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Event-triggered control for stochastic nonlinear systems ☆

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ABSTRACT

In this work, we investigate the problem of event-triggered stabilization for a class of stochastic nonlinear systems. An event-triggered control (ETC) approach is proposed by introducing an additional internal dynamic variable. The presented event-triggered mechanism (ETM) can guarantee the existence of a positive lower bound on inter-event times (or called inter-execution times). In addition, the presented technique can ensure the second moment asymptotic stability of the closed-loop stochastic nonlinear system.

1. Introduction

For physical systems, feedback control is often implemented by embedded digital microprocessors which sometimes are small and have limited communicating capability and low computing power. In order to save communication and computation resources, different from the traditional digital control executed periodically, the ETC is proposed (see Årzén, 1999; Heemels, Johansson, & Tabuada, 2012; Heemels, Sandee, & Van De. Bosch, 2008; Postoyan, Tabuada, Nešić, & Anta, 2015; Tabuada, 2007; Wang & Lemmon, 2011 and references therein). In the event-triggered control systems, the control tasks are not executed periodically but only when some triggered conditions are satisfied.

A basic issue for the ETM is to avoid that the triggered conditions are satisfied infinite times in finite time, i.e., a Zeno phenomenon, which makes the ETM infeasible for practical implementation. When one considers the case of stochastic nonlinear systems, this task becomes quite difficult. For stochastic nonlinear systems, due to the existence of additive white noise disturbances, the straightforward adaptation of the existing ETMs used in deterministic nonlinear systems (e.g., Girard, 2015; Tabuada, 2007) may lead to a Zeno phenomenon. The reason is that the states of stochastic systems may exceed any bound in an arbitrarily small amount of time, which may make the adapted event-triggered conditions satisfied in an arbitrarily small amount of time, i.e., a minimum positive inter-execution time may not be guaranteed. Furthermore, the Hessian terms, which arise from the differential of stochastic processes, make it difficult to analyze the existence of a positive upper bound on inter-execution times for a designed ETM using certain quantities, e.g., the differential of $\|e(t)\|/\|x(t)\|$ in Tabuada (2007). Hence, there are few results for triggered control for stochastic nonlinear systems. The work (Quevedo, Gupta, Ma, & Yüksel, 2014) proposes a fixed threshold-based event-triggered anytime control method under packet drops. Nevertheless, Quevedo et al. (2014) only applies to discrete-time stochastic nonlinear systems. A triggered sampling policy is proposed for state-feedback controlled stochastic differential equations in Anderson, Milutinović, and Dimarogonas (2015). However, in order to exclude the sampling Zeno phenomenon, the sampling policy in Anderson et al. (2015) needs a strict condition that an upper bound of the norm of the sampling value is known.

In this note, by introducing an additional internal dynamic variable, an efficient ETM is proposed for a class of state-feedback stochastic nonlinear control systems. Different from the deterministic case (Girard, 2015), the internal dynamic variable is constructed by the bounded functions of certain statistics of the states distribution of the stochastic nonlinear systems. Under the presented ETM, we can prove the existence of the positive minimum inter-execution time and the second moment asymptotic stability of the corresponding closed-loop ETC system. In classical ETM (Tabuada, 2007) for the deterministic nonlinear case, by solving some special scalar differential equations, one can exclude the sampling Zeno phenomenon for a given compact set containing the initial value. In this note, different from Tabuada (2007), an

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explicit formula of a positive lower bound of inter-execution times is presented for any initial value by solving a constructed equation.

