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Brief paper

Robust output consensus of networked heterogeneous nonlinear systems by distributed output regulation☆

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ABSTRACT

In this paper, we study the output consensus problem of networked heterogeneous nonlinear systems. In contrast to existing results on similar problems, output consensus of all systems is achieved with some robustness not only to heterogeneous dynamics with uncertainties but also to unreliable network communication in terms of network connectivity and communication delays. In particular, we allow the time-varying network topology and the presence of time-varying communication delays in network, which are among the most challenging issues in cooperative control. To solve the problem, we propose a hierarchical control scheme and design a distributed dynamic output feedback control law by the distributed output regulation approach. The proposed control approach offers great flexibility to incorporate the output regulation approach, the separation principle and the certainty equivalence principle to stability analysis of the closed-loop system, thus setting a stage for systematically studying the robust output consensus problem. By establishing some technical lemmas and resorting to ISS Lyapunov technique, we solve the robust output consensus problem of nonlinear multi-agent systems subject to system uncertainties, jointly connected directed network topologies, and arbitrarily bounded and nonuniform time-varying communication delays.

1. Introduction

As one of the most fundamental problems in cooperative control, the consensus problem has attracted a great deal of attention. Research of the consensus problem has evolved from linear multi-agent systems, see, e.g., Li, Duan, Chen, and Huang (2010), Olfati-Saber and Murray (2004), Ren and Beard (2008), and Tuna (2008), to nonlinear multi-agent systems, see Chen (2014), Lin, Francis, and Maggiore (2007), Lu and Huang (2016), Meng, Chen, Zhu, and Middleton (2016), Su, Lin, and Garcia (2016), Tang, Hong, and Wang (2015), Wang, Xu, and Ji (2016), Xu, Hong, and Wang (2014), Zhang and Lewis (2012), Zhu, Chen, and Middleton (2016), and the references therein.

To handle heterogeneous system dynamics and system uncertainties, the distributed output regulation approach has been introduced to the robust output consensus problem of nonlinear multi-agent systems, see Dong and Huang (2014), Isidori, Marconi, and Casadei (2014), Liu (2015), Su and Huang (2015), Xu et al. (2014), Zhu et al. (2016) and the references therein. First, the static and undirected network topology is considered in the robust output consensus problem of nonlinear multi-agent systems (Dong & Huang, 2014; Liu, 2015; Su & Huang, 2013; Tang et al., 2015). In particular, a distributed output regulation approach is proposed in Dong and Huang (2014) to study the robust output consensus problem of nonlinear multi-agent systems in output feedback form. Then, the robust output consensus problem of nonlinear multi-agent systems is further studied for the static and directed network topology (Su & Huang, 2015; Xu et al., 2014; Xu, Wang, Hong, & Jiang, 2016). In particular, for nonlinear multi-agent systems in output feedback form, a class of non-quadratic Lyapunov functions is designed in Xu et al. (2014, 2016) to handle the robust output consensus problem. Meanwhile, for nonlinear multi-agent systems in lower triangular form, another class of non-quadratic Lyapunov functions is constructed in Su and Huang (2015) to address the robust output consensus problem.

It is known that the time-varying network topology is one of the most challenging issues in cooperative control of multi-agent systems. In this paper, we consider the directed and time-varying network topology in the robust output consensus problem. In particular, we just require the time-varying network topology to be jointly connected (Jadababaie, Lin, & Morse, 2003; Ni & Cheng, 2010; Su & Lin, 2016), which is perhaps the mildest condition

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on network connectivity. In contrast to existing results, it greatly relaxes the requirement on network connectivity. However, it also brings great challenges to the solvability of the current problem. More precisely, under the directed and jointly connected condition, a matrix associated with the network topology is neither symmetric nor nonsingular, which is necessary for the solution of the consensus problem in Dong and Huang (2014), Liu (2015), Liu and Huang (2015), Long and Zhao (2014), Su and Huang (2013, 2015), Tang et al. (2015) and Xu et al. (2014). As a consequence, the existing approaches in all these references cannot be used to tackle the great challenges in this paper.

