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Brief paper

# Periodic event-triggered robust output feedback control for nonlinear uncertain systems with time-varying disturbance ☆

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ABSTRACT

This paper investigates the periodic event-triggered robust output feedback control problem for a class of nonlinear uncertain systems subject to time-varying disturbance. By means of the feedback domination approach and disturbance compensation technique, a new framework of periodic event-triggered robust control strategy is developed in the form of output feedback, which encompasses a discrete-time event-triggering transmission scheme that is only intermittently monitored at sampling instants and a discrete-time output feedback controller consisting of a set of linear difference equations. The proposed robust method may reduce the communication resource utilization as compared to the non-event triggering schemes while maintaining a desirable closed-loop system performance even in the presence of a general class of time-varying disturbance and nonlinear uncertainties. The closed-loop system under the proposed control scheme is actually modeled as a hybrid system, and it is shown that the global practical stability of the closed-loop hybrid system is guaranteed by selecting a sufficiently large scaling gain and a sufficiently small sampling period. Finally, the experimental results on a DC–DC buck power converter are presented to illustrate the effectiveness of the proposed control approaches.



## 1. Introduction

In networked control systems, the bandwidth of communication network and computation resources are generally limited (Chen, Ho, & Huang, 2015; Guo, Ding, & Han, 2014; Lehman & Lunze, 2012; Wang & Lemmon, 2011), therefore the research on event-triggered control strategy has attracted a great deal of attentions in recent decades (Liu & Jiang, 2015; Postoyan, Tabuada, Nešić, & Anta, 2015; Tabuada, 2007; Velasco & Marti, 2003; Xing, Wen, Liu, Su, & Cai, 2017; Xiong, Yu, Patel, & Yu, 2016). Quite different from the conventional time-triggered control (Fridman, Seuret, & Richard, 2004; Khalil, 2004; Mao, Jiang, & Shi, 2010; Qian & Du, 2012; Zhang, Xin, & Xu, 2013), the main idea of the event-triggered control is to execute control calculation and communication tasks only when a pre-defined state-dependent condition is verified. Instead of only pursuing better control performance while ignoring resources utilization in the time-triggered control, the aim of event-triggered control is to reduce the resources utilization while retaining a satisfactory closed-loop control performance. Hence, the event-triggered control is more suitable in applications where low energy consumption is sought, or the communication is costly or limited (Guo et al., 2014; Mazo & Tabuada, 2011; Selivanov & Fridman, 2016; Sun, Yu, Chen, & Xing, 2015; Zhu, Jiang, & Feng, 2014).

Due to the significance claimed above, various event-triggered control strategies have been developed, such as continuous-time event-triggered control (Fang & Xiong, 2014; Lunze & Lehmann, 2010; Tabuada, 2007; Xing et al., 2017; Zhang, Feng, Yan, & Chen, 2014), self-triggered control (Anta & Tabuada, 2010; Dimarogonas, Frazzoli, & Johansson, 2012; Tahir & Mazumder, 2015; Velasco & Marti, 2003; Wang & Lemmon, 2009) and sampling-based event-triggered control (Heemels, Donkers, & Teel, 2013; Heemels, Postoyan, Donkers, Teel, Anta, Tabuada, & Nešić, 2015; Peng & Han, 2013; Peng & Yang, 2013; Wang, Postoyan, Nešić, & Heemels, 2016). Compared with the continuous-time event-triggered control and the self-triggered control, the sampling-based event-triggered control scheme does not need to continuously monitor the transmission scheme, and thereby, the Zeno-behavior can be naturally excluded. Toward that end, the sampling-based event-triggered control has received considerable attentions of investigations recently. For example, the sampling-based event-triggered output feedback control can be found in the literatures for linear

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systems (Heemels & Donkers, 2013; Zhang & Feng, 2014), stochastic systems (Wu, Gao, Liu, & Li, 2017). However, it is noticed that most of aforementioned works are concentrated on linear systems or nonlinear systems in the absence of disturbances and uncertainties. In practical industrial systems, various disturbances and plant uncertainties inevitably deteriorate the static and dynamic performances (Chen, Ballance, Gawthrop, & O'Reilly, 2000; Li & Liu, 2009; Sun, Yang, Zheng, & Li, 2016). Since the event-triggering condition is generally in the form of an inequality with respect to the norm of state or a well-defined state measurement error, the communication task is possibly executed frequently when the variation of the testing variable between two successive sampled instants cannot be neglected owing to the influence of disturbances and plant uncertainties. Consequently, the disturbances and plant uncertainties not only deteriorate the closed-loop system performances, but also cause unnecessary waste of communication resource. It has been reported that even when the disturbances are extremely small, there may be no positive minimum inter-event time for event-triggering mechanisms (Borgers & Heemels, 2014). As such, this paper aims to present an effective periodic event-triggered robust output feedback control approach for nonlinear systems particularly subject to nonlinear uncertainties and external time-varying disturbance.

By virtue of the disturbance observation/compensation technique and feedback domination approach, the problem of periodic event-triggered robust output feedback control is addressed in this paper for a class of nonlinear uncertain system with time-varying disturbance. A discrete-time observer consisting of a set of linear difference equations is put forward with a discrete-time transmission scheme to determine whether or not to transmit and compute the newest state estimates and control signal. The proposed transmission scheme is intermittently monitored at constant sampling instants. It is shown that the global practical stability of the hybrid closed-loop system is guaranteed by choosing a sufficiently large scaling gain and a sufficiently small sampling period.

