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Brief paper

An LMI approach for $\mathscr{H}_2$ and $\mathscr{H}_\infty$ reduced-order filtering of uncertain discrete-time Markov and Bernoulli jump linear systems[☆]

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ABSTRACT

This paper proposes a new approach based on parameter-dependent linear matrix inequality (LMI) conditions associated with a scalar parameter that are sufficient to provide robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ reduced-order mode-dependent, partially mode-dependent or mode-independent filters for discrete-time Markov jump linear systems (MJLS) with time-invariant uncertain transition probabilities. Time-invariant uncertainties in the state–space matrices of the modes can be handled as well. As main difference with respect to the existing approaches in the literature, the filter matrices are obtained directly from the slack variables introduced in the conditions. Moreover, the proposed conditions become also necessary for a particular choice of the scalar parameter when mode-dependent full-order filters are designed for systems without uncertainties. Additionally, for precisely known generalized Bernoulli jump systems (i.e., the case where all the rows of the transition probability matrix are equal), optimal solutions are obtained for both mode-dependent and mode-independent full-order filters. Examples (including one motivated by a practical application) are presented to illustrate the proposed approach.

1. Introduction

Concerning the design of $\mathscr{H}_2$ and $\mathscr{H}_\infty$ filters for Markov jump linear systems (MJLS), several results were published in the literature considering different scenarios for the plant and distinct formulations for the synthesis conditions. For instance, the synthesis of strictly proper mode-dependent optimal filters was addressed via coupled algebraic Riccati equations in Costa and Tuesta (2004) and by means of linear matrix inequalities (LMIs) in Fioravanti, Gonçalves, and Geromel (2008). The problem of designing $\mathscr{H}_2$ mode-independent optimal filters was solved in Fioravanti, Gonçalves, and Geromel (2015) for the particular case of Bernoulli jump systems and for generic transition probabilities through augmented matrices based on the Kronecker product in Costa and Guerra (2002a, b), treating only the strictly proper case and formulating the solution in terms of higher order Nn filter matrices (where N is the number of operation modes and n represents the number of states of the plant).

Considering a structure based on the internal model of the plant, $\mathscr{H}_\infty$ optimal mode-dependent strictly proper filters were given in de Souza and Fragoso (2003), while necessary and sufficient LMI conditions were proposed in Gonçalves, Fioravanti, and Geromel (2009) to provide full-order proper mode-dependent $\mathscr{H}_\infty$ filters. In the mode-independent case, the $\mathscr{H}_\infty$ optimal filtering problem for Bernoulli jump systems was solved in Fioravanti et al. (2015) using LMIs. Nevertheless, when the transition probability matrix associated with the jumps between modes is not precisely known, only sufficient conditions (i.e., suboptimal results) were published (Gonçalves, Fioravanti, & Geromel, 2011; Morais, Braga, Lacerda, Oliveira, & Peres, 2014a; Zhang & Boukas, 2009a). Note that the extension of the mentioned LMI strategies of optimal $\mathscr{H}_2$ and $\mathscr{H}_\infty$ filtering to cope with uncertainties is not immediate, since the optimal filter is obtained from the partitions of the Lyapunov matrices.

In the particular scenario of network filtering design, one has to consider the problem of loss of packets containing the measurement signals. If the process is modeled by a Markov chain, then the methods mentioned above could be, in principle, useful. However, most of the existing approaches establish a strong and non-realistic hypothesis: the probabilities associated to the Markov chain are precisely known. This assumption may not coincide with the reality, for instance, in the case where general purpose wireless networks are employed (as Wi-Fi or EEE 802.15.4), giving rise to several problems that cannot be neglected, as intense traffic, delay and packet loss (Angrisani, Bertocco, Fortin, & Sona, 2008). Thus,

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a more realistic assumption is that the transition probability matrix is not precisely known, taking into account the uncertainties inherent to the communication networks.

