inertial vectors. Examples of this can be found in satellites, which often navigate by tracking known stars, and in marine, terrestrial, and aerial applications, in which body-fixed measurements of Earth's gravitational and magnetic field are commonly used. The principle behind attitude estimation from vector measurements is the relationship $r^i = R(q_b^i)r^b$ where a unit reference vector r is known in two frames, an inertial frame denoted $\{i\}$ and a body-fixed frame denoted $\{b\}$. The two vectors only differ by the rotation $R(q_b^i)$, here parametrized by the quaternion q_b^i. With at least two nonparallel vector pairs (r_1^i, r_1^b) and (r_2^i, r_2^b), the attitude can be determined.

One of the first attitude estimation algorithms was the TRIAD algorithm presented by Black (1964), which finds the rotation matrix explicitly for two nonparallel vector pairs. The main weakness of the TRIAD algorithm is its sensitivity to noise. Bar-Itzhack and Harman (1997) improve on this by calculating a weighted average of several different TRIAD solutions, but the achieved estimate is still not optimal in the sense of minimum variance. Wahba (1965) posed the problem of finding the rotation matrix R_i^b that minimizes the cost function $J = \sum_{j=1}^{N} \|r_j^b - R_i^b r_j^i\|_2^2$ for N measurements. This problem has received great interest and inspired many solutions. For a review and comparison of several of these solutions, see Markley and Mortari (1999).

Attitude estimation often combine vector measurements with angular rate measurements from an angular rate sensor (ARS). These measurements are often corrupted by a biased noise, which requires the estimation of an ARS bias. The workhorse of nonlinear state estimation is the extended Kalman filter (EKF). For attitude estimation, both the additive EKF (AEKF) and the multiplicative EKF (MEKF) have received great attention (Karlgaard & Schaub, 2010; Markley, 2003; Zanetti & Bishop, 2006; Zanetti, Majji, Bishop, & Mortari, 2009). The AEKF expresses the quaternion correction as an addition, which violates the quaternion norm constraint if implemented naively. The MEKF expresses the correction as a quaternion product, thus maintaining the unit norm constraint. With the correct accommodations, however, Shuster (1993) showed that the two methods are identical. In order to gain a unique representation of the attitude error, the four-dimensional multiplicative correction is often mapped to a representation of *minimal degree*, i.e. using three variables to represent the three rotations. Other Kalman filter (KF) solutions to the attitude estimation problem include the unscented KF (UKF) (Crassidis & Markley, 2003), the invariant EKF (IEKF) (Bonnabel, Martin, & Salaun, 2009), and the

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* Corresponding author.
E-mail addresses: bard.b.stovner@ntnu.no (B.N. Stovner), tor.arne.johansen@itk.ntnu.no (T.A. Johansen), thor.fossen@ntnu.no (T.I. Fossen), ingrid.schjolberg@ntnu.no (I. Schjølberg).

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geometric EKF (GEKF) (Andrle & Crassidis, 2015). These have been shown improve upon the performance of the MEKF.

Nonlinear observers (NLOs) have in recent years received increasing attention for the attitude estimation problem. They often come with global or semi-global stability properties that can be verified a priori, which generally lacks for the EKF-based methods. Salcudean (1991) presented an angular velocity observer with global convergence properties. Thienel and Sanner (2003) proposed a nonlinear observer with globally exponentially stable (GES) estimation of attitude and bias, provided constant reference vectors, under a persistency-of-excitation (POE) requirement. Later, this requirement was lifted by e.g. Mahony, Hamel, and Pflimlin (2008) and Batista, Silvestre, and Oliveira (2012). Observers for time-varying reference vectors with semi-global stability results were developed by Hua (2010) and Grip, Fossen, Johansen, and Saberi (2012, 2015). Batista et al. (2012) presented an attitude observer using a single time-varying and persistently nonconstant reference vector.

In Johansen and Fossen (2016a), a new way of using the linearized KF is presented, called the exogenous KF (XKF). This idea has been used in Johansen and Fossen (2016b); Johansen, Fossen, and Goodwin (2016) on the position estimation problem with biased range measurements, in Stovner, Johansen, Fossen, and Schjølberg (2016), Jørgensen, Johansen, and Schjølberg (2016), and Stovner, Johansen, and Schjølberg (2017) for underwater position estimation, and in Stovner and Johansen (2017) for joint position and attitude estimation. The XKF is explained conceptually in Section 2.1.

Scope and contribution

In this paper, a novel GES quaternion-based attitude and ARS bias filter is developed. Using a GES NLO and building on the results of Johansen and Fossen (2016a), the filter is developed and its global stability is proven. Beyond the theoretically guaranteed robustness and transient performance, the MXKF is both in simulations and experiments shown to outperform the NLO and an MEKF with identical tuning; The MXKF is shown to have better steady-state performance than the NLO, while better transient performance than, and at least as good steady-state performance as, the MEKF.

