Automatica 94 (2018) 45–54

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive control of time-delay cascade systems with unknown parameters by partial state feedback☆

Xu Zhang [a], Wei Lin [b,c,*,**], Yan Lin [a]

[a] School of Automation, Beihang University, Beijing, 100191, China
[b] School of Electrical Engineering, Dongguan University of Technology, Guangdong, China
[c] Department of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH, USA

ARTICLE INFO

Article history:
Received 6 November 2016
Received in revised form 9 December 2017
Accepted 9 March 2018

Keywords:
Time-delay systems
Nonlinear parameterization
Global adaptive regulation
Partial state feedback
Dynamic gains

ABSTRACT

This paper shows that under an ISS-like condition on the zero-dynamics with time-delay, it is possible to achieve global adaptive regulation by dynamic partial state feedback for certain classes of time-delay cascade systems with nonlinear parameterization. The main contribution is the development of a delay-independent, partial state adaptive control scheme that employs a dynamic gain-based method, together with the adaptive feedback domination design and the technique of changing supply rate. By constructing suitable dynamic gain-based Lyapunov–Krasovskii functionals, we prove that all the states of the time-delay cascade system can be regulated to the origin while maintaining boundedness of the closed-loop system.

1. Introduction

In this paper, we consider the following class of uncertain cascade systems with an unknown time-delay

$$
\begin{aligned}
\dot{z} &= f_0(z, z(t-d), x_1, x_1(t-d), \theta) \\
\dot{x}_1 &= x_2 + f_1(z, z(t-d), x_1, x_1(t-d), \theta) \\
&\vdots \\
\dot{x}_{r-1} &= x_r + f_{r-1}(z, z(t-d), x_1, \ldots, x_{r-1}, \\
&\qquad\qquad x_1(t-d), \ldots, x_{r-1}(t-d), \theta) \\
\dot{x}_r &= u + f_r(z, z(t-d), x, x(t-d), \theta), \\
z(s) &= \zeta(s), \quad x(s) = \mu(s), \quad s \in [-d, 0],
\end{aligned}
\tag{1}
$$

where $x = [x_1, \ldots, x_r]^T \in \mathbb{R}^r$ is the measurable state, $z \in \mathbb{R}^{n-r}$ is the unmeasurable state, $u \in \mathbb{R}$ is the control input and $\theta \in \mathbb{R}^s$ is the unknown constant parameter. The constant $d \geq 0$ represents an unknown time-delay of the system, $\zeta(s) \in \mathbb{R}^{n-r}$ and $\mu(s) \in \mathbb{R}^r$ are continuous functions defined on $[-d, 0]$, $f_0 : \mathbb{R}^{n-r} \times \mathbb{R}^{n-r} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^s \to \mathbb{R}^{n-r}$ and $f_i : \mathbb{R}^{n-r} \times \mathbb{R}^{n-r} \times \mathbb{R}^i \times \mathbb{R}^i \times \mathbb{R}^s \to \mathbb{R}$ are C^1 mappings with $f_0(0, 0, 0, 0, \theta) = 0$ and $f_i(0, 0, 0, 0, \theta) = 0$, $\forall \theta \in \mathbb{R}^s$.

When no time-delay exists in the controlled plant (i.e., $d = 0$), global adaptive regulation of the cascade system (1) with unknown parameters has been investigated by partial state feedback; see, for instance, Lin and Pongvuthithum (2003) and the references therein. A globally stabilizing adaptive controller was designed by means of changing supply rate (Sontag & Teel, 1995) and the feedback domination method (Lin & Qian, 2002; Qian & Lin, 2001a). The domination design (Lin & Qian, 2002; Qian & Lin, 2001a) has provided an effective tool for dealing with nonlinear parameterization without requiring convex/concave or linear parameterization conditions.

The goal of this paper is to show that using the dynamic gain-based design (Zhang & Lin, 2014; Zhang, Lin, & Lin, 2017b), together with the feedback domination control method (Lin & Pongvuthithum, 2003; Lin & Qian, 2002), one can find, under a suitable condition on the z-subsystem of (1), a delay-free, dynamic partial-state adaptive controller (using only the measurable state x)

$$
\begin{aligned}
\dot{L} &= \psi_1(L, \hat{\Theta}, x), \quad L \in \mathbb{R}^{r-1}, \\
\dot{\hat{\Theta}} &= \psi_2(L, \hat{\Theta}, x), \quad \hat{\Theta} \in \mathbb{R}, \\
u &= \beta(L, \hat{\Theta}, x),
\end{aligned}
\tag{2}
$$

☆ This work was supported in part by NSFC under Grants 61533009 and 61773033, China Postdoctoral Science Foundation under Grants 2016M600029 and 2017T100025, 111 Project (B08015), Projects JCY20130329152125731, JCY20150403161923519 and KCYKYQD2017005. The material in this paper was partially presented at the 20th World Congress of the International Federation of Automatic Control, July 9–14, 2017, Toulouse, France. This paper was recommended for publication in revised form by Associate Editor Xiaobo Tan under the direction of Editor Miroslav Krstic.