Another particular feature of interest that usually arises when applying stochastic models to networked control systems (NCS) (Palma, Carvalho, Gonçalves, Galarza, & de Oliveira, 2015) and transmission of images (Lecuire et al., 2006; Yu, Sahinoglu, & Vetro, 2004) is the generalized Bernoulli distribution, where the success or failure in the reception of the messages are uncorrelated, i.e., not depending on the previous transmission. Even in particular cases where the bit error rate between hops depends on the previous instant, the loss from source to destination can be considered as a Bernoulli process (Costa & Guedes, 2012). An advantage of using these models in the NCS context is that the entries of the probability matrix modeling the loss process are directly associated with the packet loss rate (PLR), a measure available in all communication protocols (accurate or with some degree of uncertainty). Particularly in network filtering design, another important issue that must be taken into account is that the signals composing the measurement vector (for example, velocity or position of different objects) can be sent by distinct sources and failure or success of the reception of each one of them is associated to a distinct operation mode of the Markov chain. Actually, when the source of the failure cannot be determined, a simpler alternative is to design filters independent of the Markov chain, such that a single filter optimizes the performance criterion for all the operation modes.

Two classes of LMI based conditions for the design of filters and controllers in the context of MJLS can be identified in the literature. As first alternative, one has the design conditions that provide the optimal $\mathscr{H}_2$ or $\mathscr{H}_\infty$ cost, but having as main drawback the fact that the synthesized filter matrices or control gains depend on the system matrices or probabilities (Fioravanti et al., 2015; Geromel, Gonçalves, & Fioravanti, 2009; Gonçalves et al., 2009). In the few situations where these methods can be extended to cope with uncertainties, the Lyapunov matrices, used to certify the closed-loop stability with $\mathscr{H}_2$ or $\mathscr{H}_\infty$ norm bounds, or partitions of those matrices, must be kept parameter-independent, making the resulting conditions conservative. On the other hand, boosted by the so called *slack variable paradigm*, more recent LMI based methods (Li, Lam, Gao, & Xiong, 2016; Morais et al., 2014a; Morais, Braga, Lacerda, Oliveira, & Peres, 2014b) were proposed specially to treat uncertainties, having as main advantage the fact that the Lyapunov matrix can be parameter-dependent, in general being more effective than the previous methods when dealing with MJLS affected by time-invariant uncertainties. The drawback in this case is the non optimality of the conditions if the system under investigation is not subject to uncertainties. The purpose of this paper is to close the gap between these two classes of LMI methods, with special attention to the Bernoulli distribution. More precisely, this paper proposes new sufficient LMI based conditions of full-order $\mathscr{H}_2$ and $\mathscr{H}_\infty$ robust filtering that become also necessary for precisely known discrete-time MJLS considering complete availability of the operation modes, in the Markov and Bernoulli cases, and partial or null availability for generalized Bernoulli systems. In other words, the optimality of the conditions is assured in the following cases: (i) synthesis of full-order mode-dependent filters for precisely known MJLS with known transition probability matrix; (ii) synthesis of full-order mode-independent filters for precisely known MJLS with known generalized Bernoulli distribution (transition probability matrix with identical rows). The following main features can be highlighted in the proposed method. The conditions are generic enough to cope with the design of full- and reduced-order filters, handling also the uncertain case and mode-independent MJLS filtering while maintaining characteristics of low conservativeness when compared to other methods. An important point is that the construction of the filter matrices is made only in terms of slack variables, facilitating the task of designing reduced-order filters. Thanks to these properties, the conditions can be extended to handle the presence of time-invariant uncertain parameters in the state–space matrices of the modes. Another distinction of the proposed method is that, differently from similar approaches (Li et al., 2016; Morais et al., 2014a, b) that use several unconstrained scalar parameters to obtain filters with improved performance, the proposed conditions are associated with a single scalar parameter belonging to a known and bounded interval, which ease the task of searching for better solutions in terms of $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance. The proposed conditions are formulated as parameter-dependent matrix inequalities with a scalar, becoming robust LMIs (infinite dimensional problems) for fixed values of the scalar parameter. Thanks to recent software developments, programmable LMIs based on polynomial approximations of increasing precision can be automatically constructed using specialized toolboxes.

