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Brief paper

Necessary and sufficient conditions for Pareto optimality of the stochastic systems in finite horizon☆

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ABSTRACT

This paper is concerned with the necessary and sufficient conditions for the Pareto optimality in the finite horizon stochastic cooperative differential game. Based on the necessary and sufficient characterization of the Pareto optimality, the problem is transformed into a set of constrained stochastic optimal control problems with a special structure. Utilizing the stochastic Pontryagin minimum principle, the necessary conditions for the existence of the Pareto solutions are put forward. Under certain convex assumptions, it is shown that the necessary conditions are also sufficient ones. Next, we study the indefinite linear quadratic (LQ) case. It is pointed out that the solvability of the related generalized differential Riccati equation (GDRE) provides the sufficient condition under which all Pareto efficient strategies can be obtained by the weighted sum optimality method. Two examples shed light on the effectiveness of theoretical results.

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1. Introduction

Game theory (Basar & Olsder, 1999; Neumann & Morgenstern, 1944) deals with the problem of cooperation or competition between/among players, whose key feather is the presence of two or more players in a situation where each player designs his/her strategy by taking into account the decisions of the other players. It has been widely applied to study various problems in many fields, such as industry, economics, ecology, management, see Dockner, Jørgensen, Long, and Sorger (2000) and the references therein. In general, according to whether the players can reach a binding agreement, the game is divided into cooperative game and noncooperative game. In a noncooperative game, the players act independently in the pursuit of their own best interests. Nash, minmax and leader–follower are main strategies in dealing with the noncooperative game. In contrast to noncooperative game, cooperate game is far less developed. Pareto optimality plays a crucial role in analyzing cooperative differential game. In the past decades, the Pareto optimality has been widely used in various economic theories such as optimal economic growth, environmental economics and engineering (Acemoglu, 2008; Basar & Olsder, 1999; Dockner et al., 2000; Ramsey, 1928). This problem has been extensively studied for the deterministic systems (Engwerda, 2008, 2010; Reddy & Engwerda, 2013, 2014). Engwerda (2008) determined the set of Pareto efficient strategies for the regular convex cooperative differential game of linear affine systems, in which the time horizon may be either finite or infinite. Engwerda (2010) presented necessary and sufficient conditions for the existence of the Pareto solutions for the finite horizon cooperative differential game of nonlinear systems. Furthermore, the obtained results were used to analyze the LQ case and the scalar case, respectively. Reddy and Engwerda (2013) derived the conditions for the existence of the Pareto optimal solutions for the LQ infinite horizon cooperative differential games, and further clarified the relationship between the Pareto optimality and the weighted sum minimization. Reddy and Engwerda (2014) extended the existing finite horizon framework (Engwerda, 2010) to the infinite horizon case.

In recent years, there is an increasing interest in the consideration of the Pareto optimality for a wider range of systems (Chen & Ho, 2016; Chen, Lee, & Wu, 2015; Mukaidani, 2013; Mukaidani & Xu, 2009; Zhang, Lin, & Xue, 2017). For the multiobjective H_2/H_∞ filtering design problem, by using a stochastic T–S fuzzy system to approximate the original nonlinear signal systems, Chen et al. (2015) developed an LMI-based multiobjective evolution

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algorithm (MOEA) to derive the Pareto optimal solutions of the nonlinear signal systems. In order to achieve the simultaneous optimization, Chen and Ho (2016) used the LMI-based MOEA to efficiently search the set of Pareto optimal solutions for the multiobjective H_2/H_∞ tracking controller design of the stochastic T–S fuzzy systems. Mukaidani and Xu (2009) obtained the decentralized stochastic Pareto optimal static output feedback strategy for a class of weakly coupled systems with state-dependent noise in infinite horizon. Mukaidani (2013) discussed Pareto and Nash games for a class of linear stochastic delay systems governed by Itô's stochastic differential equation, respectively. Zhang et al. (2017) considered the finite horizon LQ Pareto optimal control problem of the stochastic singular systems. It should be noted that most of the existing works only research the Pareto optimality for the regular convex LQ case. Therefore, Pareto optimality should be considered for more general case.

