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Brief paper

Mean square consensus of multi-agent systems over fading networks with directed graphs☆

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ABSTRACT

This paper studies the mean square consensus problem of discrete-time linear multi-agent systems (MASs) over analog fading networks with directed graphs. Compressed in-incidence matrix (CIIM), compressed incidence matrix (CIM) and compressed edge Laplacian (CEL) are firstly proposed to facilitate the modeling and consensus analysis. It is then shown that the mean square consensusability is solely determined by the edge state dynamics on a directed spanning tree. As a result, sufficient conditions are provided for mean square consensus over fading networks with directed graphs in terms of fading parameters, the network topology and the agent dynamics. Moreover, the role of network topology on the mean square consensusability is discussed. In the end, simulations are conducted to verify the derived results.

1. Introduction

The rapid development of technology has enabled wide applications of multi-agent systems (MASs). The consensus problem, which requires all agents to agree on certain quantity of common interest, builds the foundations of other cooperative tasks. One question arises before control synthesis: whether there exist distributed controllers such that the multi-agent system can achieve consensus. This problem is usually referred to as consensusability of multi-agent systems. Previously, the consensusability problem with perfect communication channels has been well studied. For example, Ma and Zhang (2010) and You and Xie (2011) study consensus conditions for continuous-time MASs and discrete-time MASs under perfect communication channels, respectively. To ensure the consensus, a (directed) spanning tree on the underlying (directed) graph is required. For consensus of discrete-time MASs, the product of unstable eigenvalues of the system matrix should additionally be upper bounded by a function of the eigen-ratio of the undirected graph. Since wireless communication is commonly used in MASs, and fading is unavoidable in urban, indoor and underwater environments, we are interested in knowing how fading affects the consensusability problem of MASs. When there exist some fading channels, the stabilization of a single system is considered by Elia (2005) and Xiao, Xie, and Qiu (2012). Elia (2005) demonstrates that to ensure mean square stability, the mean square capacity of the fading channel should be greater than the instability degree of the single-input single-output linear discrete-time dynamics. Xiao et al. (2012) further extends the results to multi-input multi-output systems with multiple fading channels.

In our previous work (Xu, Xiao, & Xie, 2016), we consider MASs over fading channels with an undirected graph setting. For consensus over identical fading networks, a decomposition method is used and the mean square consensus problem is transformed to a simultaneous mean square stabilization problem. For consensus over non-identical fading networks, the edge Laplacian defined for undirected graphs by Zelazo and Mesbahi (2011) is introduced to model the consensus error dynamics. Then sufficient mean square consensus conditions are developed. We demonstrate how the system dynamics, the communication channels and the network topological structure interplay with each other to allow the existence of a linear distributed consensus controller. However, since there is no appropriate definition of edge Laplacian for directed graphs, the method used in non-identical fading networks for undirected graphs (Xu et al., 2016) cannot be applied to directed graph cases either, which complicates the consensusability analysis due to the coupling between the channel fading and the network topology. Recently, Zeng, Wang, and Zheng (2016a, b) propose a definition of directed edge Laplacian (DEL) for directed graphs to study robust and quantized consensus problems, where in-incidence matrix (IIM) and incidence matrix (IM) are introduced to characterize the information flow in directed graphs. However,

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since Zeng et al. (2016a, b) treat every bidirectional edge as two directed edges with opposite directions, for an undirected graph, the dimension of DEL is doubled compared with that of the edge Laplacian defined in Zelazo and Mesbahi (2011). As a result, the DEL (Zeng et al., 2016a, b) cannot include the existing edge Laplacian in Zelazo and Mesbahi (2011) for undirected graphs as a special case, which may lead to inconsistency of results derived for directed and undirected graphs. In this paper, we distinguish bidirectional edges from non-bidirectional edges and introduce the compressed incidence matrix (CIIM), compressed incidence matrix (CIM) and compressed edge Laplacian (CEL) to avoid inconsistency. Based on those definitions, the mean square consensus over fading networks with directed graphs is studied.

In this paper, we are mainly concerned with the mean square consensus problem of MASs over fading networks with directed graphs, which extends our previous results (Xu et al., 2016) on undirected graphs to directed graphs. The main contributions of this paper are as follows: (1) CIIM, CIM and CEL are proposed and their properties are analyzed; (2) by defining edge states and modeling the consensus error dynamics using CIIM, CIM and CEL, sufficient conditions are provided for consensus over fading networks with directed graphs; (3) the role of network topology on the mean square consensusability is discussed.

This paper is organized as follows. The problem formulation is provided in Section 2. The definitions and properties of CIIM, CIM and CEL are discussed in Section 3. The consensus problem over fading networks is studied in Section 4. Simulations are provided in Section 5 followed by some concluding remarks in Section 6.

Notation: All matrices and vectors are assumed to be of appropriate dimensions that are clear from the context. $\mathbb{R}(\mathbb{C})$, $\mathbb{R}^n(\mathbb{C}^n)$ and $\mathbb{R}^{m\times n}(\mathbb{C}^{m\times n})$ represent the sets of real (complex) scalars, n-dimensional real (complex) column vectors, and $m \times n$-dimensional real (complex) matrices, respectively. For $c \in \mathbb{C}$, $\mathrm{Re}(c)$ and $|c|$ represent the real part and the magnitude of c, respectively. For a set $\mathcal{A}$, $|\mathcal{A}|$ denotes its cardinality. Denote by $\mathbf{1}$, I_n and $\mathbf{0}_{m\times n}$ a column vector with all entries being 1, an identity matrix with dimension $n \times n$ and a $m \times n$ matrix with all elements being zero, respectively. The subscripts m, n are dropped when the dimension is clear from the context. A', A^*, A^{-1}, $\rho(A)$ and $\mathrm{null}(A)$ are the transpose, the conjugate transpose, the inverse, the spectral radius and the null space of matrix A, respectively. $[A]_{ij}$, $[A]_{\mathrm{row}i}$ and $[A]_{\mathrm{column}j}$ represent the ijth element, the ith row and the jth column of matrix A, respectively. $\otimes$ and $\odot$ represent the Kronecker product and the Hadamard product, respectively. For a real symmetric matrix A, $A > 0$ ($A \geq 0$) means that matrix A is positive definite (semi-definite) and $\lambda_{\min}(A)$ is used to represent the minimal eigenvalue of A. $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

