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Multi-Period Vehicle Routing with Limited Period Load

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Abstract: We investigate a vehicle routing problem, in which the total workload associated with a given request portfolio must be distributed over a sequence of periods. The realization of economies of scale by compiling routes serving a high number of requests is compromised by an upper bound of the payload quantity that is allowed to be served in one period. We propose a mixed-integer linear problem for the representation of this complicated decision task. Within computational experiments, we analyze the impacts of differently mixed request portfolios with respect to the minimization of the total sum of travel distances over all periods and other performance indicators.

Keywords: Periodic vehicle routing, transport, computational methods, mathematical models, mathematical programming, load dispatching,

1. INTRODUCTION

In a multi-period (or periodic) vehicle routing problem (Gulczynski et al., 2011) it is necessary to partition a given portfolio of transport requests into sub-portfolios. Each sub-portfolio is assigned to one fulfillment period. A set of vehicle routes serving the requests of the corresponding sub-portfolio must be compiled for each period. This article addresses a multi-period routing problem with collection requests. Each period the fleet brings quantities to a central facility for further processing. The facility can handle only a limited quantity per period. Consequently, a combined routing and load distribution problem has to be solved. The primary goal of the here reported research comprises the setup and evaluation of a mixed-integer linear program of this combined decision problem.

The reported decision problem is a generalization of the capacitated vehicle routing problem (CVRP) surveyed in Golden et al. (2008). Since the one-period CVRP is already known to be computational challenging, the consideration of several interdependent routing tasks over several periods likely increases the problem complexity.

Locations might be visited several times over the periods. It is therefore necessary to consider the physical network level as well as a request level. Since it is a priori unknown if a request is fulfilled in a given period all associated routing decision in this period must be modeled in dependance of the assignment of this request into this period.

The available request portfolio impacts the opportunity to compile routes that fulfill the maximal inbound load limitation per period. However, it is a priori unclear whether portfolios with a balanced or with an unbalanced distribution of the total system load cope better with the additional restriction related to the period selection.

An informal problem statement is presented in Section 2. The setup of the mathematical model is subject of Section 3. In order to validate the proposed model, we conduct several computational experiments whose setup as well as the corresponding results are reported in Section 4.

2. PROBLEM STATEMENT

2.1 Related Work

The here investigated vehicle routing setup addresses two CVRP-extending issues: The distribution of requests among different fulfillment periods (periodicity) and the balancing of the assigned load. Multi-period vehicle routing is addressed e.g. in Angelelli and Speranza (2002). Schwarze and Voß (2013) survey load balancing issues of vehicle routing.

Multi-period vehicle routing is triggered by customers requiring a regular visit but the requested time between two consecutive visits is longer than a single planning period (e.g. like in waste collection from private households (Angelelli and Speranza, 2002)). If the available fleet capacity is insufficient to fulfill the complete portfolio then shifting requests into other planning periods remedies the capacity shortage (Wen et al., 2010). To avoid excessive right-shifting of requests a maximal period number between the release period and the fulfillment period must not be exceeded (Archetti et al., 2015). Multi-period routing models are also used to identify suitable customer service frequencies (Francis and Smilowitz, 2006) over a longer time span.

In some applications the preservation of residual capacity to hedge against unexpected demand (Wen et al., 2010) as well as fairness issues require a balanced load distribution among resources. Employee satisfaction (Schwarze and Voß, 2013) and the need to realize a high utilization degree

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for expensive resources (Keskinturk and Yildirim, 2011) motivate the balancing of loads.

In vehicle routing the load balancing among vehicles is compromised by the need to minimize the total travel costs. Without any additional limitation to be considered it is tried to serve as much load by one vehicle in order to keep travel costs as low as possible (Schwarze and Voß, 2013). In case that requests can be shifted between several periods a similar problem occurs while assigning as much requests as possible into a single period in order to serve these requests by few vehicles. A quite imbalanced load distribution over the available periods occurs. However, the movement of a complete route without modification into another period sometimes contribute to the achievement of a more balanced load distribution without causing additional costs.

Compared to the situation of load balancing among vehicles (*resource balancing*), it is necessary to modify the underlying decision model in order to ensure load balancing over periods (*temporal balancing*). Typically, load balancing issues are addressed in the objective function formulation: deviations from the average load are penalized (Wen et al., 2010; Keskinturk and Yildirim, 2011) or the minimization of the maximal load assigned to one of the vehicles in the fleet is addressed (Schwarze and Voß, 2013). Another approach is to specify load balancing constraints but for heterogeneous request portfolios a suitable "target" load can hardly be specified prior to the route composition. Therefore, a constraint-based load balancing can only be realized by specifying period-specific lower and upper bounds on the total payload assigned to a period as realized in inventory routing problems (IRP) survey by Bertazzi and Speranza (2012). In the IRP a carrier is eligible to decide on the quantity to be delivered to a customer site in a period. The carrier has to take care that the maximal storage capacity at the customer site is not exceeded and stock-out situations are prevented (minimal load). However, the lower as well as the upper bound is specified for a customer (site) but not for a period.

