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Brief paper

Convergence of gradient observer for rotor position and magnet flux estimation of permanent magnet synchronous motors☆

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ABSTRACT

In Bernard and Praly (2017), we introduced a new sensorless rotor position observer for permanent magnet synchronous motors which does not require the knowledge of the magnet's flux : only electrical measurements and knowledge of the resistance and inductance are needed. In fact, this observer extends the gradient observer from Lee et al. (2010) with the estimation of the magnet's flux. In this paper, we prove its asymptotic stability provided the voltages/intensities (and some of their derivatives) are bounded, and the rotation speed remains away from zero. The proof relies on finding appropriate changes of coordinates allowing the construction of a weak Lyapunov function by backstepping, and the study of its invariant sets.

1. Introduction

1.1. Context

To minimize the cost and increase the reliability of Permanent Magnet Synchronous Motors (PMSM), it is still important to make progress on estimating their state variables, in particular the rotor position and speed, with a minimum of sensors and fast algorithms. To this end, studies have been made for a long time on the so-called "sensorless" control which uses no mechanical variables measurement, only electrical ones. A review of the first used methods was given in Acarnley and Watson (2006), then a Luenberger observer was proposed in Poulain, Praly, and Ortega (2008). More recently, a very simple gradient observer, proposed in Lee et al. (2010) and analyzed in Ortega, Praly, Astolfi, Lee, and Nam (2011), has been shown to be extremely effective in practice as rotor position estimator. From the theoretical view point it is only conditionally convergent but it was shown in Malaizé, Praly, and Henwood (2012) how, via a very minor modification, it can be made globally convergent thanks to convexity properties.

These observers typically require the knowledge of the resistance, magnet flux and inductance. Unfortunately while the latter may be considered as known and constant (as long as there is no magnetic saturation), the other two do vary significantly with the temperature and these variations should be taken into account in the observer. For example, for a given injected current, when the magnet's temperature increases, its magnetic flux decreases, and the produced torque becomes smaller. Therefore, an online estimation of the magnet's flux would enable to adapt the control law in real time and thus ensure a torque control which is robust to the machine's temperature, and also have an estimation of the rotor's temperature and magnet's magnetization degradation with time.

That is why efforts have been made to look for observers which do not rely on the knowledge of those parameters. The case where the magnet flux is unknown but resistance and inductance are known is addressed in Henwood, Malaizé, and Praly (2012) with the design of a Luenberger observer (see Henwood, 2014 for a much more detailed analysis), and in Bobtsov, Bazylev, Pyrkin, Aranovsky, and Ortega (2016), Bobtsov, Pyrkin, and Ortega (2015) and Bobtsov, Stankovic et al. (2015), with the design of an observer based on tools from parameter linear identification. In Bernard and Praly (2017), we proposed, for the same case, another observer which is a direct extension, with estimation of the magnet flux, of the gradient observer obtained in Lee et al. (2010). We claimed its convergence, and compared it to the other aforementioned observers in terms of sensitivity to errors in the parameters and to the presence of saliency. In particular we have shown that, when the currents in the rotating frame and the rotation speed are constant, an error in the values of the resistance and the inductance induces a bias on the estimated flux and rotor position that we have quantified. We have also reported on the performances achieved in open-loop via simulations using real data. In this paper, as

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a complement of Bernard and Praly (2017), we concentrate our attention only on the proof, not provided in Bernard and Praly (2017), of convergence of the new observer in ideal conditions.

1.2. System model and problem statement

Using Joule's and Faraday's laws, a simple PMSM model expressed in a fixed $\alpha\beta$-frame reads

$$\dot{\Psi} = u - R\,i \tag{1}$$

where Ψ is the total flux generated by the windings and the permanent magnet, (u, i) are the voltage and intensity of the current in the fixed frame and R the stator winding resistance. The quantities u, i and Ψ are two dimensional vectors, and, for the case of a non-salient PMSM, the total flux may be expressed as

$$\Psi = Li + \Phi \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{2}$$

where L is the inductance, Φ the magnet's flux, and θ the electrical phase. This relation implies

$$|\Psi - Li|^2 - \Phi^2 = 0 \tag{3}$$

and the electrical phase θ is nothing but the argument of $\Psi - Li$. It follows that, in the case where L and i are known, θ can be recovered simply through an estimate of the total flux Ψ.

Therefore, our interest in this work is about observers of Ψ using measurements of u and i, knowledge of R and L but not of Φ. In fact, we go further and look for observers for the augmented system

$$\begin{cases} \dot{\Psi} = u - Ri \\ \dot{\Phi} = 0 \\ y = |\Psi - Li|^2 - \Phi^2 \end{cases} \tag{4}$$

with inputs (u, i), known parameters (R, L), state (Ψ, Φ) and output y which is known to be constantly zero according to (3).