* Corresponding author at: School of Electrical Engineering, Dongguan University of Technology, Guangdong, China.
** Dept. of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH, USA.
E-mail address: linwei@case.edu (W. Lin).

https://doi.org/10.1016/j.automatica.2018.04.007
0005-1098/

globally regulating the time-delay cascade system (1) with unknown parameters. That is, the state (z, x) of the cascade system (1) can be steered to the origin while maintaining boundedness of the closed loop system (1)–(2), where $\psi_1 : \mathbb{R}^{r-1} \times \mathbb{R} \times \mathbb{R}^r \to \mathbb{R}^{r-1}$ is a continuous function and $\psi_2 : \mathbb{R}^{r-1} \times \mathbb{R} \times \mathbb{R}^r \to \mathbb{R}$ and $\beta : \mathbb{R}^{r-1} \times \mathbb{R} \times \mathbb{R}^r \to \mathbb{R}$ are smooth functions, with $\psi_1(0, 0, 0) = 0$, $\psi_2(0, 0, 0) = 0$ and $\beta(0, 0, 0) = 0$.

Time-delay phenomena can be found in a variety of applications and physical systems such as chemical processes, networked control systems, communication-based systems and biological systems. To address control problems of time-delay systems, various analysis and synthesis approaches have been developed in the literature. Among them, the Lyapunov–Krasovskii and Lyapunov–Razumikhin methods are two powerful tools in the stability analysis of time-delay systems (Gu, Kharitonov, & Chen, 2003; Jankovic, 2001; Pepe, 2014; Richard, 2003). There are primarily three types of time-delay systems that have received considerable attention. One class includes the delay in the system state (Hua, Liu, & Guan, 2009; Karafyllis & Jiang, 2010; Nguang, 2000) and the other one contains the delay in the control input (Bresch-Pietri & Krstic, 2014; Krstic, 2010; Mazenc, Mondie, & Niculescu, 2003). Of course, a more complex situation involves time-delays in both states and actuators of controlled plants (Bekiaris-Liberis & Krstic, 2013). In Mazenc et al. (2003), a saturation controller was proposed for global asymptotic stabilization of a chain of integrators with a delay in the input. In Krstic (2010), control of a class of nonlinear systems with input delay was investigated with the condition that the system is forward complete. For a strict feedback system with state delay, an attempt was first made in Nguang (2000) to design a delay-independent, smooth state feedback controller. Later, it was found that the paper (Nguang, 2000) is false due to the use of a circular argument in the static state feedback design. This problem was partially addressed in subsequent work (Hua et al., 2009; Karafyllis & Jiang, 2010), and satisfactorily solved in Zhang and Lin (2014), Zhang, Lin, and Lin (2017a) and Zhang et al. (2017b), using *dynamic* instead of static state feedback.

It should be noticed that the afore-mentioned works only concentrated on the time-delay systems without zero-dynamics (Hua et al., 2009; Karafyllis & Jiang, 2010; Nguang, 2000; Zhang & Lin, 2014; Zhang et al., 2017b) and unknown parameter (Zhang et al., 2017a). The purpose of this paper is to move beyond the time-delay systems with *trivial zero-dynamics* and *uncertainty whose bound is known*, by studying the adaptive control problem for the time-delay cascade system (1) with unknown parameters. In this work we focus our attention on the first category of time delay problems, namely, the class of cascade systems under consideration contains only delays in the state. The main contribution of this paper is to prove that under a suitable condition on the zero-dynamics, global adaptive regulation of the time-delay cascade system (1) with parameter uncertainty is possible by a delay-independent, *dynamic partial state* adaptive controller. A delay-free, dynamic partial state adaptive control law can be designed recursively, by using the dynamic gain-based design approach (Zhang & Lin, 2014; Zhang et al., 2017b), coupled with the adaptive control strategy for nonlinearly parameterized systems (Lin & Pongvuthithum, 2003; Lin & Qian, 2002) and the idea of changing supply rate (Sontag & Teel, 1995). Another worth-mentioning contribution (may find useful in other applications) is the construction of appropriate Lyapunov–Krasovskii functionals involving the dynamic gains, which play an important role in proving the global stability as well as adaptive state regulation of the time-delay cascade system with unknown parameters.

