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Test for additive interaction in proportional hazard model applied to Pitot sensors reliability and survivability

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Abstract: Cox proportional hazards model is one of the most common methods used in analysis of time to event data. Estimation of proportional hazards models when covariates are time independent increase the power of significance of the model. Techniques to release this assumption of proportionality allow researchers to test whether the effects of covariates change over time and also permit a more nuanced understanding of the phenomenon being studied. The current paper presents an extension for the Cox model in case of non-proportional risk. This new model will be applied in the Pitot Sensor's reliability function to study the covariates interaction with time. A simulation study is used to illustrate the model and to show its new performances. This paper focuses in the determination of the Pitot sensor reliability and survivability Function. However, the same process can be generalized to study the reliability function of other standby systems.

Keywords: Pitot sensor, hazard model, interaction, time dependant, estimation

1. INTRODUCTION

Air France Flight 447 (AF447/AFR447) was a scheduled passenger flight from Rio de Janeiro, Brazil to Paris, France, which crashed on 1 June 2009. The Airbus A330, operated by Air France, entered aerodynamic from which it did not recover and crashed into the Atlantic Ocean , killing all 228 passengers, aircrew and cabin crew aboard the aircraft The BEA's Bureau d'Enqutes et d'Analyses (BEA) pour la Scurit de l'Aviation civile final report, concluded that the aircraft crashed after temporary inconsistencies between the airspeed measurements caused the autopilot to disconnect, after which the crew reacted incorrectly and ultimately led the aircraft to an aerodynamic stall from which they did not recover. To avoid such situations and accidents one should define the reliability funstion of the Pitot sensors to increase the accuracy its measurements.

Survival analysis is a class of statistical methods which characteristically focuses on time to event data. In the most general sense, it presents techniques for positive valued random variable.

In literature, many proposals can be found for modelling the reliability and the aging of equipment (Vesely and Wolford (1988)) and (Samanta et al. (1991)). In some cases, the reliability model considers only the effect of the operational conditions. Instead, others consider only the effect of maintenance action on the equipment. The underlying cause of most common failures involves not only operational and environmental causes, but also interactions between the latter two. The Cox proportional-hazards model is widely used in addition to introducing above operational and environmental factors into the components reliability model.

David Cox (Wiley Cox (1972), Cox (1975)) introduces a stimulation procedure for the regression analysis of censored failure time data. This procedure was widespread use in the analysis of time-to-event data. The author specified a semi parametric model for the instantaneous failure rate, or hazard function. The model specifies a parametric regression function, of exponential form, for the ratio of failure rates between study subjects, while allowing an arbitrary baseline function. If the choice of the baseline function is arbitrary, the cox model is sufficiently flexible for many applications. In addition to this flexibility, Cox's class of models becomes very general upon release the relative risk to be a function of time. Such a release is typically impractical for failure time models outside the Cox model family.

Regression modelling time to event data was the interest of many studies. Many proposals can be found in literature for studying extensions of the Cox model for proportional and non-proportional purposes. Kalbfleisch and Prentice (2011) considers a generalization for the construction of the likelihood function to introduce time dependent covariates. The author illustrates numerical examples and some comments on the released model efficiency.

Hosmer and Lemeshow (2000) introduces methods for examining and testing the proportional hazards assumption, evaluating subject-specific diagnostic statistics that measure leverage and influence on the fit of the proportional hazards model and computing summary measures of goodness-of-fit. Li and Chambless (2007) describes a method for testing and estimating a two-way additive in-

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teraction between two categorical variables, each of which has greater than or equal to two levels. Borucka (2014) proposes different ways of Cox model modification in case of non-proportional hazards introducing interactions of selected covariates with function of time and stratification model.

In this paper, we have described methods to test for an additive interaction with time to estimate the parameters of the Pitot sensor reliability and survivability function using Cox Proportional Hazard model. Our goal is to outline and demonstrate a critical methodological refinement that improves the application of event analysis to the study of the Pitot sensors reliability function. In this paper, we illustrate how to release the Proportional Hazards assumption. We provide a detailed description of the Proportional Hazards assumption in event analysis and the use of what refer to non-proportional analysis. We then demonstrate how the extension of proportional hazard model can be applied to the description of the Pitot sensor reliability and survivability function. Specifically, we reanalyse data from previously work studying the same application but only with time independent covariates.

