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Brief paper

Decentralized uniform input-to-state stabilization of hierarchically interconnected triangular switched systems with arbitrary switchings[☆]

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ABSTRACT

We consider a class of large-scale systems composed of hierarchically interconnected switched nonlinear triangular form subsystems affected by external disturbances with arbitrarily varying switching signals. For any system of this class, we design a decentralized feedback controller which renders the entire large-scale closed-loop system globally ISS with respect to the external disturbances uniformly and regardless of the unknown switching signals. To solve the problem, we use a certain modification of the classical small gain theorems formulated in terms of the ISS Lyapunov functions and combine it with our version of the backstepping approach with a suitable gain assignment.

1. Introduction

The theory of control and stability of nonlinear large-scale and multi-agent systems has been gaining popularity over the past 15 years and it is important in many applications (Abdessameud, Tayebi, & Polushin, 2012; Dashkovskiy, Kosmykov, Mironchenko, & Naujok, 2012; Dashkovskiy, Rüffer, & Wirth, 2007; Liu & Jiang, 2013). The essential characteristics of multi-agent control systems are autonomy and decentralization: each agent should be self-aware whereas it is not always possible to observe the entire system due to its complexity, for instance. This naturally leads to the problem of decentralized control (Krishnamurthy & Khorrami, 2003; Mehraeen, Jagannathan, & Crow, 2011a, b).

One efficient tool for solving such problems is the small gain approach based on small gain theorems (Liu & Jiang, 2013). In Jiang, Teel, and Praly (1994), the classical small gain theorems for two interconnected systems were obtained, which led to many fruitful results, e.g. Ito, Pepe, and Jiang (2010) and Karafyllis and Tsinias (2004). Next significant breakthrough was getting new small gain theorems for $N \geq 2$ interconnected nonlinear systems (Dashkovskiy et al., 2007; Dashkovskiy, Rüffer, & Wirth, 2010; Jiang & Wang, 2008), which were extended in various directions (Dashkovskiy et al., 2012; Geiselhart & Wirth, 2015; Liu & Jiang, 2013).

Another topic, which has been gaining popularity over the past decade, is motivated by the theory of switched systems (Efimov, Loria, & Panteley, 2011; Liberzon, Hespanha, & Morse, 1999; Liberzon & Morse, 1999; Sun & Ge, 2005) and by some fundamental results on the uniform stability of switched systems (Mancilla-Aguilar & Garcia, 2001). It is known that some trajectories of a switched system can diverge while each constant switching signal produces some globally asymptotically stable system of ODE (Liberzon & Morse, 1999). Then, it is natural to explore whether the classical designs of stabilizers (e.g. backstepping) can be extended to the problem of uniform stabilization of switched systems by means of switching-independent stabilizers (Dashkovskiy & Pavlichkov, 2012; Long & Zhao, 2014; Ma & Zhao, 2010). Dashkovskiy and Pavlichkov (2012) deal with "centralized" uniform switching-independent stabilization of large-scale interconnected switched systems in general triangular form, i.e., each agent should know all the components of all the states of the other agents. For switched systems in strict-feedback form with dynamic uncertainties, the problem of stabilization was tackled in Long and Zhao (2014). However, first, Long and Zhao (2014) deal with interconnections of *two* switched subsystems only similarly to the classical result from Jiang et al. (1994) devoted to the ODE case, and, second, Long and Zhao (2014) address *stabilization under certain dwell-time conditions.* Contrarily, the main result of our current work provides the uniform stabilization in presence of arbitrary switching signals without any dwell-time conditions and

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our tool will be a suitable modification of Dashkovskiy et al. (2010) *for* $N \geq 2$ *interconnected switched systems*. As in many other works devoted to decentralized control (Krishnamurthy & Khorrami, 2003; Mehraeen et al., 2011a, b), we assume that our network has a certain specific structure of interconnections. This structure is the same as in Mehraeen et al. (2011a) and its engineering motivation is given, for instance, in Mehraeen et al. (2011b). However, while Mehraeen et al. (2011a, b) deal with ODE systems without switchings, our problem formulation addresses the case of *uniform, switching-independent ISS stabilization of interconnected switched systems,* which is more general, and our main tool differs from Mehraeen et al. (2011a, b).

