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Brief paper

Minimal inputs/outputs for subsystems in a networked system ☆

Tong Zhou

Department of Automation, Tsinghua University, Beijing, 100084, China

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ABSTRACT

Minimal input/output selection is investigated in this paper for each subsystem of a networked system. Some novel sufficient conditions are derived, respectively, for the controllability and observability of a networked system, as well as some necessary conditions. These conditions only depend separately on parameters of each subsystem and its in/out-degrees. It is proven that in order to be able to construct a controllable/observable networked system, it is necessary and sufficient that each subsystem is controllable/observable. In addition, both sparse and dense subsystem connections are helpful in making the whole system controllable/observable. An explicit formula is given for the smallest number of inputs/outputs for each subsystem required to guarantee controllability/observability of the whole system.

1. Introduction

With the increment of the dimension of a system, which is mainly due to technology developments in sensors, communications, etc., as well as more complicated and demanding tasks expected for a system, computation costs and numerical stability emerge as essential issues in system analysis and synthesis (Siljak, 1978; Van Schuppen et al., 2011; Zhou, 2015; Zhou & Zhang, 2016). It is now widely recognized that with the increment of its subsystem number, direct applications of results about a lumped system to a networked system may often result in an exponential increment of computation time and storage requirements (Hendrickx, Olshevsky, & Tsitsiklis, 2011; Liu, Slotine, & Barabasi, 2011; Siljak, 1978; Van Schuppen et al., 2011; Zhou, 2015). To make things worse, these direct applications are usually numerically unreliable. Well known examples include the computation of the eigenvalues and/or eigenvectors for a square matrix, which is often required in analyzing system properties and designing a controller (Horn & Johnson, 1991; Zhou, 2015; Zhou, Doyle, & Glover, 1996). To overcome these difficulties, various efforts have emerged recently for the analysis and synthesis of a networked system. Among which, an extensively studied problem is about its controllability/observability verifications, and construction of a controllable/observable networked system (Commault & Dion, 2013; Liu et al., 2011; Olshevsky, 2014; Pequito, Kar, & Aguiar, 2016; Rahimian & Aghdam, 2013; Summers, Cortesi, & Lygeros, 2016; Tzoumas, Rahimian, Pappas, & Jadbabaie, 2016; Zhou, 2015, 2016).

Many results have now been obtained for this important theoretical issue in systems and control. For example, robustness of structural controllability, input addition, decentralized controllability, etc., have been investigated, respectively, in Commault and Dion (2013) and Rahimian and Aghdam (2013). In Pasqualetti, Zampieri, and Bullo (2014), clustered networks are found easier to be controlled. It is declared in Olshevsky (2014) that finding the sparest input/output matrix such that a networked system is controllable/observable is NP-hard, and some algorithms are suggested in Pequito et al. (2016) and Summers et al. (2016) to approximately solve this minimal controllability/observability problem. A minimal actuator placement problem is also proven in Tzoumas et al. (2016) to be NP-hard, and a best approximation is suggested which has a polynomial computational complexity. Structural controllability and the cavity method are used in Liu et al. (2011) to derive a set of driver nodes for assuring system controllability. In Zhou (2015), we have obtained a necessary and sufficient condition for an arbitrarily connected networked system to be controllable/observable, which depends separately on parameters of each subsystem. These results have been extended to various situations, such as the full column normal rank (FCNR) condition adopted in Zhou (2015) is not satisfied, there are constraints on system inputs and states, etc. (Zhang and Zhou, 2015, 2017; Zhou, 2016). It has been discovered in Simon and Mitter (1968), Yuan, Zhao, Du, Weng, and Lai (2013) and Zhou (2017) that, when the state transition matrix (STM) of a networked system is given, in order to guarantee its controllability/observability, the minimal

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E-mail address: tzhou@mail.tsinghua.edu.cn.

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number of its inputs/outputs is equal to the maximum geometric multiplicity of that matrix.

