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Brief paper

Stability analysis for positive singular systems with distributed delays[☆]

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ABSTRACT

This paper is concerned with the stability analysis for continuous-time positive singular systems with distributed delays. By introducing an auxiliary system, a necessary and sufficient positivity condition is proposed for singular systems with distributed lags. Based on this condition, we present a sufficient stability condition for positive singular systems with distributed delays and extend the result to systems with distributed delays over a bounded time-varying interval. In addition, this stability condition is also necessary when strictly positive initial conditions are available. Numerical simulations are utilized to illustrate the effectiveness of our results.

1. Introduction

In the past decade, increasing attention of researchers has been drawn to singular systems (also referred to as generalized state–space systems, descriptor systems, differential-algebraic systems, or implicit systems) due to their broad application. Singular systems are widely used in practical engineering, such as chemical reactions, economic models, and flexible robot control (Hale, 1993). Many results focus on extending the well developed theories of state–space systems to singular systems (Wu, Park, Su, & Chu, 2013). In this paper, we study the positivity and stability of singular systems with distributed delays. Systems are called positive if for any nonnegative initial condition and input signals, their states and output signals stay in the nonnegative orthant. Variables have no physical meaning if they are endowed with negative values, like the population of a particular species, the mass of different reactants in a chemical process, and the probabilities in a stochastic model. A typical example is the mathematical model representing the relationship between two competing species populations. The dynamic system is positive when the populations of animals are chosen as the state variables; when one of the state variables is the difference of their populations, the dynamic system is not positive. Owing to the intensive application background, positive systems have attracted an ever-increasing interest and development in recent years (Ait Rami, Tadeo, & Helmke, 2011; Dong, 2015; Liu, Yu, & Wang, 2009, 2010; Ngoc, 2013; Shen & Lam, 2015; Zhao, Liu, Yin, & Li, 2014; Zhao, Zhang, Shi, & Liu, 2012; Zhu, Li, & Zhang, 2012; Zhu, Meng, & Zhang, 2013).

In contrast with the abundant results of standard positive time-delay systems, positive singular systems with delays received relatively less attention in the literature. The exponential stability criteria for continuous-time singular systems with constant delays and their discrete-time counterparts were presented in Liu, Wu, and Tong (2015) and Phat and Sau (2014), respectively. However, both of them dealt with constant delays and required the existence of a monomial matrix in order to establish the positivity and stability criteria for singular time-delay systems. In Li and Xiang (2016), the stability of positive switched singular systems with delays were studied. It should be emphasized that the results in Li and Xiang (2016), Liu et al. (2015) and Phat and Sau (2014) are based on such a common assumption that is unnecessary for positive singular systems, as demonstrated in Ait Rami and Napp (2012). It was pointed out that the matrix representing the projector on the set of admissible initial conditions is not necessary to be nonnegative, which gave rise to a less restrictive result. The positivity and stability conditions for discrete-time and continuous-time positive singular systems were presented in Ait Rami and Napp (2012, 2014), respectively, without the unnecessary assumption.

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As a generalization of the standard state–space description, singular systems can be applied to many practical systems. Systems with distributed delays can be applied to the controller design in a liquid monopropellant rocket motor with pressure feeding (Xie, Fridman, & Shaked, 2001). In order to modeling of an economic "world" where some parts of the behavior are governed by distributed-lag schemes, positive singular systems with distributed delays were introduced in Bear (1966), Shao, Liu, Sun, and Feng (2014) and Tarr (1975). The stability condition of positive systems with distributed delays has been established in Ngoc (2013) and it is natural to ask how distributed delays affect the stability of their singular form counterparts. In this paper, instead of the slow-fast decomposition method widely used in Li and Xiang (2016), Liu et al. (2015), and Phat and Sau (2014), a different and effective transformation is applied to the singular system, which leads to an equivalent augmented system and reduces conservatism. By using the auxiliary system, a necessary and sufficient positivity condition is proposed for positive singular systems with distributed delays, which can be treated as an extension result of Ait Rami and Napp (2012). Finally, via reductio ad absurdum, a sufficient stability condition of positive singular systems with distributed delays over a bounded time-varying interval is derived and shown also necessary when strictly positive initial conditions is available.

Notation: The notation used in the paper is standard. $\mathbb{R}^n$ denotes n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ denotes set of $n \times m$ real matrices. $\bar{\mathbb{R}}_+$ denotes the set of nonnegative real numbers and $\mathbb{R}_+$ denotes set of positive real numbers. a_{ij} denotes the (i, j)th entry of matrix A. A real matrix $A \in \mathbb{R}^{m\times n}$ with all of its entries nonnegative (respectively, positive) is called nonnegative (respectively, positive) matrix and is denoted by $A \succeq 0$ (respectively, $A \succ 0$) and $A \in \bar{\mathbb{R}}_+^{n\times m}$ (respectively, $A \in \mathbb{R}_+^{n\times m}$). For two matrices $A, B \in \mathbb{R}^{m\times n}$, $A \succeq B$ means that $A - B$ is a nonnegative matrix, or equivalently, $a_{ij} \geq b_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The matrix exponential e^{At} is a nonnegative matrix for all $t \geq 0$ if and only if A is a Metzler matrix (i.e., all its off-diagonal elements are nonnegative). The ∞-norm of a vector $x \in \mathbb{R}^n$ is the maximal of components, that is, $\|x\|_\infty = \max_{1\leq i\leq n}|x_i|$. $C([-d, 0], \mathbb{R}^n)$ is the Banach space of all vector-valued continuous functions defined on $[-d, 0]$ with norm $\|\phi\| = \max_{t\in[-d,0]}\|\phi\|_\infty$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following continuous-time singular systems with distributed delays:

$$\begin{cases} E\dot{x}(t) = Ax(t) + \displaystyle\int_{-d(t)}^{-\underline{d}} A_d(s)x(t+s)ds, \\ x(s) = \phi(s),\ s \in [-\bar{d}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; A is a known constant real matrix and $A_d(s)$ is a continuous matrix-valued function defined on $[-\bar{d}, 0]$. The delay $d(t)$ satisfies $0 < \underline{d} \leq d(t) \leq \bar{d}$ for $t \geq 0$. The matrix $E \in \mathbb{R}^{n\times n}$ is assumed to be singular, that is, $\mathrm{rank}(E) = r < n$; $\phi(\cdot)$ is the admissible initial condition. Several definitions and lemmas will be employed in the proof of the main results.

Definition 1 (*Xu & Lam, 2006*).

(i) The pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero.
(ii) The pair (E, A) is said to be impulse-free if $\deg\{\det(sE-A)\} = \mathrm{rank}(E)$.

Definition 2 (*Shu & Lam, 2008*).

(i) System (1) is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free.
(ii) System (1) is said to be exponentially stable if there exists a positive number $N > 0$ such that, for any initial conditions $\phi(\cdot)$, the solution $x(t|\phi)$ satisfies

$$\|x(t;\phi)\| \preceq Ne^{-\beta t}\|\phi\|,\ \forall t \geq 0.$$

Definition 3. System (1) is said to be asymptotically stable, if for any $\varepsilon > 0$, a scalar $\delta(\varepsilon) > 0$ exists such that for any compatible initial condition $\phi(t)$ satisfying $\sup_{-\bar{d}\leq t\leq 0}\|\phi(t)\| \leq \delta(\varepsilon)$, the solution $x(t)$ of (1) satisfies $\|x(t)\| \leq \varepsilon$ for $t \geq 0$ and, furthermore, $x(t) \to 0$, when $t \to \infty$.

Definition 4 (*Farina & Rinaldi, 2000*). System (1) is positive if for any admissible initial condition $\phi(\cdot)$ satisfying $\phi(s) \succeq 0$, $s \in [-\bar{d}, 0]$, the corresponding trajectory $x(t) \succeq 0$ for all $t \geq 0$.

Definition 5 (*Kunkel & Mehrmann, 2006*). For any matrix $E \in \mathbb{R}^{n\times n}$, there always exists a unique matrix E^D, called the Drazin inverse of E, such that

$$EE^D = E^DE,\ E^DEE^D = E^D,\ E^DE^{\nu+1} = E^\nu,$$

where ν is the smallest nonnegative integer such that $\mathrm{rank}(E^\nu) = \mathrm{rank}(E^{\nu+1})$ and is denoted by $\nu = \mathrm{ind}(E)$.

Lemma 1 (*Campbell, 1980*). *Let $E, A \in \mathbb{R}^{n\times n}$ with the pair (E, A) regular and $\eta \in \mathbb{R}$ be chosen such that the matrix $\eta E - A$ is nonsingular. Then, the matrices*

$$\hat{E} := (\eta E - A)^{-1}E,\ \hat{A} := (\eta E - A)^{-1}A$$

commute.

Lemma 2 (*Duan, 2010*). *The pair (E, A) is impulse-free for singular system if and only if $\mathrm{rank}(\hat{E}) = \mathrm{rank}(\hat{E}^2)$, that is, $\nu = 1$.*

Lemma 3 (*Kunkel & Mehrmann, 2006*). *Let $E, A \in \mathbb{R}^{n\times n}$ with $EA = AE$. Then,*

$$EA^D = A^DE,\ E^DA = AE^D,\ E^DA^D = A^DE^D.$$

Lemma 4 (*Kunkel & Mehrmann, 2006*). *Let $E, A \in \mathbb{R}^{n\times n}$ with $EA = AE$ and suppose that (E, A) is regular. Then,*

$$(I - E^DE)A^DA = (I - E^DE).$$

Then, we introduce a useful technical proposition in the following analysis.

Lemma 5. *Suppose that the pair (E, A) is regular, impulse-free and define $x_1(t) = Mx(t)$, $x_2(t) = (I - M)x(t)$ with $M = \hat{E}^D\hat{E}$. Then, for $t \geq 0$, $x_1(t)$ and $x_2(t)$ satisfy*

$$\dot{x}_1(t) = A_1x_1(t) + \int_{-d(t)}^{-\underline{d}} A_{d1}(s)\{x_1(t+s) + x_2(t+s)\}ds, \tag{2}$$

$$0 = -x_2(t) + \int_{-d(t)}^{-\underline{d}} A_{d2}(s)\{x_1(t+s) + x_2(t+s)\}ds, \tag{3}$$

where, for $s \in [-\bar{d}, 0]$,

$$A_1 = \hat{E}^D\hat{A},\ A_{d1}(s) = \hat{E}^D\hat{A}_d(s),\ A_{d2}(s) = (M - I)\hat{A}^D\hat{A}_d(s),$$
$$\hat{A} = (\eta E - A)^{-1}A,\ \hat{A}_d(s) = (\eta E - A)^{-1}A_d(s),$$
$$\hat{E} = (\eta E - A)^{-1}E,$$

with any $\eta \in \mathbb{R}$ such that $(\eta E - A)^{-1}$ exists.