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Brief paper

Guaranteed cost control of periodic piecewise linear time-delay systems☆

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ABSTRACT

This paper is concerned with the guaranteed cost control problem for continuous-time periodic piecewise linear systems with time delay. Sufficient delay-dependent conditions of closed-loop asymptotic stability are presented based on an improved formulation, which uses a novel Lyapunov–Krasovskii functional with relaxed requirement in positive definiteness of the involved symmetric matrices. The corresponding optimization problems aiming at the mixed performance involving an upper bound of H_2 guaranteed cost and an H_∞ performance index for disturbance attenuation are established. By designing an iterative algorithm subject to the proposed conditions, the periodic guaranteed cost controller gains over each sub-interval are tractable for the resulting closed-loop time-delay system. The effectiveness and reduced conservatism of our proposed criteria are validated and illustrated via numerical simulations.

1. Introduction

Periodic systems with time-delay phenomena are common in various fields. The delays arising from biochemical reactions, mechanics, communication or measurement can lead to instability and poor system performance. The stability and control problems of periodic systems with delay terms have been considered from various aspects (Butcher, Dabiri, & Nazari, 2017; Deshmukh, Ma, & Butcher, 2006; Zhou & Li, 2015). It has been found difficult to derive direct stability conditions from Floquet theory for continuous-time systems formulated by periodic delay differential equations (Bittanti & Colaneri, 2008; Insperger & Stépán, 2002). Periodic time-delay systems have been studied based on different approximation or transformation methods, such as the discretization of periodicity (Shao & Sheng, 2014), and the polynomial representation of characteristic function (Lampe & Rosenwasser, 2011). In Gomez, Ochoa, and Mondié (2016), the exponential stability for continuous-time periodic time-delay systems has been investigated without system approximation, but the results are necessary conditions that cannot be directly extended to controller synthesis.

Recently, the periodic piecewise formulation has been found effective as an approximation of more complex periodic systems that do not necessarily have closed-form expressions, providing the possibility to implement stability analysis and controller synthesis based on a sequential Lyapunov approach (Li, Lam, & Cheung, 2015; Zhou & Qian, 2017). A periodic piecewise linear system can be regarded as a special case of switched systems, since it is modeled by a finite number of periodically repeating subsystems with prescribed dwell time. Within one fundamental period, each subsystem is approximated by the average model over the corresponding sub-interval, or directly calculated based on the values of periodically constant variables. In practice, typical examples include springs with periodic piecewise constant stiffness in vehicle suspensions, periodically forced piecewise linear oscillators, and periodic piecewise invariant loads of conveyor systems. For delay-free periodic piecewise systems, the linear time-invariant subsystems have provided efficient alternatives to employ time-domain Lyapunov or Lyapunov-like approaches for H_∞ controller design (Li, Lam, & Cheung, 2017; Xie, Lam, & Li, 2017), and frequency-domain responses framework for stability analysis (Zhou & Qian, 2017). In recent studies, guaranteed cost control problems for systems with hybrid characteristics have been considered under different conditions (Hu, Jiang, & Yang, 2016; Li, Wang, Wu, Lam, & Gao, 2018). The main idea is to find the minimal upper bound of maximum control cost guaranteed for an infinite range of inputs. In terms of time-delay systems, delay-dependent stability conditions, which usually can be derived by constructing appropriate Lyapunov–Krasovskii functional candidates, have been proved to be effective and less conservative in previous studies (Briat, 2013, 2015, 2016a). For periodic piecewise time-delay systems, the guaranteed cost control with mixed H_2/H_∞ performance has received very little attention, which motivates our study.

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In this paper, the guaranteed cost control problem with the optimization of two performance indexes is investigated for a class of continuous-time periodic piecewise linear time-delay systems. Delay-dependent conditions of the asymptotic stability are proposed based on a novel periodic piecewise Lyapunov–Krasovskii functional. The stability analysis and controller synthesis with mixed H_2/H_∞ performance can be summarized into optimization problems that can be solved under the framework of an iterative algorithm. By optimizing a weighted objective constituted by the upper bounds of the H_2 guaranteed cost and H_∞ disturbance attenuation level, the closed-loop system is designed to be asymptotically stable. The maximum control cost can be guaranteed by the optimized upper bound. The contributions and novelties of this paper include: (i) The proposed Lyapunov–Krasovskii functional contains a time-varying matrix function, and one of the involved symmetric matrices may not be positive definite. (ii) The designed controller can introduce less conservatism into the optimal solutions of mixed H_2/H_∞ performance, which can be simultaneously computed under a multi-objective optimization framework.