In cooperative control of multi-agent systems, the presence of the communication delay is inevitable (Hu & Lin, 2017; Hu & Hong, 2007; Moreau, 2004; Olfati-Saber & Murray, 2004; Zhu & Jiang, 2015). In existing literature, consensus of multi-agent systems subject to time-delays mainly focuses on linear dynamics, see, Hu and Hong (2007), Moreau (2004), Olfati-Saber and Murray (2004) and Zhu and Jiang (2015). In this paper, communication delays are considered in the consensus problem of nonlinear multi-agent systems. As a result, the existing approaches in Hu and Hong (2007), Moreau (2004), Olfati-Saber and Murray (2004), Zhu and Jiang (2015) cannot be used directly to solve the current problem. Recently, the consensus problem of a class of second-order nonlinear multi-agent systems subject to constant communication delays and static networks is studied in Meng et al. (2016). In contrast, communication delays considered in this paper can be time-varying and arbitrarily bounded. Thus, the control approach in Meng et al. (2016) cannot be adopted.

In this paper, time-varying network topologies and time-varying communication delays are taken into consideration in output consensus of nonlinear uncertain multi-agent systems. As mentioned above, these settings make our problem formulation more practical and at the same time, cause great challenges to the solvability of the problem. To the best of our knowledge, there exists no relevant result tackling these challenging issues. To overcome these difficulties, we propose a hierarchical control scheme and design a distributed dynamic output feedback control law, which is composed of a distributed internal model and a distributed dynamic compensator. By introducing a technical lemma, we convert the consensus problem into a robust stabilization problem of an augmented system composed of original multi-agent systems, the distributed internal model and the dynamic compensator. Then, we establish some technical lemmas to lay the foundation for the solution of the robust stabilization problem. It is noted that the employment of hierarchical control scheme greatly facilitates the utilization of the separation principle and certainty equivalence principle to stability analysis of the closed-loop system. By means of constructing Lyapunov function and resorting to the input-to-state stability (ISS) Lyapunov technique, we show that the robust stabilization problem of the augmented system can be solved. Finally, we achieve robust output consensus of nonlinear multi-agent systems subject to system uncertainties, time-varying network topologies and time-varying communication delays.

The rest of this paper is organized as follows. Section 2 formulates the robust output consensus problem. Section 3 presents the main result. Cooperative control of multiple controlled Lorenz systems is used to illustrate our design in Section 4. Finally, some conclusions are made in Section 5.

Notation. For any column vectors x_i, $i = 1, \ldots, m$, denote $\mathrm{col}(x_1, \ldots, x_m) = [x_1^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}]^{\mathrm{T}}$. For some positive scalar h, denote by $\mathcal{C}([-h, 0], \mathbb{R}^n)$ the Banach space of continuous functions mapping the interval $[-h, 0]$ into $\mathbb{R}^n$ endowed with the supremum norm. $\mathrm{diag}(M)$ denotes the diagonal matrix obtained from M by setting all off-diagonal entries equal to zero. Denote by $\mathrm{diag}\{d_i\}$ the diagonal matrix D of appropriate dimension with its ith diagonal element being d_i. Denote by $\hat{I}_i$ the row vector of appropriate dimension with its ith element being 1 and all the other elements being 0. Denote by $\check{\mathcal{I}}_i$ the square matrix of appropriate dimension with its ith row being $\hat{I}_i$ and all the other rows being zero row vector. For simplicity, the dimension of $\check{\mathcal{I}}_i$ is not stated if not misleading. Denote a piecewise constant switching signal by a right continuous function $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, 2, \ldots, \rho_0\}$ for some positive integer ρ_0, where the switching instants t_i, $i = 0, 1, 2, \ldots$, satisfy $t_0 = 0$ and $t_{i+1} - t_i \geq \tau_D$ for some positive real number τ_D. The set $\mathcal{P}$ is called the switching index set and τ_D is called the dwell time.

