ARTICLE IN PRESS
Automatica ▮ (▮▮▮▮) ▮▮▮–▮▮▮
Contents lists available at ScienceDirect
Automatica
journal homepage: www.elsevier.com/locate/automatica

Technical communique

Event-triggered control for robust set stabilization of logical control networks☆

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ARTICLE INFO

Article history:
Received 21 July 2017
Received in revised form 10 April 2018
Accepted 4 June 2018
Available online xxxx

Keywords:
Logical control network
Event-triggered control
Robust control
Set stabilization
Algebraic state space representation

ABSTRACT

This paper addresses the robust set stabilization problem of k-valued logical control networks (KVLCNs) via the algebraic state space representation approach, and proposes an event-triggered control scheme. Based on the algebraic form of KVLCNs, a necessary and sufficient condition is presented for the robust set stabilization of KVLCNs via time-variant state feedback control. Moreover, the event-triggered control design problem is formulated, and a sufficient condition is presented to design state feedback event-triggered controllers for the robust set stabilization of KVLCNs.

1. Introduction

Event-triggered control was firstly proposed in the late 1990s (Aström & Bernhardsson, 1999). It consists of two basic elements: (i) a feedback controller which determines the control input, and (ii) a triggering mechanism which decides when the control input should be updated again Heemels, Donkers, and Teel (2013). The main advantage of event-triggered control is that it can greatly reduce the control execution times and save the computation costs. In the last two decades, event-triggered control has been investigated in many research fields such as nonlinear systems (Postoyan, Tabuada, Nesic, & Anta, 2015; Tabuada, 2007), networked control systems (Li, Modares, Lewis, Chai, & Xie, 2017; Wu, Meng, Xie, Lu, Su, & Wu, 2017), and so on.

As a special kind of networked control systems, k-valued logical control networks (KVLCNs) are widely used in gene regulatory networks (GRNs) (Kauffman, 1969), networked evolutionary games (Cheng, He, & Qi, 2015; Zhu, Xia, & Wu, 2016) and finite automata (Xu & Hong, 2013). Recently, an algebraic state space representation (ASSR) approach has been established for the analysis and control of KVLCNs via the semi-tensor product (STP) of matrices (Cheng, Qi, & Li, 2011; Lu, Li, Liu, & Li, 2017). The main feature of this approach is that one can convert the dynamics of KVLCNs into a bilinear form, which facilitates the use of classical control theory in KVLCNs. In the last decade, many excellent results have been obtained for the analysis and control of KVLCNs by using ASSR, which include stability and stabilization (Chen, Li, & Sun, 2015; Meng, Liu, & Feng, 2017), controllability and observability (Fornasini & Valcher, 2013; Liu, Chen, Lu, & Wu, 2015), invertibility and nonsingularity (Zhang, Zhang, & Xie, 2015), Kalman decomposition (Zou & Zhu, 2015), optimal control (Laschov & Margaliot, 2011; Wu & Shen, 2015), synchronization (Zhong, Lu, Liu, & Cao, 2014), etc.

Among the above issues in the study of KVLCNs, stabilization is a basic one and has some important applications such as gene therapeutic (Li, Yang, & Chu, 2013). There exist two kinds of stabilization problems in KVLCNs, that is, stabilization to an equilibrium (Liang, Chen, & Liu, 2017; Tian, Zhang, Wang, & Hou, 2017) and set stabilization (Guo, Wang, Gui, & Yang, 2015; Li, Li, Xie, & Zhou, 2017). When disturbance inputs are considered in the dynamics of KVLCNs, robust stabilization problem is proposed (Li, 2017; Li & Wang, 2017; Li, Xie, & Wang, 2016). It should be pointed out that all the existing results on the (robust) stabilization of KVLCNs just focus on the traditional feedback control method. As was pointed out in Li, Shen, Liu, and Alsaadi (2016); Yue, Guan, Tao, Liao,

☆ The work was supported by the National Natural Science Foundation of China under grant 61503225, the Natural Science Fund for Distinguished Young Scholars of Shandong Province under grant JQ201613, the Natural Science Foundation of Shandong Province under grant ZR2015FQ003, and the Research Innovation Fund for Graduate Student of Shandong Normal University under grant SCX201812. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Rifat Sipahi under the direction of Editor André L. Tits.