The paper is organized as follows. Section 2 describes the model of periodic piecewise time-delay systems and gives the preliminary requirements of the guaranteed cost control problem. In Section 3, analysis on closed-loop stability, H_2 guaranteed cost performance and the mixed H_2/H_∞ performance are proposed. The corresponding criteria and a tractable iterative algorithm is designed. In Section 4, the results of numerical simulations are presented with comparisons. Section 5 concludes the paper.

Notation: $\mathbb{R}^n$ stands for the n-dimensional Euclidean space; $|\cdot|$ represents the absolute value; $\|\cdot\|$ denotes the Euclidean norm of a vector, or the spectral norm of a matrix; I and 0 represent the identity matrix and a zero matrix with appropriate dimensions, respectively. $P > 0$ (≥ 0) denotes that P is a real symmetric and positive definite (semi-definite) matrix. P^T and P^{-1} denote the transpose and the inverse of matrix P, respectively. $\bar{\lambda}(P)$, $\underline{\lambda}(P)$ refer to the maximum, minimum eigenvalues of a real symmetric matrix P. $\mathbf{diag}(\cdot)$ denotes a diagonal matrix constructed by the given diagonal elements and zero non-diagonal elements. In block symmetric matrices, the symbol "$*$" is used as an ellipsis for the terms introduced by symmetry. For convenience of description, we use a non-negative integer l to represent the nominal number of fundamental periods, that is, $l = 0, 1, 2, \ldots$.

2. System description and preliminaries

Consider a class of continuous-time T_p-periodic piecewise time-delay systems given as

$$\begin{aligned}
\dot{x}(t) &= A(t)x(t) + A_d(t)x(t-d) + B(t)u(t) + B_w(t)w(t),\\
z(t) &= C(t)x(t) + C_d(t)x(t-d) + D(t)u(t) + D_w(t)w(t),\\
x(t) &= \varphi(t),\ \forall t \in [-2d, 0],
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{n_u}$, $z(t) \in \mathbb{R}^{n_z}$ and d are the state vector, control input, system output and time delay, respectively. The exogenous disturbance $w(t) \in \mathbb{R}^{n_w}$ belongs to $L_2[0, \infty)$. For all $t \geq 0$, $A(t+T_p) = A(t)$, $A_d(t+T_p) = A_d(t)$, $B(t+T_p) = B(t)$, $B_w(t+T_p) = B_w(t)$, $C(t+T_p) = C(t)$, $C_d(t+T_p) = C_d(t)$, $D(t+T_p) = D(t)$, $D_w(t+T_p) = D_w(t)$, and the periodic system (1) is supposed be equivalently represented by the following piecewise linear formulation with S subsystems:

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + A_{di}x(t-d) + B_i u(t) + B_{wi}w(t),\\
z(t) &= C_i x(t) + C_{di}x(t-d) + D_i u(t) + D_{wi}w(t),\\
&\quad t \in [lT_p + t_{i-1}, lT_p + t_i),\\
x(t) &= \varphi(t),\ \forall t \in [-2d, 0],
\end{aligned} \tag{2}$$

where the time-invariant matrices $(A_i, A_{di}, B_i, B_{wi}, C_i, C_{di}, D_i, D_{wi})$, $i \in \mathcal{S} \triangleq \{1, 2, \ldots, S\}$ are known with appropriate dimensions, and constitute the ith subsystem activated over sub-interval $[t_{i-1}, t_i)$ given by dwell time $T_i = t_i - t_{i-1}$, $\Sigma_{i=1}^{S} T_i = T_p$ with $t_0 = 0$ and $t_S = T_p$. The time delay is a scalar satisfying $d \in (0, \min_{i\in\mathcal{S}}\{T_i/2\})$. Considering the periodicity of system (2), the initial condition $\varphi(t)$ over $[-2d, 0]$ is given by