The notation used in the paper is summarized as follows. The set of natural numbers is denoted by $\mathbb{N}$ and the nth dimensional Euclidean space with the usual norm $\|\cdot\|$ is expressed by $\mathbb{R}^n$. The fundamental probability space is described by $(\Omega, \mathscr{F}, \{\mathscr{F}_k\}, \Gamma)$. The finite set with all the σ Markov operation modes is represented by $\mathbb{K} = \{1, \dots, \sigma\}$ and the mathematical expectation is symbolized by $\mathscr{E}[\cdot]$. The initial probability distribution of the Markov chain is $\mu = [\mu_1, \dots, \mu_\sigma]$ such that $\Pr(\theta(0) = i) = \mu_i$. To deal with clusters, consider the set $\mathbb{Q} = \{1, 2, \dots, \sigma_c\}$, $\sigma_c \le \sigma$, that contains the indexes ℓ of system clusters, and the set $\mathbb{U}_\ell$, that gathers the modes belonging to the cluster ℓ, such that $\mathbb{K} \equiv \cup_{\ell\in\mathbb{Q}}\mathbb{U}_\ell$ and $\cap_{\ell\in\mathbb{Q}}\mathbb{U}_\ell \equiv \varnothing$. The symbol $'$ stands for transpose of a matrix or a vector. The superscript indexes $(\cdot)^{-1}$ and $(\cdot)^{-T}$ stand, respectively, for inverse and inverse transpose of a matrix. In any square matrix, $\star$ denotes a block induced by symmetry and $\mathrm{He}(X)$ is used to represent the sum $X + X'$. The space of discrete-time signals that are square-integrable is denoted by ℓ_2.

2. Preliminaries

A discrete-time MJLS $\mathscr{G}$ is described, on the probabilistic space $(\Omega, \mathscr{F}, \{\mathscr{F}_k\}, \Gamma)$, by the following equations

$$\mathscr{G} = \begin{cases} x(k+1) = A(\theta_k)x(k) + E(\theta_k)w(k) \\ z(k) = C_z(\theta_k)x(k) + E_z(\theta_k)w(k) \\ y(k) = C_y(\theta_k)x(k) + E_y(\theta_k)w(k) \\ k \ge 0, \quad w \in \ell_2^{n_w}, \quad \mathscr{E}\left[\|x(0)\|^2\right] < \infty, \quad \theta_0 \sim \mu, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $w(k) \in \mathbb{R}^{n_w}$ is the external perturbation, $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated and $y(k) \in \mathbb{R}^{n_y}$ is the measured output. The operation modes of system $\mathscr{G}$ given by $(\theta(k);\ k \ge 0)$ assume values in the finite state–space $\mathbb{K} = \{1, \dots, \sigma\}$ which is associated to a transition probability matrix $\Gamma = [p_{ij}]$, $\forall i, j \in \mathbb{K}$, where $p_{ij} = \Pr(\theta(k+1) = j \mid \theta(k) = i)$, $\forall k \ge 0$ satisfying the constraints $p_{ij} \ge 0$ and $\sum_{j=1}^{\sigma} p_{ij} = 1$ for each $i \in \mathbb{K}$. For conciseness, the notation $A_i \in \mathbb{R}^{n_x\times n_x}$, $E_i \in \mathbb{R}^{n_x\times n_w}$, $C_{zi} \in \mathbb{R}^{n_z\times n_x}$, $E_{zi} \in \mathbb{R}^{n_z\times n_w}$, $C_{y_i} \in \mathbb{R}^{n_y\times n_x}$, $E_{y_i} \in \mathbb{R}^{n_y\times n_w}$ is used whenever $\theta(k) = i$, $\forall i \in \mathbb{K}$.

One definition for the stability of the MJLS (1) is the mean-square stability (MSS), stated as $\mathscr{E}[\|x(k)\|] \to 0$ as $k \to \infty$ for any initial condition $x(0) \in \mathbb{R}^{n_x}$, $\theta_0 \in \mathbb{K}$. Necessary and sufficient conditions to verify MSS in terms of LMIs were demonstrated in Costa and Fragoso (1993) and Costa, Fragoso, and Marques (2005).

The aim in this paper is to design a fixed-order robust mode-dependent linear filter described by

$$\begin{cases} x_f(k+1) = A_f(\theta_k)x_f(k) + B_f(\theta_k)y(k) \\ z_f(k) = C_f(\theta_k)x_f(k) + D_f(\theta_k)y(k) \end{cases} \tag{2}$$

where $x_f(k) \in \mathbb{R}^{n_f}$, $n_f \le n_x$, is the estimated state, $z_f(k) \in \mathbb{R}^{n_z}$ is the estimated output and the matrices $A_{f_i} \in \mathbb{R}^{n_f\times n_f}$, $B_{f_i} \in \mathbb{R}^{n_f\times n_y}$,