In actual engineering, however, it is generally preferable to have inputs/outputs directly and separately affecting/measuring the states of each individual subsystem and/or their functions (Pequito et al., 2016; Siljak, 1978; Summers et al., 2016; Van de Wal & De Jager, 2001; Zhou, 2015). Under this restriction, it is still not clear how many inputs/outputs are required for each subsystem to make the whole system controllable/observable. To emphasize this characteristic, the associated problem is called in this paper local input/output selections.

To settle this problem, we at first investigate relations among subsystem observability/controllability, subsystem out/in-degree and system observability/controllability. It has been made clear that in order to guarantee the observability/controllability of a networked system, each of its subsystems must be observable/controllable. A sufficient condition is derived for system observability which depends separately only on parameters of each subsystem and its out-degree. This condition reveals that both sparse and dense subsystem connections are helpful to make the whole system observable/controllable. Based on these results, it is further proven that a necessary and sufficient condition for being able to build an observable/controllable networked system is that, each subsystem is observable/controllable. It has also been proven that in order to guarantee system controllability/observability, the number of inputs/outputs in each subsystem must at least be equal to the maximum geometric multiplicity of its state transition matrix.

The outline of this paper is as follows. At first, Section 2 gives a precise problem formulation and some preliminary results. Relations between controllability/observability of a networked dynamic system and subsystem out/in-degree are investigated in Section 3. The minimal input/output problem is discussed in Section 4. Finally, some concluding remarks are given in Section 5. An Appendix is included to give proofs of some technical results. Some numerical examples are provided to illustrate the obtained theoretical conclusions.

The following notation and symbols are adopted. $\mathcal{R}^{m\times n}$ and $\mathcal{C}^{m\times n}$ are utilized, respectively, to represent the sets of $m \times n$ dimensional real and complex matrices. When m and/or n are equal to 1, they are usually omitted. $\mathbf{diag}\{X_i|_{i=1}^{L}\}$ denotes a block diagonal matrix with its ith diagonal block being X_i, while $\mathbf{col}\{X_i|_{i=1}^{L}\}$ the vector/matrix stacked by $X_i|_{i=1}^{L}$ with its ith row block vector/matrix being X_i. 0_m and $0_{m\times n}$ represent, respectively, the m dimensional zero column vector and the $m \times n$ dimensional zero matrix. The superscripts T and H stand, respectively, for the transpose and the conjugate transpose of a matrix/vector, while $\|\cdot\|_2$ the Euclidean norm of a vector.

2. Problem formulation and some preliminaries

Consider the networked system $\mathbf{\Sigma}$ adopted in Zhang and Zhou (2015, 2017) and Zhou (2015, 2016), which consists of N linear time invariant (LTI) dynamic subsystems. In this system, the dynamics of its ith subsystem $\mathbf{\Sigma}_i$ is described by

$$\begin{bmatrix} x(t+1,i) \\ z(t,i) \\ y(t,i) \end{bmatrix} = \begin{bmatrix} A_{\mathbf{TT}}(i) & A_{\mathbf{TS}}(i) & B_{\mathbf{T}}(i) \\ A_{\mathbf{ST}}(i) & A_{\mathbf{SS}}(i) & B_{\mathbf{S}}(i) \\ C_{\mathbf{T}}(i) & C_{\mathbf{S}}(i) & D(i) \end{bmatrix} \begin{bmatrix} x(t,i) \\ v(t,i) \\ u(t,i) \end{bmatrix} \tag{1}$$

and interactions among its subsystems are described by

$$v(t) = \Phi z(t) \tag{2}$$

Here, $z(t) = \mathbf{col}\left\{z(t,i)|_{i=1}^{N}\right\}$ and $v(t) = \mathbf{col}\left\{v(t,i)|_{i=1}^{N}\right\}$. Moreover, t and i stand, respectively, for the temporal variable and the index number of a subsystem, $x(t,i)$ represents the state vector of the ith subsystem $\mathbf{\Sigma}_i$ at time t, $z(t,i)$ and $v(t,i)$ represent, respectively, its outputs affecting other subsystems and inputs denoting influences from other subsystems, $y(t,i)$ and $u(t,i)$ represent, respectively, its output and input vectors. Similar to Zhou (2015, 2016), in order to distinguish them, $z(t,i)$ and $v(t,i)$ are called internal output/input vectors, while $y(t,i)$ and $u(t,i)$ are called external output/input vectors.