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https://doi.org/10.1016/j.automatica.2018.06.030
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Liu, and Lai (2017), in the study of GRNs, the traditional feedback control may lead to some undesirable phenomena such as continuous transmission of information between mRNA and protein, Zeno behavior, and so on, which will consume a large number of control execution times and computation costs. Motivated by this, Li et al. (2016) and Yue et al. (2017) developed the event-triggered control scheme for GRNs. Since logical control network is a classic model of GRNs, it is natural to introduce the event-triggered control for KVLCNs in order to reduce the control execution times and save the computation costs. To our best knowledge, there are fewer results on the robust set stabilization of KVLCNs by event-triggered control.

In this paper, using ASSR, we investigate the robust set stabilization problem of KVLCNs, and present a number of new results. The main contributions of this paper are as follows.

- An event-triggered control scheme is developed for the robust set stabilization of KVLCNs, which is effective in reducing the computation costs (see Remark 3).
- A constructive procedure is proposed to design state feedback event-triggered controllers for the robust set stabilization of KVLCNs. The procedure is computationally tractable via MATLAB.

The rest of this paper is organized as follows. Section 2 studies the robust set stabilization problem of KVLCNs. Section 3 formulates the triggering mechanism, and investigates how to design state feedback event-triggered controllers. Section 4 is a brief conclusion.

Notations: $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{Z}_+$ denote the sets of real numbers, natural numbers and positive integers, respectively. $\mathcal{D}_k := \{0, 1, \ldots, k-1\}$. $\mathcal{D}_k^n := \underbrace{\mathcal{D}_k \times \cdots \times \mathcal{D}_k}_{n}$. $\Delta_n := \{\delta_n^i : i = 1, \ldots, n\}$, where δ_n^i denotes the ith column of the identity matrix I_n. An $n \times t$ matrix M is called a logical matrix, if $M = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_t}]$, which is briefly expressed as $M = \delta_n[i_1\ i_2\ \cdots\ i_t]$. Denote the set of $n \times t$ logical matrices by $\mathcal{L}_{n\times t}$. $Col(A)$ denotes the set of columns of a matrix A. $Blk_i(A)$ denotes the ith $n \times p$ block of an $n \times mp$ matrix A. Throughout this paper, the default matrix product is STP ("$\ltimes$"). For the definition and properties of STP, please refer to (Cheng et al., 2011). Since STP is a generalization of the conventional matrix product, we omit the symbol "$\ltimes$" if no confusion arises in the following.

2. Robust set stabilization of KVLCNs

A k-valued logical control network with n nodes, m control inputs, p outputs and q disturbances is described as follows:

$$\begin{cases} x_i(t+1) = f_i(X(t), U(t), \Xi(t)),\ i = 1, \ldots, n; \\ y_j(t) = h_j(X(t)),\ j = 1, \ldots, p, \end{cases} \tag{1}$$

where $X(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in \mathcal{D}_k^n$, $U(t) = (u_1(t), \ldots, u_m(t)) \in \mathcal{D}_k^m$, $\Xi(t) = (\xi_1(t), \ldots, \xi_q(t)) \in \mathcal{D}_k^q$ and $Y(t) = (y_1(t), \ldots, y_p(t)) \in \mathcal{D}_k^p$ are states, control inputs, disturbance inputs and outputs at time t, respectively, and $f_i : \mathcal{D}_k^{n+m+q} \to \mathcal{D}_k$, $i = 1, \ldots, n$ and $h_j : \mathcal{D}_k^n \to \mathcal{D}_k$, $j = 1, \ldots, p$ are k-valued logical functions. Given an initial state $X(0) \in \mathcal{D}_k^n$, a control sequence $\{U(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^m$ and a sequence of disturbance inputs $\{\Xi(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^q$, denote the trajectory of system (1) by $X(t; X(0), U, \Xi)$.

The definition of robust set stabilization for system (1) is given as follows.

Definition 1. Given a nonempty set $A \subseteq \mathcal{D}_k^n$ and an initial state $X(0) \in \mathcal{D}_k^n$. System (1) is said to be robustly stabilizable to A, if there exist a control sequence $\{U(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^m$ and a positive integer τ such that

$$X(t; X(0), U, \Xi) \in A \tag{2}$$

holds for $\forall\, t \geq \tau$ and $\forall\, \{\Xi(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^q$.