2.2 The Investigated Multi-Period Collection Problem

The considered part of the time axis is partitioned into N^P equidistant intervals (periods). A facility processes material provided by several supply sides in the neighborhood. Each individual supply side provides a certain quantity per period. The service facility maintains a homogeneous fleet of vehicles (truck capacity is L^{max}) to collect the raw materials from the supply sides and bring them to the central facility. Routes originating and terminating at the central service facility and visiting each customer location must be compiled like in the CVRP for each period.

A severe variation of the inbound load at the central service facility from period to period reduces its longer term efficiency (reconfiguration costs as well as extra payments for overtimes). For this reason, the collection processes have to take care of balancing the inbound load at the central processing facility over periods.

Instead of picking up supply quantities immediately after their provision at the beginning of the first period selected supply sites are visited some periods later. The postponement of selected pickups enables the consideration of a maximal inbound load quantity accepted by the central facility per period. Whenever the sum of all released supply quantities exceeds this threshold, then some requests are right shifted into a later period. However, each supplier receives the guarantee that a specified supply quantity is finally collected within the N^P periods.

The planning goal is to minimize the sum of the travel distances of the routes specified for all periods. All pickup requests have to be served but the maximal inbound load per period at the central facility must not be exceeded in any period.

3. MATHEMATICAL MODEL

We define $\mathcal{P} := \{1, 2, \ldots, N^P\}$ to be the set of the N^P periods to be covered and $p \in \mathcal{P}$ denotes a period. Vehicle routes are compiled from the node set $\mathcal{C} := \{0, 1, 2, \ldots, N^C, N^C + 1\}$. All customer locations are collected in the set $\mathcal{C}^* := \mathcal{C} \setminus \{0, N^C + 1\}$. The depot from which all vehicle routes originate is represented by node $i = 0$ but node $i = N^C + 1$ represents the (same) depot in which all vehicle routes terminate. The homogeneous fleet $\mathcal{F}$ comprises the vehicles $f \in \{1, 2, \ldots, N^F\}$. Let $\mathcal{M}$ be a sufficiently large integer number.

An individual collection request r is specified by the ordered pair $r = (i_r; q_r)$. The first component determines the node $i_r \in \mathcal{C}$ where the quantity q_r is available. All requests are collected in the set $\mathcal{R}$. It takes $s(i_r)$ time units to pickup the quantity q_r at i_r. The travel time between node i and node j is stored in $t(i; j)$. The maximal duration of a route is D^{max} time units. The maximal inbound load per period that is carried to the central facility (depot) is IBL^{max}, and the travel distance between node i and node j is stored in $d(i; j)$. Two major decisions must be made. The fulfillment period must be determined for all requests (*shifting decisions*) and the vehicle routes to be executed in each period must be compiled (*routing decisions*). Both the shifting as well as the routing decisions are interdependent. In case that the fulfillment of a request is shifted into a certain period p then a vehicle has to visit the associated location in p and vice versa. In order to ensure consistent routing and shifting decisions we deploy the following five decision variable families.

- $x_{ijfp} \in \{0; 1\}$: $x_{ijfp} = 1 \Leftrightarrow$ vehicle f travels between node $i \in \mathcal{C}$ and node $j \in \mathcal{C}$ in period p.
- $pt_{rp} \in \{0; 1\}$: $pt_{rp} = 1 \Leftrightarrow$ request r is picked up in period p.
- $y_{ip} \in \{0; 1\}$: $y_{ip} = 1 \Leftrightarrow$ customer location i is visited in period p.
- $z_{rfp} \in \{0; 1\}$: $z_{rfp} = 1 \Leftrightarrow$ vehicle f is assigned to request r to be served in period p.
- $at_{ifp} \in R_{\geq 0}$: arrival time of vehicle f at customer location i in period p.
- $lt_{ifp} \in R_{\geq 0}$: arrival time of vehicle f from customer location i in period p.

The following linear constraints (1)-(16) determine the set of feasible solutions for the here investigated multi-period vehicle routing scenario.

$$\sum_{p \in \mathcal{P}} pt_{rp} = 1 \ \forall r \in \mathcal{R} \tag{1}$$