$$\varphi(t) = \begin{cases} \phi_s(t), & t \in [-2d, -d] \\ \phi(t), & t \in [-d, 0] \end{cases} \tag{3}$$

where $\phi_s(t)$ is an arbitrary start-up continuous function that can be used to determine $\phi(t)$, $t \in [-d, 0]$ based on the known dynamics of the Sth subsystem and $\phi(-d) = \phi_s(-d)$. The initial condition $\varphi(t)$ is hence continuous for all $t \in [-2d, 0]$, and the state $x(t)$ is continuous for $t \geq 0$.

Remark 1. The initial condition $\varphi(t)$, $t \in [-2d, 0]$ in (3) is specifically defined for periodic piecewise time-delay systems, where the fixed sub-intervals provide helpful information to extend the time interval of initial condition to $[-2d, 0]$ and determine the delayed state $x(t-d)$ for $t \in [0, d]$ via $\phi(t)$, $t \in [-d, 0]$. Actually, since the previous subsystem can be easily known once t_0 is given, we can start analyses and syntheses from a time at any switching point of system (2). For convenience, the starting time in this study is chosen as $t_0 = 0$.

For stabilization of system (2), consider a periodic feedback control law:

$$u(t) = K_i x(t),\ t \in [lT_p + t_{i-1}, lT_p + t_i), \tag{4}$$

where K_i, $i \in \mathcal{S}$, is the controller gain to be determined for each subsystem. Applying the control law to system (2) will result in the following closed-loop system:

$$\begin{aligned}
\dot{x}(t) &= A_{ci}x(t) + A_{di}x(t-d) + B_{wi}w(t),\\
z(t) &= C_{ci}x(t) + C_{di}x(t-d) + D_{wi}w(t),\\
&\quad t \in [lT_p + t_{i-1}, lT_p + t_i),\\
x(t) &= \varphi(t),\ \forall t \in [-2d, 0],
\end{aligned} \tag{5}$$

where $A_{ci} = A_i + B_i K_i$, $C_{ci} = C_i + D_i K_i$, $i \in \mathcal{S}$. Define a cost function for system (5) as

$$\mathcal{J}_c = \sum_{l=0}^{\infty} \sum_{i=1}^{S} \int_{lT_p+t_{i-1}}^{lT_p+t_i} \left[x^T(t)R_i x(t) + u^T(t)Q_i u(t)\right] \mathrm{d}t, \tag{6}$$

where $R_i > 0$, $Q_i > 0$, $i \in \mathcal{S}$, are prescribed constant weighting matrices. When $t \to \infty$, the number of periods $l \to \infty$. This paper aims to design a periodic state feedback controller described by (4) with mixed H_2/H_∞ performance, and the following requirements are simultaneously achieved:

(R1) The closed-loop periodic piecewise time-delay system (5) is asymptotically stable.
(R2) When $w(t) = 0$, the guaranteed cost function (6) satisfies $\mathcal{J}_c \leq \mathcal{J}_c^*$, where $\mathcal{J}_c^* > 0$ is the upper bound of H_2 guaranteed cost.
(R3) For a scalar $\gamma > 0$, under zero-initial conditions with disturbance $w \in L_2[0, \infty)$, the controlled output $z(t)$ satisfies

$$\int_0^{\infty} z^T(t)z(t)\mathrm{d}t \leq \gamma^2 \int_0^{\infty} w^T(t)w(t)\mathrm{d}t. \tag{7}$$

To cope with the aforementioned problem, the controller design consists of two steps: (i) analyzing the asymptotic stability and H_2 guaranteed cost performance of closed-loop system (5) with $w(t) = 0$, and (ii) synthesizing the periodic guaranteed cost controller to obtain a minimal upper bound $\mathcal{J}_c^*$ of the optimal H_2 guaranteed cost, and the corresponding upper bound γ^* of the H_∞ disturbance attenuation level. A lemma facilitating the subsequent analysis is given below.