Motivated by the above discussion, in this paper, we study the Pareto game of the stochastic Itô systems in finite horizon. It may be viewed as an extensive research of Engwerda (2010), in which the Pareto optimality was studied for the deterministic systems. Due to the definition of the solution of Itô equations, the adaptability of the solution with respect to the information flow $\{\mathcal{F}_t\}_{t\geq 0}$ should be considered. Thus, compared with Engwerda (2010), the Pareto game of the stochastic systems becomes substantially more difficult to be solved. In addition, the presence of the control in the diffusion term makes the Pareto game of the stochastic systems significantly different from the deterministic one. In the Pareto game of the deterministic systems, the control weighting matrix in the cost functional has to be positive definite. However, in stochastic Itô systems, the control weighting matrix can even be negative definite. Hence, this work is not the routine extension of the deterministic counterpart at all. The main contributions of this paper are as follows: (i) For the nonlinear case, in view of the equivalent characterization of the Pareto optimality, the necessary conditions for the existence of the Pareto solutions are put forward by means of the stochastic Pontryagin minimum principle and the Lagrange multipliers technique. (ii) Conversely, under certain convex assumptions, it is shown that the necessary conditions are also sufficient. It should be noted that, in the historical development of the Pareto game, there are few results established on the existence conditions of the Pareto solutions for the stochastic systems. (iii) For the LQ case, we provide the sufficient conditions, under which, the Pareto efficient strategy is equivalent to the weighted sum optimal control. In addition, a generalized Lyapunov equation (GLE) is introduced and all Pareto solutions are obtained based on the solution of the GLE. Different from the regular requirements in Engwerda (2010, Section 3.2), the control weighting matrices in the cost functionals are allowed to be indefinite.

The rest of the paper is organized as follows. Section 2 presents both the necessary and sufficient conditions for the existence of the Pareto solutions for the nonlinear stochastic systems. Section 3 is devoted to exploring the indefinite LQ case. It gives the sufficient condition for us to calculate all Pareto efficient strategies by the weighted sum optimality method. Two examples are provided to illustrate the effectiveness of the obtained conclusions. Finally, Section 4 concludes the paper with some remarks.

Notation. $\mathcal{R}^n$: the space of all real n-dimensional vectors. $\mathcal{R}^{m\times n}$: the space of all $m\times n$ real matrices. $A > B$ (resp. $A \geq B$): $A-B$ is a real symmetric positive definite (resp. positive semi-definite) matrix. A^T: the transpose of matrix A. $\mathbb{E}(x)$: the mathematical expectation of x. e_i: the n-dimensional identity vector in which the ith entry is 1 and the others are 0. $\|x\|$: the Euclidean norm of vector x. $\mathcal{A} := \{\alpha = (\alpha_1,\ldots,\alpha_N) | \ 0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^N \alpha_i = 1\}$. $\bar{N} := \{1,\ldots,N\}$. $\bar{N}\backslash i$: the set $\bar{N}$ where i is lacking. $\mathcal{L}^2_{\mathcal{F}}(0,T;\mathcal{R}^n)$: the space of nonanticipative stochastic process $\varphi(t)\in\mathcal{R}^n$ with respect to (w.r.t.) an increasing σ-algebra $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying $\mathbb{E}\int_0^T \|\varphi(t)\|^2 dt < \infty$.

2. Necessary and sufficient conditions for the nonlinear case

In this section, we consider the cooperative differential game that N players decide to coordinate their actions with an intent to minimize their cost functionals. For player i, $i\in\bar{N}$, the cost functional

$$\begin{aligned} & J_i(u_1,\ldots,u_N,x_0) \\ & = \mathbb{E}\left\{\int_0^T f_i(t,x(t),u_1(t),\ldots,u_N(t))dt + h_i(x(T))\right\}, \end{aligned} \tag{1}$$

where $f_i : [0,T]\times\mathcal{R}^n\times\mathcal{R}^{m_1}\times\cdots\times\mathcal{R}^{m_N}\to\mathcal{R}$, $h_i:\mathcal{R}^n\to\mathcal{R}$, $i\in\bar{N}$ and $x(t)\in\mathcal{R}^n$ is the state vector of the following nonlinear stochastic system