2. Preliminaries and main definitions

Throughout the paper, $\langle\cdot,\cdot\rangle$ denotes the scalar product in $\mathbb{R}^N$ and $|\xi| := \langle\xi,\xi\rangle^{\frac{1}{2}}$ denotes the quadratic norm of $\xi \in \mathbb{R}^N$. All vectors from $\mathbb{R}^N$ are treated as columns, i.e., $\mathbb{R}^N \cong \mathbb{R}^{N\times 1}$. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{K}$, if it is continuous, strictly increasing and $\alpha(0) = 0$, and it is said to be of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A continuous function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to be positive definite, if $\alpha(r) = 0$ implies $r = 0$. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is said to be of class $\mathcal{KL}$ if for each fixed $t \geq 0$ we have $\beta(\cdot, t) \in \mathcal{K}$ and for each fixed $s \geq 0$ we have $\beta(s, t) \to 0$ as $t \to +\infty$ and $t \mapsto \beta(s, t)$ is strictly decreasing for each $s > 0$ and $\forall t \geq 0$ $\beta(0, t) = 0$. We say that $V : \mathbb{R}^n \to \mathbb{R}_+$ is positive definite if $V(x) = 0 \Leftrightarrow x = 0 \in \mathbb{R}^n$, and it is radially unbounded, if, in addition, $\exists\alpha \in \mathcal{K}_\infty$ s.t. $\forall\, x \in \mathbb{R}^n$ $V(x) \geq \alpha(|x|)$.

Consider the following nonlinear switched system

$$\dot{x}(t) = F_{\sigma(t)}(t, x(t), D(t)), \quad t \in \mathbb{R}, \tag{1}$$

with states $x \in \mathbb{R}^n$, piecewise constant switching signals $\mathbb{R} \ni t \mapsto \sigma(t) \in \{1, \ldots, M\}$, and external disturbance inputs $D(\cdot) \in L_\infty(\mathbb{R}; \mathbb{R}^N)$, where each F_σ is continuous w.r.t. (t, x, D) and is locally Lipschitz continuous w.r.t. (x, D) for each fixed $\sigma \in \{1, \ldots, M\}$. Let us note that our main result and its proof are the same in two cases: piecewise constant switching signals and just measurable switching signals. For all $(t_0, x^0) \in \mathbb{R} \times \mathbb{R}^n$, $D(\cdot) \in L_\infty$, and piecewise constant $\sigma(\cdot)$ by $t \mapsto x(t, t_0, x^0, D(\cdot), \sigma(\cdot))$ we denote the trajectory of (1) with $x(t_0) = x^0$, $D = D(t)$, $\sigma = \sigma(t)$.

Definition 1. System (1) is said to be uniformly input-to-state stable (UISS) (at the origin $x^* = 0 \in \mathbb{R}^n$) if there are $\beta \in \mathcal{KL}$, and $\gamma \in \mathcal{K}$ such that for each $t_0 \in \mathbb{R}$, each $x^0 \in \mathbb{R}^n$, each $D(\cdot) \in L_\infty(\mathbb{R}; \mathbb{R}^N)$ and each piecewise constant $t \mapsto \sigma(t) \in \{1, \ldots, M\}$ we obtain

$$|x(t, t_0, x^0, D(\cdot), \sigma(\cdot))| \leq \max\{\beta(|x^0|, t - t_0),$$
$$\gamma(\|D(\cdot)\|_{L_\infty[t_0,+\infty[})\} \qquad \text{for all } \ t \geq t_0. \tag{2}$$

For ODE case, the notion of ISS was introduced in Sontag (1989) and Definition 1 for switched systems was given in Mancilla-Aguilar and Garcia (2001).

Remark 1. It is easy to show that one of the sufficient conditions for the UISS property is as follows: there are a positive definite and radially unbounded ISS Lyapunov function $V(x)$ of class C^1 and a gain $\hat{\gamma}(\cdot) \in \mathcal{K}$ s.t.

$$\forall x \in \mathbb{R}^n \quad \forall t \in \mathbb{R} \quad \forall D \in \mathbb{R}^N \quad V(x) \geq \hat{\gamma}(|D|) \Rightarrow$$
$$\forall\sigma \in \{1, \ldots, M\} \quad \nabla V(x) F_\sigma(t, x, D) \leq -\alpha(V(x)) \tag{3}$$

with some continuous and positive definite $\alpha(\cdot) : [0, +\infty[\ \to [0, +\infty[$. If each F_σ is time-invariant, i.e., $\forall\sigma$ $F_\sigma = F_\sigma(x, u)$, then one should omit the quantifier $\forall t \in \mathbb{R}$ in (3) and in Definition 1, and one can put $t_0 = 0$ in (2).

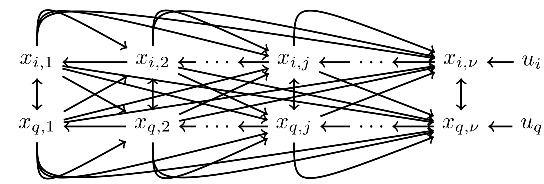

Fig. 1. Structure of interconnections of system (4) for $\nu_i = \nu$.

Remark 2. By Hadamard's lemma, we call the following simple fact: if $F \in C^{\mu+1}(\mathbb{R}^N; \mathbb{R})$, then $F(\xi) - F(\eta) = \Phi(\xi, \eta)(\xi - \eta)$, $\xi \in \mathbb{R}^N$, $\eta \in \mathbb{R}^N$, where $\Phi(\xi, \eta) = \int_0^1 \nabla F(\eta + s(\xi - \eta))ds$ is of class C^μ. (Because $F(\xi) - F(\eta) = \int_0^1 \left[\frac{d}{ds}F(\eta + s(\xi - \eta))\right] ds$.)

