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Brief paper

Useful redundancy in parameter and time delay estimation for continuous-time models ☆

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ABSTRACT

In this paper we propose an algorithm to estimate the parameters, including time delay, of continuous time systems based on instrumental variable identification methods. To overcome the multiple local minima of the cost function associated with the estimation of a time delay system, we utilize the useful redundancy technique. Specifically, the cost function is filtered through a set of low-pass filters to improve convexity with the useful redundancy technique exploited to achieve convergence to the global minimum of the optimization problem. Numerical examples are presented to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

The goal of system identification is to estimate the parameters of a model in order to analyse, simulate and/or control a system. Recently, continuous-time (CT) identification methods have received attention due to advantages such as providing insights to the physical system and being independent of the sampling time (Garnier, 2015; Garnier & Young, 2013; Unbehauen & Rao, 1990; Young, 1981). For example, with irregular sample time in CT identification, the system is time-invariant. In reality, irregular sampling occurs in many cases, e.g. when the sampling is event-triggered, when the measurement is manual or in the case of missing data (Åström & Bernhardsson, 2003).

In some CT identification problems, one needs to estimate the system parameters and any unknown time delay. Many practical examples exist including, chemical processes, economic systems and biological systems, that possess time delays. It is important to estimate the delay accurately since a poor estimate can result in unsatisfactory model order selection and inaccurate estimates of the system parameters. There are many approaches to estimate a system time delay (Björklund, 2003). A simple approach is to consider the impulse response data, e.g. estimate the time delay by finding where the impulse response becomes nonzero (Carlemalm, Halvarsson, Wigren, & Wahlberg, 1999) or by noting the delay where the correlation between input and output is maximum (Carlemalm et al., 1999; Carter, 1987). Another approach is to model the delay by a rational polynomial transfer function using a Padé or similar approximation and then estimate the time delay as part of the system parameters (Agarwal & Canudas, 1987; Ahmed, Huang, & Shah, 2006; Gawthrop & Nihtil, 1985). In Baysse, Carrillo, and Habbadi (2011, 2012) and Yang, Iemura, Kanae, and Wada (2007), the time delay and system parameters of a Multiple Input Single Output CT system are estimated in a separable way using an iterative global nonlinear least-squares or instrumental variable method.

Recently, a method of estimating the parameters and time delay of CT systems has been suggested in Chen, Garnier, and Gilson (2015) which is based on a gradient technique. The parameters and the time delay are estimated separately, i.e. when one is estimated, the other is fixed which is then repeated in an iterative manner. In this approach, the Simplified Refined Instrumental Variable (SRIVC) method is used to estimate the parameters whilst the time delay is estimated using the Gauss–Newton method. In addition, due to the effects of multiple minima in the cost function to be minimized for the time delay (Björklund, Nihtilä, & Söderström, 1991; Kaminskas, 1979; Pupeikis, 1989), a low-pass filter is employed to increase convexity. As shown in Björklund et al. (1991), Eckhard, Bazanella, Rojas, and Hjalmarsson (2017) and Ferretti, Maffezzoni, and Scattolini (1996), a suitable low-pass filtering operation on the estimation data can help to extend the global convergence region of the cost function, hence improve the accuracy of the time delay estimate.

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In this paper, we adopt the idea of using a low-pass filter to increase convexity, where instead of using only one filter, we suggest to use multiple low-pass filters and incorporate the useful redundancy technique (Alamir, 2008; Alamir, Welsh, & Goodwin, 2009). Useful redundancy is a technique to avoid local minima when solving a nonlinear inverse problem. The concept is to generate a family of cost functions that have different local minima, but share the same global minimum with the original cost function of the optimization problem. Whenever the algorithm is trapped in a minimum, the solver path is switched to another solver path in a way such that the minimum found using the new solver path corresponds to a decrease in the original cost function. This allows the algorithm to cross local minima and converge to the global minimum, hence improving the accuracy of the estimated parameters. In this paper, multiple cost functions are generated by filtering the original time delay cost function through a number of low-pass filters with different cut-off frequencies that span the system bandwidth. Note that we also published another algorithm to estimate time delay and system parameters that utilizes the multiple filtering idea (Ha & Welsh, 2016). However, the proposed algorithm in Ha and Welsh (2016) requires some priori knowledge of the initial time delay, which we overcome in this paper, by implementing the useful redundancy technique.

The paper is organized as follows. Section 2 describes the model setting and formulates the problem. Section 3 introduces the useful redundancy technique and Section 4 describes the proposed method and provides analysis on its effectiveness. Section 5 presents numerical results for both regular and irregular sampling schemes. Finally, the conclusion will be drawn in Section 6.