2. BASIC CONCEPT OF PROPORTIONAL HAZARD MODEL

Several survival distributions are used to model the survival experience of homogeneous data. However, there are explanatory variables upon which failure time may depend. Therefore, it becomes interesting to consider generalizations of these models to take into account the concomitant information on the individuals sampled.

The no-need to parameterize time dependency is a significant advantage in most scientific applications.

Let $\lambda(t, X)$ represent the hazard function at time t for an individual with covariates. The proportional model specifies that:

$$\lambda(t, X) = \lambda_0(t) * exp(\beta X) \tag{1}$$

Where X denotes a row vector of n measured covariates, is a column vector of n regression parameters and t is the associated failure time.

These models are typically estimated via a quasi or partial likelihood procedure, in which the term for the baseline hazard is considered as a nuisance parameter and integrated out of the likelihood.

The Cox model is based on several restrictive assumptions which the most restrictive is the proportional hazards assumption.

The proportional hazards assumption is necessary to the interpretation and use of a fitted hazards model. There are several methods to verify the proportional hazard assumption. For example, we can consider graphical method which is based on the plot of the log hazard function (Kalbfleisch and Prentice (2011)). The author considers that the proportional hazards model has a log-hazard function of the form:

$$\ln(\lambda(t, X)) = \ln(\lambda_0(t)) + \beta X \tag{2}$$

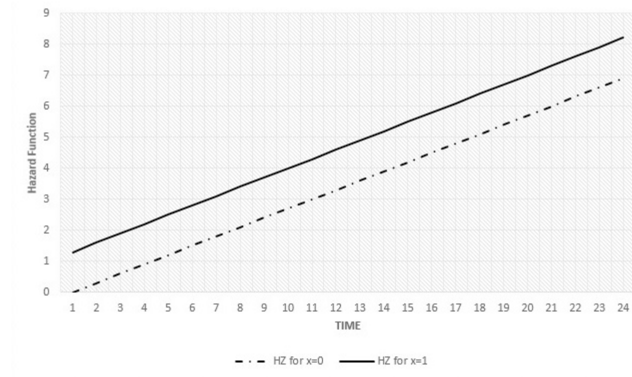

Fig. 1. Hazard Function Plot

This function has two parts, the log of the baseline hazard function, $\ln(\lambda_0(t))$, and the linear predictor, βX. Assume for the moment that the model contains a single dichotomous covariate.

The plot of the log-hazard function over time would produce two continuous curves, one for $x = 0$, $\ln(\lambda_0(t))$ and the other for $x = 1$, $\ln(\lambda_0(t)) + \beta$.

The figure 1 illustrates the plot of the hazard function with $\lambda_0(t) = exp(-0.3t)$ and $\beta = 0.4$:

Figure 1 shows that the difference between these curves at any point in time is β, regardless of the baseline hazards function (simple or complicated). In this case, the estimated hazard ratio HR is expressed by:

$$\mathrm{HR} = \frac{\lambda(t, X_1)}{\lambda(t, X_2)} \tag{3}$$

$$= \frac{\lambda_0(t) * exp(\beta X_1)}{\lambda_0(t) * exp(\beta X_2)} \tag{4}$$

$$= exp[\beta(X_1 - X_2)] \tag{5}$$

Where:

- X_1: vector of covariates of subject I
- X_2: vector of covariates of subject II

In this proportional hazards function, the hazard ratio is constant over time which means that the effect of the given covariate on the hazard level is the same at all time points.

3. INTERACTION WITH TIME

Estimating proportional hazards models for covariates whose hazard ratios are non-constant over time results in biased coefficient estimates and decreased power of significance tests. The power of corresponding tests decreases because of sub-optimal weights for combining the information provided by the risk sets of times where failures occur. In addition, the extension of proportional hazards model is recommended to increase model performances.

The first method uses interactions with time for covariates for which assumption is not satisfied. This method is in fact both the way to identify such covariates in the model and solution of the problem at the same time (Borucka (2014)).