The research motivations of the proposed approach are three folds: (1) the uncertain nonlinearities under consideration, which do not necessarily satisfy the Lipschitz conditions, are different from most of the existing works on event-triggered control; (2) since the disturbance observation/compensation technique is utilized, the closed-loop system shows not only strong disturbance rejection performance, but also the possibility to save communication resource in the presence of disturbances/uncertainties, and (3) by accurately discretizing the proposed continuous-time observer, the proposed robust output feedback controller can be written as a set of linear difference equations. Such a control design strategy is intuitive and straightforward for practical implementation via digital computers.

Even though the state and disturbance observer design and analysis are straightforward, the stability as well as performance analysis of the closed-loop hybrid systems consisting of discrete-time state/disturbance observer, composite control law subject to event-triggering scheme constraints, and the continuous-time nonlinear systems suffering from nonlinear uncertainties and time-varying external disturbance are indeed nontrivial. In particular, how the controller parameters will affect the stability as well as the ultimate bounds of the closed-loop hybrid systems is very crucial for practical controller parameter tuning, and also quite difficult due to lack of available analysis tools in the presence of time-varying disturbances and nonlinear uncertainties under consideration. As such, we strive to investigate the qualitative relationship between control parameters (including the sampling period $T$ and the scaling gain $h$) and the stability of the closed-loop hybrid systems. Furthermore, the ultimate bounds of the states of the closed-loop hybrid systems are finally expressed as a function of the control parameters. This provides an explicit indication on how the ultimate bounds will vary as the sampling period $T$ and the scaling gain $h$ change.

## Notations

Throughout this paper, the superscript $T$ represents the transpose. The symbol $I_{n\times n}$ represents the identity matrix with dimensions $n \times n$. The symbols $1_{n\times m} \in \mathbb{R}^{n\times m}$ and $0_{n\times m} \in \mathbb{R}^{n\times m}$ stand for matrices, where all elements are 1 and 0, respectively. The set of real numbers is denoted by $\mathbb{R}$. The set of nonnegative integers is denoted by $\mathbb{N}$. For a scalar $r \in \mathbb{R}$, its absolute value is denoted by $|r|$. Given a vector $a = (a_1, \dots, a_n)$, where $a_i \in \mathbb{R}$ for each $i = 1, \dots, n$, $\mathrm{diag}(a)$ denotes a diagonal matrix having the entries of $a$ on the main diagonal. Both the Euclidean norm of a vector and the corresponding induced matrix norm are denoted by $\|\cdot\|$. Given a symmetric matrix $P$, $\lambda_M(P)$ and $\lambda_m(P)$ denote the maximum and minimum eigenvalues of matrix $P$, respectively.

## 2. Preliminaries

In this part, some important lemmas are introduced. Firstly, Gronwall–Bellman Inequality is presented in the following lemma.

**Lemma 1** (Apostol, 1974)**.** *Let $\lambda : [a, b] \to \mathbb{R}$ be continuous and $\mu : [a, b] \to \mathbb{R}$ be continuous and nonnegative. If a continuous function $w : [a, b] \to \mathbb{R}$ satisfies*

$$w(t) \le \lambda(t) + \int_a^t \mu(s)w(s)ds$$

*for $a \le t \le b$, then in the same interval*

$$w(t) \le \lambda(t) + \int_a^t \lambda(s)\mu(s)e^{\int_s^t \mu(\tau)d\tau} ds.$$

Using Lemma 1, we have the following result.

**Lemma 2.** *Consider the following system*

$$\dot{\zeta}(\tau) = F(\zeta(\tau), \zeta(\tau_k)),\ \forall\, \tau \in [\tau_k, \tau_{k+1}),\ \tau_k = kT,\ k \in \mathbb{N}, \tag{1}$$

*where $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ . If $F(\zeta(\tau), \zeta(\tau_k))$ satisfies*

$$\|F(\zeta(\tau), \zeta(\tau_k))\| \le a_1\|\zeta(\tau) - \zeta(\tau_k)\| + a_2\|\zeta(\tau_k)\| + a_3(\tau_k), \tag{2}$$

*where $a_1$, $a_2$ are two positive constants, and $a_3(\tau_k)$ is a nonnegative function with respect to $\tau_k$. Then the following inequality holds*

$$\|\zeta(\tau) - \zeta(\tau_k)\| \le \frac{a_2\|\zeta(\tau_k)\| + a_3(\tau_k)}{a_1}\left(e^{a_1(\tau-\tau_k)} - 1\right),\quad \forall\, \tau \in [\tau_k, \tau_{k+1}),\ k \in \mathbb{N}.$$

**Proof.** See Appendix A.1. □

## 3. Main results

Consider the following dynamic system subject to unmatched nonlinear uncertainties and time-varying disturbances, depicted by

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) + \phi_i(t, x(t), u(t), w(t)), & i = 1, \dots, n-1, \\ \dot{x}_n(t) = u(t) + d(t) + \phi_n(t, x(t), u(t), w(t)), \\ y(t) = x_1(t), \end{cases} \tag{3}$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n$, $y(t) \in \mathbb{R}$ and $u(t) \in \mathbb{R}$ are the state vector, the measurement output and the control input, respectively. $\phi_i$ $(i = 1, \dots, n)$ are continuous nonlinear uncertainties with disturbances $w(t) \in \mathbb{R}^m$, which are composed of matched term $\phi_n$ and unmatched terms $\phi_i$ $(i = 1, \dots, n-1)$. An "unmatched" condition means that the uncertainty enters the system through different channels from those of the control