Notations: For a given vector x, x^T and $\|x\|$ denote the transpose of x and its Euclidean norm, respectively. For a given matrix X, X^T and $\|X\|$ denote the transpose of X and its trace norm, respectively. For real numbers a and b, $a \vee b$ and $a \wedge b$ denote the maximum and minimum of a and b, respectively. $\mathcal{C}^i$ is the space of all functions which have continuous ith order derivatives in their arguments. In this paper, the function classes $\mathcal{K}$, $\mathcal{K}_\infty$, $\mathcal{KL}$ are defined as usual. The notation $id(\cdot)$ stands for the identity mapping from $\mathbb{R}$ to $\mathbb{R}$. Let $\mathcal{F}_n^m$ denote the function set $\{K : \mathbb{R}^n \to \mathbb{R}^m \big| K \in \mathcal{C}^0 \text{ and } K(0) = 0\}$. If there is no ambiguity, we will write with x, e, x_k and e_k instead of $x(t)$, $e(t)$, $x(t_k)$ and $e(t_k)$ for any $t \geq 0$ and $k \in \mathbb{N}$.

2. Problem statement

Consider the following stochastic nonlinear control systems:

$$dx = f(x, u)dt + g(x, u)dw, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u = u(x) : \mathbb{R}^n \to \mathbb{R}^m$ is the input, w is an r-dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\geq 0}$ being a filtration, and P being a probability measure, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n\times r}$ are measurable functions with $f(0, 0) = 0_{n\times 1}$ and $g(0, 0) = 0_{n\times r}$.

Definition 1 (*Anderson et al., 2015*)**.** System (1) is said to be second moment asymptotically stable if there exists a function $\beta \in \mathcal{KL}$ such that

$$E(\|x(t)\|^2) \leq \beta(E(\|x(t_0)\|^2), t - t_0), t \geq t_0, x_0 \in \mathbb{R}^n.$$

Assume that there exists a state feedback controller $K \in \mathcal{F}_n^m$ such that the control signal $u = K(x)$ stabilizes the system (1) in second moment. The controller is implemented on a digital platform so that the actual input of system (1) is given by

$$u(t) = K(x_k), \quad t \in [t_k, t_{k+1}), \tag{2}$$

where t_k, $k = 0, 1 \cdots$, are the execution times determined by the ETM.

Let $e(t) = x_k - x(t)$, $t \in [t_k, t_{k+1})$. Then the closed-loop system (1)–(2) is transformed into

$$dx(t) = f(x, e)dt + g(x, e)dw, \quad t \in [t_k, t_{k+1}), \tag{3}$$

where $f(x, e) := f(x, K(x + e))$ and $g(x, e) := g(x, K(x + e))$.

The following assumption is given to guarantee the existence and uniqueness of the solution of system (3).

Assumption 1. (i) For every integer $n \geq 1$, there exists a positive constant L_n such that for all $x', x'', e', e'' \in \mathbb{R}^n$ with $\|x'\| \vee \|x''\| \vee \|e'\| \vee \|e''\| < n$,

$$\begin{aligned}&\|g(x', e') - g(x'', e'')\|^2 \vee \|f(x', e') - f(x'', e'')\|^2 \\ &\leq L_n(\|x' - x''\|^2 + \|e' - e''\|^2).\end{aligned} \tag{4}$$

(ii) There exists a positive constant H such that for all $x, e \in \mathbb{R}^n$,

$$\left|x^T f(x, e) + \frac{1}{2}\|g(x, e)\|^2\right| \leq H(\|x\|^2 + \|e\|^2), \tag{5}$$

$$\left| - e^T f(x, e) + \frac{1}{2}\|g(x, e)\|^2\right| \leq H(\|x\|^2 + \|e\|^2). \tag{6}$$

Remark 1. By (5) and (6), we obtain that $x^T f(x, e) - e^T f(x, e) + \|g(x, e)\|^2 \leq 2H(\|x\|^2 + \|e\|^2)$. Hence, Assumption 1 can be referred to as monotone growth and local Lipschitz conditions of the following stochastic system

$$\begin{aligned}&dx(t) = f(x, e)dt + g(x, e)dw, t \in [t_i, t_{i+1}) \\ &de(t) = -f(x, e)dt - g(x, e)dw, t \in [t_i, t_{i+1}) \\ &x(t_i) = x(t_i^-), \quad e(t_i) = 0, \quad i = 1, 2, \ldots\end{aligned} \tag{7}$$

These conditions are often used to obtain the global existence and uniqueness of the solutions of stochastic nonlinear systems (see Mao, 1997, Theorem 3.6).