$$\begin{cases} dx(t) = b(t,x(t),u_1(t),\ldots,u_N(t))dt \\ \qquad\quad + \sigma(t,x(t),u_1(t),\ldots,u_N(t))dw(t), \\ x(0) = x_0, \end{cases} \tag{2}$$

where $b,\sigma: [0,T]\times\mathcal{R}^n\times\mathcal{R}^{m_1}\times\cdots\times\mathcal{R}^{m_N}\to\mathcal{R}^n$, $u_i(t)\in\mathcal{R}^{m_i}$ is the control vector of player i, $i\in\bar{N}$, $w(t)$ is one-dimensional standard Wiener process that is defined on the complete filtered probability space $(\Omega,\mathcal{F},\mathcal{P};\mathcal{F}_t)$ with $\mathcal{F}_t=\sigma(w(s):0\leq s\leq t)$ and $x_0\in\mathcal{R}^n$ is the initial condition which is deterministic.

To have a well-posed problem, we introduce the following assumptions:

Hypothesis 1. *(A1) b, σ are continuously differentiable w.r.t. $(x,u_1,\ldots,u_N)$. b, σ are bounded by $C_1(1+\|x\|+\|u_1\|+\cdots+\|u_N\|)$. b_x, σ_x, b_{u_i}, σ_{u_i}, $i\in\bar{N}$ are bounded.*

(A2) f_i, $i\in\bar{N}$ are continuously differentiable w.r.t. $(x,u_1,\ldots,u_N)$, h_i, $i\in\bar{N}$ are continuously differentiable w.r.t. x. f_i, $i\in\bar{N}$ are bounded by $C_2(1+\|x\|+\|u_1\|+\cdots+\|u_N\|)^2$, h_i, $i\in\bar{N}$ are bounded by $C_2(1+\|x\|)^2$. f_{ix}, f_{iu_j}, $i,j\in\bar{N}$ are bounded by $C_2(1+\|x\|+\|u_1\|+\cdots+\|u_N\|)$, h'_i, $i\in\bar{N}$ are bounded by $C_2(1+\|x\|)$.

Since the players coordinate their actions, we denote the joint action by $u(t) := (u_1(t),\ldots,u_N(t))\in\mathcal{R}^m$ with $m=\sum_{i=1}^N m_i$. The set of all admissible controls is denoted by $\mathcal{U}$. In this section, we consider $\mathcal{U} = \mathcal{L}^2_{\mathcal{F}}(0,T;\mathcal{R}^{m_1})\times\cdots\times\mathcal{L}^2_{\mathcal{F}}(0,T;\mathcal{R}^{m_N})$, which is a convex subspace. Since we are interested in the joint minimization of the objectives of all players, the cost incurred by a single player cannot be minimized without increasing the cost incurred by other players. So, we consider solutions which cannot be improved upon by all the players simultaneously, i.e., the so-called Pareto optimal solutions.

Definition 2 (*Engwerda, 2008, Definition 1.1*)**.** Let $\mathcal{U}$ denote the set of admissible controls. Then $\hat{u}\in\mathcal{U}$ is called Pareto efficient if the set of the following inequalities

$$J_i(u,x_0)\leq J_i(\hat{u},x_0),\ i\in\bar{N},$$

do not allow for any solution $u\in\mathcal{U}$, where at least one of the inequalities is strict. The corresponding point $(J_1(\hat{u},x_0),\ldots,J_N(\hat{u},x_0))\in\mathcal{R}^N$ is called a pareto solution. The set of all Pareto solutions is called the Pareto frontier.

The objective of this section is to find the set of Pareto efficient strategies of the finite horizon stochastic cooperative differential game (1)–(2), which is denoted by problem (P). Lemma 3, given below, provides us an easy way to find Pareto efficient strategies. It points out that every control minimizing a weighted sum (where all weights are strictly positive and the sum is one) of the cost functionals of all players is Pareto efficient. So, varying the positive weights over the unit simplex, one obtains, in principle, different Pareto efficient strategies.

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