2. Model setting and problem formulation

Consider a continuous-time linear, time invariant, single input single output system,

$$\begin{aligned} x(t) &= G_0(p)u(t-\tau_0) = \frac{B(p)}{A(p)}u(t-\tau_0), \\ y(t) &= x(t) + e(t). \end{aligned} \tag{1}$$

with

$$B(p) = b_0p^m + b_1p^{m-1} + \cdots + b_m,$$
$$A(p) = p^n + a_1p^{n-1} + \cdots + a_n, \quad n \geq m,$$

where τ_0 is the time delay, $u(t)$, $x(t)$ are the input and deterministic output of the system respectively and p is the differential operator, i.e. $p^{(i)}x(t) = d^ix(t)/dt^i$. In addition, we make the following assumptions:

Assumption 2.1. Polynomials $B(p)$ and $A(p)$ are coprime.

Assumption 2.2. The system is asymptotically stable.

Assumption 2.3. The high frequency gain of $G_0(p)$ is 0, i.e. $G_0(p)$ is strictly proper.

Assumption 2.4. $e(t)$ is a white random process uncorrelated with $u(t)$ having intensity λ.

Furthermore, we consider the sampling time of the input, $u(t_k)$, and output, $y(t_k)$, data as either regular or irregular. The time-varying sampling interval is denoted as,

$$h_k = t_{k+1} - t_k, \qquad k = 1, 2, \ldots, N-1, \tag{2}$$

where N is the length of the data. The objective of a CT system identification problem is to estimate the time delay, τ_0, and the parameters $a_1, a_2, \ldots, a_n, b_0, b_1, \ldots, b_m$ of the CT model in (1), using the measured input $u(t_k)$ and output data $y(t_k)$, $k = 1, \ldots, N$.

There are many algorithms suggested to estimate CT system parameters, with one of the most common being the SRIVC method (Garnier & Wang, 2008; Garnier & Young, 2013; Young, 1981). Based on SRIVC, a technique (Chen et al., 2015) was developed to estimate both the time delay and parameters for the model (1). This technique considers the problem as a separable nonlinear least squares problem, i.e. the SRIVC algorithm is utilized to estimate the system parameters and the Gauss–Newton method is used to estimate the time delay. In this problem, the cost function for the time delay estimation has multiple minima (Chen et al., 2015), hence a low-pass filter is utilized to extend the global convergence region (Chen et al., 2015; Eckhard et al., 2017; Ferretti et al., 1996). A description of the SRIVC-based time delay estimation with filtering can be found in Chen et al. (2015). However, the problem of local minima still exists even with the use of a filter.

In this paper, we propose a new approach to avoid the local minima in the time delay estimation problem. From this, a new algorithm to estimate CT system parameters and time delay with a high accuracy is obtained.

3. The useful redundancy method

In this section we describe the useful redundancy technique (Alamir, 2008; Alamir et al., 2009) that we utilize to develop our proposed algorithm. The technique was originally proposed to avoid local minima when solving a non-linear inverse problem. We define the useful redundancy technique by quoting the definition directly from Alamir et al. (2009).

Definition 1. Consider an optimization problem,

$$\min_{\rho} J_0(\rho).$$

Then it is called *M-safely redundant* if and only if the following conditions hold:

(1) There exists a finite M cost functions J_i sharing the same global minimum $\rho^* \in \mathbb{R}^n$.
(2) There exists a solver (or an iterative scheme) ℓ and a finite number of iterations $r^* \in \mathbb{N}$ such that for some $\gamma \in [0, 1)$ and all $\rho \in \mathbb{R}$ the following inequality holds,

$$\Delta_N^{\gamma}(\rho) = \min_{i\in\{0,\ldots,M\}} [J_0(\ell^{(r^*)}(\rho, J_i)) - \gamma J_0(\rho)] \leq 0 \tag{3}$$

where $\ell^{(r^*)}(\rho, J_i)$ is the candidate solution obtained after r^* iterations of ℓ using the cost function J_i, starting from the initial guess ρ. ■

The solver path (ρ, J_i) is defined as the sequence of iterates $\ell^{(j)}(\rho, J_i)$ for the solver ℓ when the cost function J_i starts from an initial guess ρ. Condition 2 means that for any initial ρ, there always exists a solver path ℓ that corresponds to a decrease in the original cost J_0 after at most r^* iterations. It is proven in Alamir et al. (2009) that if an optimization problem is *M-safely redundant*, then it is possible to define an iterative algorithm such that convergence to the global minimum is guaranteed.

4. The useful redundancy SRIVC time delay estimation

To construct an *M-safely redundant* optimization problem for the estimation of the time delay, we need a cost function J_0 and multiple solver paths that satisfy the conditions in Definition 1. Here, the solver paths are generated by filtering the time delay estimation error using a set of low-pass filters having different cut-off frequencies. The cost function, J_0, is formulated from these