To determine an ETM for system (3) such that this system is second moment asymptotically stable, as in the deterministic case (Tabuada, 2007), we make the following assumptions:

Assumption 2. There exist class $\mathcal{K}_\infty$ functions $\alpha_1, \alpha_2, \gamma, \alpha$, and a function $V(x) \in \mathcal{C}^2$ such that $\bar{\alpha}_1^{-1}$ is a concave function, $\alpha \circ \alpha_2^{-1}$ is a convex function, and for $\forall (x, e) \in \mathbb{R}^n \times \mathbb{R}^n$, the following inequalities

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \tag{8}$$

$$\mathcal{L}V(x) \leq \gamma(\|e\|) - \alpha(\|x\|) \tag{9}$$

hold, where $\bar{\alpha}_1(s) := \alpha_1(\sqrt{s})$, and the differential operator $\mathcal{L}$ is defined as in Mao (1997).

Assumption 3. There exist a convex function $\alpha_0 \in \mathcal{K}_\infty$ and a concave function $\gamma_0 \in \mathcal{K}_\infty$ satisfying

$$\alpha_0(\|x\|^2) \leq \alpha(\|x\|), \tag{10}$$

$$\gamma_0(\|e\|^2) \geq \gamma(\|e\|) \tag{11}$$

and $\gamma_0^{-1} \circ (\theta\alpha_0)(\cdot) \geq \eta \cdot id(\cdot)$ with constants $\eta > 0$ and $0 < \theta < 1$, where α and γ are defined as in Assumption 1.

Remark 2. If one can find a control law k and a positive definite function $V(x)$ such that Assumption 2 is satisfied with $\alpha(s) = \sum_{k=2}^{N} a_k s^k$ and $\gamma(s) = b_2 s^2 + b_1 s$, where the non-negative parameters a_k, $2 \leq k \leq N$ and b_1, b_2 such that $\sum_{k=2}^{N} |a_k|^2 > 0$ and $b_1^2 + b_2^2 > 0$, respectively, then the convex function α_0 and the concave function γ_0, satisfying Assumption 3, always exist, e.g., $\alpha_0(s) = \sum_{k=2}^{N} a_k s^{\frac{k}{2}}$ and $\gamma_0(s) = b_1\sqrt{s} + b_2 s$. The above discussed conditions hold for many classes of stochastic linear/nonlinear control systems, see Sections 4 and 5 for a discussion of the linear case and a nonlinear example, respectively.

Under Assumption 2, the adaptation of the deterministic case (Tabuada, 2007) leads to a ETM for system (3):

$$t_{k+1} = \inf\{t \geq t_k \big| \gamma(\|e(t)\|) \geq \theta\alpha(\|x(t)\|)\} \tag{12}$$

with a constant $0 < \theta < 1$. However, the Zeno phenomenon may occur with this type of ETMs, because for the stochastic system (3), the value of $\gamma(\|e(t)\|)$ may exceed the value of $\theta\alpha(\|x(t)\|)$ in an arbitrarily small amount of time. Hence, the existence of a minimum lower bound of inter-execution times $\tau_k = t_{k+1} - t_k$ may not be guaranteed by the ETM (12). Therefore, the direct adaptation of the ETM of the deterministic case (Tabuada, 2007) to the stochastic system (3) may not work. In addition, due to the Hessian terms in the differential operator, the analysis of the differential of $\frac{\|e(t)\|}{\|x(t)\|}$, used in the proof of the existence of a positive upper bound on inter-execution times in Tabuada (2007), is not easy. For these reasons, to guarantee the stability of system (3) in second moment, we propose a ETM based on $E(\|x(t)\|^2)$ and $E(\|e(t)\|^2)$ in this paper.

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