In the following, we convert system (1) into an equivalent algebraic form based on STP.

Using the vector form of logical variables and setting $x(t) = \ltimes_{i=1}^n x_i(t) \in \Delta_{k^n}$, $u(t) = \ltimes_{i=1}^m u_i(t) \in \Delta_{k^m}$, $\xi(t) = \ltimes_{i=1}^q \xi_i(t) \in \Delta_{k^q}$ and $y(t) = \ltimes_{i=1}^p y_i(t) \in \Delta_{k^p}$, by STP, system (1) can be converted into the following equivalent algebraic form:

$$\begin{cases} x(t+1) = Lu(t)x(t)\xi(t), \\ y(t) = Hx(t), \end{cases} \tag{3}$$

where $L \in \mathcal{L}_{k^n \times k^{n+m+q}}$ is the state transition matrix, and $H \in \mathcal{L}_{k^p \times k^n}$ is the output matrix.

In this paper, we consider the following time-variant state feedback control:

$$u_i(t) = \psi_i(t, X(t)),\ i = 1, \ldots, m, \tag{4}$$

where $\psi_i : \mathbb{N} \times \mathcal{D}_k^n \to \mathcal{D}_k$, $i = 1, \ldots, m$ are time-variant k-valued logical functions. For each ψ_i, $i = 1, \ldots, m$, according to Cheng et al. (2011), there exists a unique structure matrix $\Psi_i(t, x(0)) \in \mathcal{L}_{k\times k^n}$ such that $u_i(t) = \Psi_i(t, x(0))x(t)$. Using the Khatri–Rao product of matrices, there exists a unique time-variant structure matrix $\Psi(t, x(0)) \in \mathcal{L}_{k^m\times k^n}$ such that

$$u(t) = \Psi(t, x(0))x(t), \tag{5}$$

where $\Psi(t, x(0)) = \Psi_1(t, x(0)) * \cdots * \Psi_m(t, x(0))$ is called the time-variant state feedback gain matrix, and "$*$" denotes the Khatri–Rao product of matrices.

Now, we give a criterion for the robust set stabilization of system (1) under the control (4).

Based on (3), using the swap matrix (Cheng et al., 2011), one can obtain the following equivalent form of system (1):

$$x(t+1) = \hat{L}\xi(t)u(t)x(t), \tag{6}$$

where $\hat{L} = LW_{[k^q, k^{m+n}]} \in \mathcal{L}_{k^n\times k^{n+m+q}}$, and $W_{[k^q, k^{m+n}]}$ is a swap matrix.

Split $\hat{L}$ into k^q equal blocks as

$$\hat{L} = [\hat{L}_1\ \hat{L}_2\ \cdots\ \hat{L}_{k^q}],$$

where $\hat{L}_i \in \mathcal{L}_{k^n\times k^{m+n}}$, $i = 1, 2, \ldots, k^q$. For any $i \in \{1, 2, \ldots, k^q\}$, split $\hat{L}_i$ into k^m equal blocks as

$$\hat{L}_i = [\hat{L}_i^1\ \hat{L}_i^2\ \cdots\ \hat{L}_i^{k^m}],$$

where $\hat{L}_i^j \in \mathcal{L}_{k^n\times k^n}$, $j = 1, 2, \ldots, k^m$.

Given a nonempty set $A \subseteq \Delta_{k^n}$ and $t \in \mathbb{Z}_+$, denote the t-step robust reachable set (Li & Wang, 2017) of A by $\Upsilon_t(A)$. Then, the structure of $\Upsilon_t(A)$ is given as follows.

$$\Upsilon_1(A) = \{\delta_{k^n}^\beta : \text{ there exists an integer } 1 \leq j_\beta \leq k^m \text{ such that } \sum_{\gamma\in A}\sum_{i=1}^{k^q}(\hat{L}_i^{j_\beta})_{\gamma,\beta} = k^q\}, \tag{7}$$

$$\Upsilon_t(A) = \{\delta_{k^n}^\beta : \text{ there exists an integer } 1 \leq j_\beta \leq k^m \text{ such that } \sum_{\delta_{k^n}^{\beta'}\in\Upsilon_{t-1}(A)}\sum_{i=1}^{k^q}(\hat{L}_i^{j_\beta})_{\beta',\beta} = k^q\},\ t \geq 2. \tag{8}$$

We have the following result.