2. Problem formulation and preliminaries

This paper addresses a class of nonlinear uncertain multi-agent systems as follows:

$$\begin{aligned} \dot{z}_i(t) &= f_i(z_i(t), y_i(t), v(t), w) \\ \dot{y}_i(t) &= g_i(z_i(t), y_i(t), v(t), w) + b_i(w)u_i(t) \\ z_i(0) &= z_i^0(0), y_i(0) = y_i^0(0), \ i = 1, \ldots, N \end{aligned} \tag{1}$$

where $z_i(t) \in \mathbb{R}^{n_{z_i}}$, $y_i(t) \in \mathbb{R}$ are system states, $u_i(t) \in \mathbb{R}$ is the control input, $z_i^0(0) \in \mathbb{R}^{n_{z_i}}$, and $y_i^0(0) \in \mathbb{R}$. The uncertain parameter $w \in \mathbb{W} \subseteq \mathbb{R}^{n_w}$ for a compact subset $\mathbb{W}$. $v(t) \in \mathbb{R}^q$ is generated by the leader system as follows:

$$\begin{aligned} \dot{v}(t) &= Qv(t) \\ y_0(t) &= h(v(t)), v(\theta) = v^0(\theta), \theta \in [-\tau, 0] \end{aligned} \tag{2}$$

where $Q \in \mathbb{R}^{q\times q}$ is a constant matrix and $y_0(t)$ is the output of the leader. $v^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^q)$, where τ denotes the upper bound of time-delays involved. As in Dong and Huang (2014), leader system (2) can be used to formulate both reference input signals and external disturbance signals. The regulated output is defined as

$$e_i(t) = y_i(t) - y_0(t). \tag{3}$$

All functions in (1) and (2) are assumed to be sufficiently smooth satisfying $f_i(0, 0, 0, w) = 0$, $g_i(0, 0, 0, w) = 0$, and $h(0) = 0$ for all $w \in \mathbb{R}^{n_w}$.

Given a piecewise constant switching signal $\sigma(t)$, a time-varying digraph $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)})$[1] can be used to describe the communication network of the multi-agent system. The node set $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$, where node 0 denotes leader (2) and node i, $i \neq 0$, denotes the ith follower (1). The edge set $\bar{\mathcal{E}}_{\sigma(t)} = \{(i, j), i, j \in \bar{\mathcal{V}}, i \neq j\}$. The edge $(j, i) \in \bar{\mathcal{E}}_{\sigma(t)}$, $j = 0, 1, \ldots, N$, $i = 1, \ldots, N$, if and only if the control law of agent i can access the information of agent j at the instant of time t. If the graph contains a sequence of edges (i_k, i_{k+1}), $k = 0, 1, \ldots, l-1$, then node i_l is said to be reachable from node i_0. Let $\bar{\mathcal{A}}_{\sigma(t)} = [a_{ij}(t)]_{i,j=0}^N$ be the weighted adjacency matrix of digraph $\bar{\mathcal{G}}_{\sigma(t)}$, i.e., for $i, j = 0, 1, \ldots, N$, $a_{ii}(t) = 0$, $a_{ij}(t) = 1$ if $(j, i) \in \bar{\mathcal{E}}_{\sigma(t)}$ and $a_{ij}(t) = 0$ otherwise. Furthermore, define a matrix $\mathcal{H}_{\sigma(t)} = [h_{ij}(t)]_{i,j=1}^N$, where $h_{ij}(t) = -a_{ij}(t)$, $i, j = 1, \ldots, N$, $i \neq j$, and $h_{ii}(t) = \sum_{j=0}^N a_{ij}(t)$. Denote the neighbor set of agent i by $\bar{\mathcal{N}}_i(t) = \{j, (j, i) \in \bar{\mathcal{E}}_{\sigma(t)}\}$.

Now, the robust output consensus problem of the multi-agent system is defined as follows.

Problem 2.1. Given the multi-agent system composed of followers (1), leader (2), a time-varying network topology $\bar{\mathcal{G}}_{\sigma(t)}$, and any compact subsets $\mathbb{V}_0 \subseteq \mathbb{R}^q$ and $\mathbb{W} \subseteq \mathbb{R}^{n_w}$ with $0 \in \mathbb{V}_0$ and $0 \in \mathbb{W}$, design a distributed dynamic output feedback control law of the form

$$\begin{aligned} u_i(t) &= \hat{k}_i(y_i(t), \eta_i(t), \eta_j(t - \tau_{ij}(t)), j \in \bar{\mathcal{N}}_i(t)) \\ \dot{\eta}_i(t) &= \hat{h}_i(y_i(t), \eta_i(t), \eta_j(t - \tau_{ij}(t)), j \in \bar{\mathcal{N}}_i(t)) \\ \eta_i(\theta) &= \eta_i^0(\theta), \theta \in [-\tau, 0] \end{aligned} \tag{4}$$

[1] See Godsil and Royle (2001) for the definition of digraph.