2. Models and preliminaries

2.1. Exogenous Kalman filter

The XKF is in many ways similar to the EKF. The EKF linearizes a nonlinear model about its own estimate of the state of the system, and employs the linear time-varying (LTV) Kalman filter (KF) on the linearized model. When the estimate is close to the true state, the linearization approximates the nonlinear model accurately, which often yields near-optimal performance. This, along with its simplicity, largely explains the success of the EKF, which is proven by its extensive use in various applications. A problem with the EKF is the lack of proven stability in the general case. This is caused by the feedback of the state estimate to the linearization, which may increase rather than decrease the estimation error in the correction step. The XKF remediates this by linearizing the nonlinear model about an exogenous signal, thus replacing the potentially destabilizing feedback with a feedforward from an auxiliary estimator. The exogenous signal should be a globally stable, but possibly suboptimal, state estimate. This cascade gains two desired properties: the global stability of the auxiliary estimator and the near-optimality of the linearized KF w.r.t. noise. The general XKF structure is shown in Fig. 1.

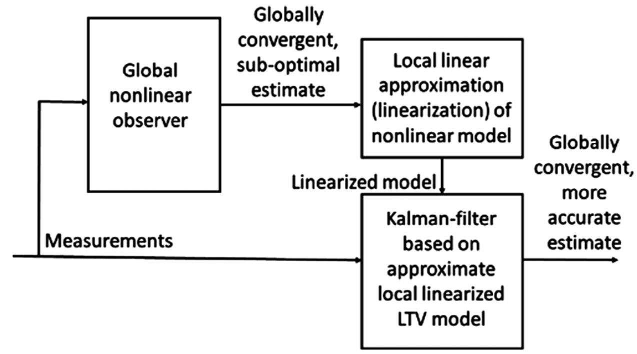

Fig. 1. The general XKF structure (Johansen & Fossen, 2016a).

2.2. State and error representation

The north-east-down (NED) frame, denoted $\{n\}$, is assumed to be an inertial frame. This implies that Earth's rotation is neglected and that the vehicle stays within a limited area such that a local flat Earth approximation is accurate. The vehicle's body-fixed (BF) frame, denoted $\{b\}$, is the frame in which the ARS and vector measurements are gathered. The interest of this paper is to estimate the rotation from the BF to NED frame. This attitude estimate defines a third coordinate frame, the estimated BF (EBF) frame, denoted $\{\hat{b}\}$. Lastly, the MXKF relies on an exogenous signal, which is the state estimate of the NLO. The NLO's attitude estimate defines the exogenous BF (XBF) frame, denoted $\{\bar{b}\}$.

In this paper, the attitude is represented by the unit quaternion $q = [\eta; \varepsilon]$ where η is the scalar part, and $\varepsilon = [\varepsilon_x, \varepsilon_y, \varepsilon_z]^\top$ is the vector part. For column vectors a and b, the ;-notation denotes the vertical concatenation $[a; b] = [a^\top, b^\top]^\top$. For a quaternion q, the corresponding rotation matrix is $R(q) = I + 2\eta S(\varepsilon) + 2S^2(\varepsilon)$ (Fossen, 2011) where $S(x)$ is the skew symmetric matrix

$$S(\varepsilon) = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix} \tag{1}$$

and I is the 3×3 identity matrix. By 0_n and $0_{n\times m}$, we denote $n \times n$ and $n \times m$ matrices of zeros, respectively.

For the coordinate frames denoted a, b, and c, a rotation from c to a can be described by $R(q_c^a) = R(q_b^a)R(q_c^b) = R(q_b^a \otimes q_c^b)$, where $\otimes$ denotes the Hamilton product

$$q_1 \otimes q_2 = \begin{bmatrix} \eta_1\eta_2 - \varepsilon_1^\top \varepsilon_2 \\ \eta_1\varepsilon_2 + \eta_2\varepsilon_1 - S(\varepsilon_1)\varepsilon_2 \end{bmatrix} = q_1\eta_2 + \Psi(q_1)\varepsilon_2 \tag{2}$$

and $\Psi(q_1) = [-\varepsilon_1^\top; \eta_1 I - S(\varepsilon_1)]$ for $q_1 = [\eta_1; \varepsilon_1]$ and $q_2 = [\eta_2; \varepsilon_2]$. The quaternions representing the rotations from the BF, XBF, and EBF frames to the NED frame are denoted q_b^n, $q_{\bar{b}}^n$, and $q_{\hat{b}}^n$, respectively. The true ARS bias is denoted b^b when it is decomposed in the BF frame. When decomposed in their respective frames, the MXKF's and NLO's bias estimates are denoted $\hat{b}^{\hat{b}}$ and $\bar{b}^{\bar{b}}$, respectively.

The true attitude q_b^n and the estimates of it, $q_{\hat{b}}^n$ and $q_{\bar{b}}^n$, can be expressed by their scalar (η) and vector (ε) parts $q_b^n = [\eta; \varepsilon]$, $q_{\hat{b}}^n = [\hat{\eta}; \hat{\varepsilon}]$, and $q_{\bar{b}}^n = [\bar{\eta}; \bar{\varepsilon}]$, respectively. We define the *additive quaternion estimation error* and the bias estimation error of the MXKF as

$$\tilde{q} \triangleq q_b^n - q_{\hat{b}}^n \tag{3a}$$

$$\tilde{b} \triangleq b^b - \hat{b}^{\hat{b}} \tag{3b}$$

respectively. Let the true state and the MXKF's and NLO's state estimates be given by $z = [q_b^n; b^b]$, $\hat{z} = [q_{\hat{b}}^n; \hat{b}^{\hat{b}}]$, and $\bar{z} = [q_{\bar{b}}^n; \bar{b}^{\bar{b}}]$,