Notations: The rotation matrix of angle θ is denoted $\mathcal{R}(\theta)$, i-e

$$\mathcal{R}(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}.$$

2. Gradient observer

2.1. Main result

Originally, in Lee et al. (2010), the authors proposed the gradient observer

$$\dot{\hat{\Psi}} = u - Ri - 2q(\hat{\Psi} - Li)\left(\left|\hat{\Psi} - Li\right|^2 - \Phi^2\right) \tag{5}$$

for System (1), with q some strictly positive real number. This observer turned out to be quite efficient in practice but it was proved in Ortega et al. (2011) that it was only conditionally convergent. In particular it may admit several equilibrium points depending on the rotation speed ω. Later in Malaizé et al. (2012), it was shown that taking rather the following "convexified" gradient observer

$$\dot{\hat{\Psi}} = u - R\,i - 2q(\hat{\Psi} - Li)\max\left(\left|\hat{\Psi} - Li\right|^2 - \Phi^2,\, 0\right) \tag{6}$$

enables to achieve global asymptotic stability.

But in Bernard and Praly (2017), we proposed rather to extend directly the gradient observer (5) with the estimation of Φ, namely

$$\begin{cases} \dot{\hat{\Psi}} = u - R\,i - 2q(\hat{\Psi} - Li)\left(\left|\hat{\Psi} - Li\right|^2 - \hat{\Phi}^2\right) \\ \dot{\hat{\Phi}} = q\,\hat{\Phi}\left(\left|\hat{\Psi} - Li\right|^2 - \hat{\Phi}^2\right) \end{cases} \tag{7}$$

where q is an arbitrary strictly positive real number. We claimed that, without any convexification, this system is an asymptotically stable observer for System (4) as soon as the input signals (u, i) (and their derivatives) are bounded, and the rotor rotation speed is lower-bounded away from zero. More precisely:

Theorem 1. *Consider (ψ, Φ) in $\mathbb{R}^2 \times (0, +\infty)$ and inputs $u, i : \mathbb{R} \to \mathbb{R}^2$ such that there exist strictly positive numbers $\overline{\omega}_1$, $\overline{\omega}_0$, and $\underline{\omega}_0$ such that the solution $(\Psi(\psi; t; u, i), \Phi)$ of (4) verifies*

$$0 < \underline{\omega}_0 \le \dot{\theta}(t) \le \overline{\omega}_0 \quad , \quad \ddot{\theta}(t) \le \overline{\omega}_1 \tag{8}$$

with

$$\theta(t) = \arg(\Psi(\psi; t; u, i) - Li(t)).$$

Then, this solution $(\Psi(\psi; t; u, i), \Phi)$ of (4) is an asymptotically stable solution of (7) with basin of attraction containing the forward invariant set $\Omega = \mathbb{R}^2 \times (0, +\infty)$.

The goal of this paper is to provide the proof of this result. The observer has a very simple expression and is cheap in terms of computing time. But as in Ortega et al. (2011), its convergence analysis has some tricky points. First, we do a change of coordinates to transform the problem of asymptotic stability of a solution into one of an equilibrium. A second transformation allows us to obtain a feedback form to which backstepping tools can be applied to obtain a (weak) Lyapunov function. This enables to establish stability, boundedness and convergence of some quantities. All these steps are very standard. But to conclude, we need a finer and ad hoc analysis of the ω-limit points.

Remark 1. According to (1) and (2), the assumption of boundedness in time of $\omega = \dot{\theta}$ and $\dot{\omega} = \ddot{\theta}$ is achieved as soon as the signals $(u, i, \dot{u}, \widehat{\dot{i}}, \widehat{\ddot{i}})$ are bounded. The values of the bounds do not matter, as long as they exist. But they do have an effect on the behavior and in particular on the convergence speed and the magnitude of the solutions.

Remark 2. The fact that the rotation speed ω should stay away from zero is quite standard and related to the observability of the system. In Bobtsov et al. (2016), Bobtsov et al. (2015) and Bobtsov, Stankovic et al. (2015), this assumption appears through the persistent excitation condition, and in Henwood et al. (2012), it is a condition for the invertibility of the Luenberger transformation.

2.2. Change of coordinates

Consider any solution (Ψ, Φ) of (4) with Φ in $(0, \infty)$ and define

$$\theta(t) = \arg(\Psi(t) - Li(t)),$$

as in Theorem 1. Since we know that $y(t) = 0$ for all t, we have

$$\Psi(t) = L\,i(t) + \Phi\begin{pmatrix}\cos\theta(t) \\ \sin\theta(t)\end{pmatrix}. \tag{9}$$

To simplify the analysis, we transform the solution $(\Psi, \Phi) = \left(L\,i + \Phi\left(\begin{smallmatrix}\cos\theta \\ \sin\theta\end{smallmatrix}\right), \Phi\right)$ into an equilibrium. To that end, we consider the coordinates

$$\begin{pmatrix}X_d \\ X_q\end{pmatrix} = \mathcal{R}(-\theta)\,(\Psi - Li), \quad \begin{pmatrix}\hat{X}_d \\ \hat{X}_q\end{pmatrix} = \mathcal{R}(-\theta)\left(\hat{\Psi} - Li\right),$$

i.e the solution (Ψ, Φ) is transformed into the constant point $(\Phi, 0, \Phi)$. In those coordinates, the dynamics of the observer (7)