2. Problem formulation

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to characterize the interaction among agents, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the node set representing N agents and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the edge set with ordered pairs of nodes denoting the information transmission among agents. An edge $(i, j) \in \mathcal{E}$ means that the ith agent can send information to the jth agent, where node i and node j are called the initial node and terminal node of this edge, respectively. The neighborhood set $\mathcal{N}_i$ of agent i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} \,|(j, i) \in \mathcal{E}\}$. A directed path on $\mathcal{G}$ from agent i_1 to agent i_l is a sequence of ordered edges in the form of $(i_k, i_{k+1}) \in \mathcal{E}$, $k = 1, 2, \ldots, l-1$. A directed cycle is a directed path starting and ending at the same node. A graph contains a directed spanning tree if it has at least one node with directed paths to all other nodes. The underlying graph of $\mathcal{G}$ is the graph obtained by treating edges of $\mathcal{G}$ as unordered pairs. The adjacency matrix A_{adj} is defined as $[A_{\mathrm{adj}}]_{ii} = 0$, $[A_{\mathrm{adj}}]_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $[A_{\mathrm{adj}}]_{ij} = 0$, otherwise. The graph Laplacian matrix $\mathcal{L}$ is defined as $[\mathcal{L}]_{ii} = \sum_{j\in\mathcal{N}_i}[A_{\mathrm{adj}}]_{ij}$, $[\mathcal{L}]_{ij} = -[A_{\mathrm{adj}}]_{ij}$ for $i \neq j$. The graph Laplacian $\mathcal{L}$ has the following property.

Lemma 1 (*Ren & Beard, 2008*)**.** *All the eigenvalues of $\mathcal{L}$ have non-negative real parts. Zero is a simple eigenvalue of $\mathcal{L}$ with a right eigenvector* $\mathbf{1}$ *if and only if $\mathcal{G}$ contains a directed spanning tree.*

The discrete-time dynamics of agent i is given by

$$x_i(t+1) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ represent the agent state and control input, respectively.

We assume that each agent obtains the relative state information from its neighbors through fading channels (Elia, 2005; Xiao et al., 2012). The block fading model (Caire, Taricco, & Biglieri, 1999) is utilized, such that the channel fading is approximately constant within each fading block represented by the index t, but is independent from block to block. Based on the received information, agent i generates the control input by the following consensus protocol

$$u_i(t) = K\sum_{j\in\mathcal{N}_i}\epsilon_{ij}(t)(x_i(t) - x_j(t)), \tag{2}$$

where ϵ_{ij} models the channel fading and K is the consensus parameter to be designed. Depending on the particular propagation environment and communication scenario, different statistical models can be used for the channel fading ϵ_{ij} (e.g., Rayleigh, Nakagami, Rician) (Goldsmith, 2005).

In this paper, we are interested in the consensusability problem, i.e., we aim to establish conditions on the fading statistics, the agent dynamics and the communication topology under which there exists K in the protocol (2) such that the MAS (1) can achieve mean square consensus, i.e., $\lim_{t\to\infty}\mathbb{E}\{\|x_i(t) - x_j(t)\|_2^2\} = 0$ for any i, j in $\mathcal{V}$. In view of results in Ren and Beard (2008) and You and Xie (2011), the following assumption is made.

Assumption 1.

1. (A, B) is controllable and all the eigenvalues of A are either on or outside the unit disk.
2. The directed graph $\mathcal{G}$ contains a directed spanning tree.

Remark 1. The relative sensing model has been widely used in the study of consensus problems (Guo & Dimarogonas, 2013; Li & Chen, 2017; Li, Duan, Chen, & Huang, 2010). An application example of the protocol (2) is the containment control of Autonomous Vehicles (AVs) (Cao, Stuart, Ren, & Meng, 2011; Zhu, Xie, Han, Meng, & Teo, 2017). Consider the scenario that only leaders are equipped with relative state measurement sensors, such as radars, to reduce cost. The follower agents can obtain the relative state information from corresponding leaders through wireless fading channels. Besides, (2) can also model the case that each agent communicates their own state with neighboring agents over intermittent channels (Hatano & Mesbahi, 2005), which can be modeled by restricting $\epsilon_{ij} \in \{0, 1\}$. If $\epsilon_{ij} = 1$, agent i successfully receives agent j's state x_j and uses $x_i - x_j$ in its consensus update. Otherwise, the transmission of x_j from agent j to agent i fails and agent i does not include $x_i - x_j$ in the consensus update. As a result, we can also use (2) to describe the consensus protocol.

Remark 2. The fading factors of MASs appear in the consensus protocol in a similar way as the coupling terms c_{ij} in Li, Liu, Ren, and Xie (2013), Li, Ren, Liu, and Fu (2013) and Li, Ren, Liu, and Xie (2013), which design adaptive updating laws for c_{ij} to achieve a fully distributed consensus control. However, they are different in the following aspects. Firstly, ϵ_{ij} in our formulation arises from