To address the state regulation problem for the time-delay cascade system (1) with unknown parameters by a delay-free, adaptive partial state feedback law, we assume that the class of time-delay cascade systems (1) considered in this paper satisfies the following condition.

Assumption 1.1. There is a C^1 Lyapunov function $U_0(z)$, which is positive definite and proper, such that

$$\frac{\partial U_0}{\partial z} f_0(z, z(t-d), x_1, x_1(t-d), \theta) \\ \le -\|z\|^2 + \alpha(x_1, x_1(t-d), \theta), \tag{3}$$

where $\alpha(x_1, x_1(t-d), \theta)$ is a C^2 non-negative function with $\alpha(0, 0, \theta) = 0\ \forall \theta \in \mathbb{R}^s$.

Assumption 1.1 implies that the z-subsystem is input-to-state-stable (ISS) when treating $(x_1, x_1(t-d))$ as its inputs. When $d = 0$, it reduces to an ISS condition for nonlinear cascade systems, under which the existence of a partial-state feedback law is guaranteed. It also implies that the zero-dynamics is asymptotically stable and hence the minimum-phase property. Thus, Assumption 1.1 can be viewed as a time-delay version of the ISS-like condition for time-delay cascade systems.

With the aid of Assumption 1.1, we shall prove in the next section that global adaptive stabilization or regulation of the time-delay cascade system (1) with parametric uncertainty is solvable by dynamic partial-state feedback. A non-smooth counterpart of the result is then presented in Section 3, for a class of time-delay uncertain systems with strong nonlinearity, which cannot be controlled by smooth feedback due to the uncontrollable unstable nature of the linearized system. Conclusions are given Section 4.

Notations: Throughout this paper, for the technical convenience, let v_d denote the time-delay term $v(t-d)$, for example, $z_d = z(t-d)$ and $x_{id} = x_i(t-d)$. Define $\bar{v}_i = [v_1, \ldots, v_i]^T \in \mathbb{R}^i$ for $i = 1, \ldots, r$. Hence, $\bar{x}_i = [x_1, \ldots, x_i]^T$, $\bar{x}_{id} = [x_{1d}, \ldots, x_{id}]^T$ and $\bar{l}_i = [l_1, \ldots, l_i]^T$.

2. Adaptive regulation by smooth partial-state feedback

To establish the first main result of this paper, we introduce three lemmas to be used in the next two sections.

Lemma 2.1 (*Lin & Qian, 2002*)**.** *Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ be a continuous function. Then, there are smooth scalar functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$ and $d(y) \ge 1$, such that*

$$|f(x, y)| \le a(x) + b(y), \qquad |f(x, y)| \le c(x)d(y). \tag{4}$$

Lemma 2.2 (*Lin & Pongvuthithum, 2003*)**.** *For any C^1 function $f : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$ with $f(0, 0) = 0$, there exist smooth functions $g(x) \ge 1$ and $h(y) \ge 1$, such that*

$$f(x, y)(\|x\| + \|y\|) \le g(x)\|x\| + h(y)\|y\|. \tag{5}$$

Lemma 2.3 (*Lin & Pongvuthithum, 2003*)**.** *Let $\alpha : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^s \to \mathbb{R}$ be a nonnegative C^2 function with $\alpha(0, 0, \theta) = 0,\ \forall \theta \in \mathbb{R}^s$. Then, there exist smooth scalar functions $b_{01}(x_1, \theta) \ge 0$ and $b_{02}(x_{1d}, \theta) \ge 0$, such that*

$$\alpha(x_1, x_{1d}, \theta) \le x_1^2 b_{01}(x_1, \theta) + x_{1d}^2 b_{02}(x_{1d}, \theta). \tag{6}$$

Using Lemmas 2.1–2.3, we prove in this section that under Assumption 1.1, it is possible to design a delay-independent, dynamic partial-state adaptive controller of the form (2), which solves the problem of global adaptive regulation for the nonlinear parameterized cascade system (1) with time-delay. The proof and the construction of a delay-free adaptive control law rely on an effective coupling of the dynamic gain-based control method (Zhang & Lin, 2014; Zhang et al., 2017b), the adaptive feedback domination design (Lin & Qian, 2002) and the idea of changing supply rate for cascade systems (Sontag & Teel, 1995).