The following three assumptions are adopted in this paper.

A.1: the dimensions of the vectors $x(t,i)$, $v(t,i)$, $u(t,i)$, $z(t,i)$ and $y(t,i)$, are, respectively, $m_{\mathbf{x}i}$, $m_{\mathbf{v}i}$, $m_{\mathbf{u}i}$, $m_{\mathbf{z}i}$ and $m_{\mathbf{y}i}$. ◇

A.2: the networked system $\mathbf{\Sigma}$ is well-posed, meaning that for an arbitrary external input series $\mathbf{col}\{u(t,i)|_{i=1}^{N}\}_{t=0}^{\infty}$, the system states $\mathbf{col}\{x(t,i)|_{i=1}^{N}\}_{t=0}^{\infty}$, as well as the external outputs $\mathbf{col}\{y(t,i)|_{i=1}^{N}\}_{t=0}^{\infty}$, are uniquely determined. ◇

A.3: the subsystem connection matrix (SCM) Φ is a constant matrix, and each of its rows has only one nonzero element which is equal to one. ◇

The first assumption is adopted only for clarifying dimensions of the associated vectors. The second one is clearly necessary for a networked system to work properly and hence physically significant (Zhou, 2015; Zhou et al., 1996), which is equivalent to the requirement that the matrix $I - \Phi\mathbf{diag}\{A_{\mathbf{SS}}(i)|_{i=1}^{N}\}$ is invertible (Zhou, 2015). The third assumption appears very restrictive, but as argued in Zhang and Zhou (2015) and Zhou (2015, 2016), it actually does not introduce any constraints on the structure of the whole system. Briefly, when this assumption is not satisfied by an original system model, it can be satisfied by a modified model with completely the same input–output relations, through simply augmenting the associated subsystem internal input/output vectors $v(t,i)/z(t,i)$ with repeated elements, and modifying the associated matrices $A_{\mathbf{ST}}(i)$, $A_{\mathbf{SS}}(i)$ and $B_{\mathbf{S}}(i)$. Note that a large scale networked system usually has a sparse structure, which implies that this augmentation generally does not increase significantly the dimensions of the associated matrices. In addition, under this assumption, each element of a subsystem's internal output vector is able to simultaneously affect more than one subsystems, and different elements of an internal output vector are able to affect different subsystems.

It is worthwhile to point out that when a networked system is densely connected, the above augmentation may lead to a high dimension of some subsystem parameter matrices, that is not attractive in system analysis and synthesis.

In this paper, the following problem is investigated.

Problem. For prescribed subsystem STMs $A_{\mathbf{TT}}(i)|_{i=1}^{N}$, find the minimal $m_{\mathbf{z}i}$ and $m_{\mathbf{y}i}$ ($m_{\mathbf{v}i}$ and $m_{\mathbf{u}i}$), such that an observable (controllable) networked system $\mathbf{\Sigma}$ can be constructed using only external outputs $\mathbf{col}\{y(t,i)|_{i=1}^{N}\}$, $t = 0, 1, 2, \ldots$ (external inputs $\mathbf{col}\{u(t,i)|_{i=1}^{N}\}$, $t = 0, 1, 2, \ldots$). ◇

A similar problem has been investigated in Simon and Mitter (1968), Yuan et al. (2013) and Zhou (2017) for a lumped system. The above problem, however, is different in the sense that it asks for the minimal number of outputs/inputs for *each* subsystem in constructing an observable/controllable networked system in the whole. This requirement reflects the fact that subsystems of a networked system are usually far away from each other geometrically, which makes it expensive in engineering practices to have a signal that simultaneously and directly affects actuators of two or more different subsystems, or have a sensor to measure an output that is an explicit function of the states of several subsystems. In other words, it is more attractive in applications to restrict each input to *directly* affect states of *only* one subsystem, as well as to restrict each output to be a *direct* linear combination of the states *only* in one subsystem. To emphasize this characteristic, an input/output meeting these restrictions is called a *local* input/output, and the associated input/output selection problem is called a minimal *local* input/output problem.