3. Main results

We consider a large-scale switched control system in the following form

$$\begin{aligned} &\dot{x}_{i,j} = f_{i,j}(x_{i,1}, \ldots, x_{i,j+1}) + \Delta_{i,j,\sigma(t)}(\theta, \overline{X}_j, D(t)),\\ &j = 1, \ldots, \nu_i - 1,\\ &\dot{x}_{i,\nu_i} = f_{i,\nu_i}(x_{i,1}, \ldots, x_{i,\nu_i}, u_i) + \Delta_{i,\nu_i,\sigma(t)}(\theta, \overline{X}_{\nu_i}, D(t));\\ &i = 1, \ldots, N, \end{aligned} \tag{4}$$

(with ν_i equations in each ith subsystem) with state vector components $x_{i,j} \in \mathbb{R}^{m_{i,j}}$ (with $m_{i,j} \leq m_{i,j+1}$), controls $u_i = x_{i,\nu_i+1} \in \mathbb{R}^{m_{i,\nu_i+1}}$, external disturbances $D(\cdot) \in L_\infty(\mathbb{R}; \mathbb{R}^{l_0})$, piecewise constant switching signals $\mathbb{R} \ni t \mapsto \sigma(t) \in \{1, \ldots, M\}$ and unknown parameters $\theta \in \mathbb{R}^\varkappa$, where $X_{i,p} := [x_{i,1}^\top, \ldots, x_{i,p}^\top]^\top$ for all $p = 1, \ldots, \nu_i$, $i = 1, \ldots, N$, and

$$\overline{X}_p := \begin{bmatrix} X_{1,\min\{p,\nu_1\}} \\ \ldots \\ X_{N,\min\{p,\nu_N\}} \end{bmatrix} \quad \text{for all } p = 1, \ldots, \max_{1\leq i\leq N} \nu_i. \tag{5}$$

As we mentioned above, some specific restrictions for the structure of interconnections are needed in such problems. For instance, if the dynamics of $x_{i,p}$ were allowed to depend on $x_{j,p+1}$ $(j \neq i)$ in (4) then we could not deal even with the problem of asymptotic stabilization. As a counterexample consider the system $\dot{x}_{1,1} = x_{1,2} - x_{2,2}$, $\dot{x}_{1,2} = u_1$, $\dot{x}_{2,1} = x_{2,2} - x_{1,2}$, $\dot{x}_{2,2} = u_2$. This system cannot be asymptotically stabilized even by a "centralized" feedback because its any trajectory satisfies $x_{1,1}(t) + x_{2,1}(t) = \text{const}$. The structure of interconnections in (4) for any two subsystems of (4) is depicted in Fig. 1 and it is the same as in Mehraeen et al. (2011a, b). (Note that Mehraeen et al. (2011a, b) address ODE systems without any switching signals.) Mehraeen et al. (2011b) provide an explicit engineering and physical motivation both for our problem formulation and for Mehraeen et al. (2011a, b).

We assume that system (4) satisfies the following conditions:

(I) All the functions $f_{i,j}$, $\Delta_{i,j,\sigma}$ are of class $C^{\nu+1}$, where $\nu := \max_{1\leq i\leq N}\{\nu_i\}$ and $f_{i,j}(0) = \Delta_{i,j,\sigma}(\theta, 0) = 0 \in \mathbb{R}^{m_{i,j}}$ for every θ s.t. $|\theta| \leq \theta^*$.

(II) For each $i = 1, \ldots, N$ and each $j = 1, \ldots, \nu_i$, the function $x_{i,j+1} \mapsto f_{i,j}(x_{i,1}, \ldots, x_{i,j}, x_{i,j+1})$ is right invertible, i.e., there is a map $(x_{i,1}, \ldots, x_{i,j}, w) \mapsto \alpha_{i,j}(x_{i,1}, \ldots, x_{i,j}, w)$ of class C^ν with $\alpha_{i,j}(0) = 0$ such that $f_{i,j}(x_{i,1}, \ldots, x_{i,j}, \alpha_{i,j}(x_{i,1}, \ldots, x_{i,j}, w)) = w$ for all $x_{i,1} \in \mathbb{R}^{m_{i,1}}$, $\ldots$, $x_{i,j} \in \mathbb{R}^{m_{i,j}}$, $w \in \mathbb{R}^{m_{i,j}}$.

(III) There exists some *known* $\theta^* \geq 0$ such that $|\theta| \leq \theta^*$.

Our main result is summarized in the following theorem.

Theorem 1. *Suppose that system (4) satisfies Assumptions (I)–(III). Then there exists a decentralized feedback controller in the form* $u_i = \hat{u}_i(x_{i,1}, \ldots, x_{i,\nu_i})$ *of class* C^1 *such that* $\hat{u}_i(0) = 0$ *and such that the*