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Brief paper

Self-triggered control with tradeoffs in communication and computation☆

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ABSTRACT

We study networked control of linear discrete-time systems using self-triggered strategies to reduce the amount of communication between the plant and the remote controller. At each transmission, the controller determines the next transmission time in advance based on the current state. We propose three self-triggered strategies which guarantee control performance based on a quadratic cost function. They have different characteristics with respect to the on-line computation load for finding the transmission times. Through a numerical example, we illustrate the effectiveness of the three strategies and, in particular, demonstrate the tradeoffs between computation loads and transmission frequencies.

1. Introduction

In recent years, the use of communication networks in control systems has drastically increased for connecting plants with controllers which may be remotely located (Bemporad, Heemels, & Johansson, 2010). Due to the shared nature of networks as well as the limited computation available in embedded devices, it is important to design such networked control systems with certain considerations to keep the communication and computation loads low. In this respect, conventional digital control techniques employing periodic sampling may not be ideal.

Reduction in communication can be achieved by activating transmissions only when it is necessary. This is the underlying idea in the strategies of event-triggered control and self-triggered control, which have lately gained much attention; see, e.g., Heemels, Johansson, and Tabuada (2012) and Mazo and Tabuada (2008) and the references therein. In event-triggered control, the state of the plant is continuously monitored, but only when the state value has sufficiently changed and satisfies certain conditions, communication is triggered for the controller to be updated (Cetinkaya, Ishii, & Hayakawa, 2017; Heemels, Donkers, & Teel, 2013; Lehmann, Henriksson, & Johansson, 2013; Tabuada, 2007). On the other hand, in self-triggered control, when the controller transmits the new control input to the plant, it is accompanied with the information regarding the next time instant when the sensor should send the state (Almeida, Silvestre, & Pascoal, 2014; Anta & Tabuada, 2010; Mahmoud & Memon, 2015; Mazo, Anta, & Tabuada, 2009; Souza, Deaecto, Geromel, & Daafouz, 2012; Wang & Lemmon, 2009). Since the transmission times are determined in advance, self-triggered control may require more communication in general compared to the event-triggered case. The advantage is however that (i) at the sensor side, continuous monitoring of the state is unnecessary, and moreover (ii) embedded devices can shut their communication until the next transmission time. This technique has also been explored in multi-agent systems for the communication to achieve coordination (De Persis & Frasca, 2013; Nowzari & Cortés, 2012).

In this paper, we study self-triggered control strategies for linear time-invariant systems in the discrete-time domain. Self-triggered state and output feedback control for discrete-time systems has been previously studied by Eqtami, Dimarogonas, and Kyriakopoulos (2010) and Zhang, Zhao, and Zheng (2015). Recently, Brunner, Gommans, Heemels, and Allgöwer (2015) and Henriksson, Quevedo, Peters, Sandberg, and Johansson (2015) explored self-triggered model predictive control. Moreover, Brunner, Heemels, and Allgöwer (2016) investigated self-triggering control under bounded disturbances, and Gommans, Antunes, Donkers, Tabuada, and Heemels (2014) studied self-triggered approach to linear quadratic control under random disturbances.

Here, we develop three self-triggered schemes that require different levels of on-line computation for finding the next transmission time at the controller during its operation. While all of

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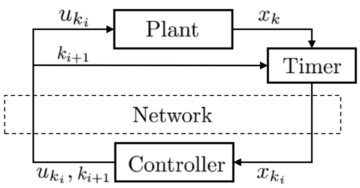

Fig. 1. Networked control system.

them are guaranteed to achieve prespecified control performance, they exhibit tradeoffs between the necessary computation and the length of waiting times before the next transmissions. Hence, depending on the system requirements and available resources, the most appropriate option should be chosen. The first strategy is based on computing the future state using the plant model for finding the next transmission times and is hence more computationally intensive. The second strategy requires much less on-line computation by using bounds on the state trajectories, but in general is more demanding in terms of communication. In the third one, referred to as the off-line type, the amount of on-line computation is further reduced by partitioning the state space into a finite number of regions, where each region has its corresponding transmission time. The idea of state space partitioning has been previously utilized for self-triggered control of continuous-time systems in Aminifar, Tabuada, Eles, and Peng (2016) and Fiter, Hetel, Perruquetti, and Richard (2012). The partitioning methods and hence the analysis in those works differ from the ones in this paper.

In all three strategies, we follow the sampled-data control method of Ishii and Francis (2002) studied in the context of quantized control. There, a Lyapunov-based approach is developed for finding the so-called dwell time in continuous time, which is in fact closely related to event-triggered control. This reference further considers quantization of the control input and how to reduce the data rate in communication. However, in this paper, we employ only the ideas for the sampling part of the results there. An interesting aspect is that the state space is projected on a two-dimensional space, which enables us to reduce the on-line computation in our strategies.

This paper is organized as follows. In Section 2, we formulate the networked control problem studied. In Section 3, we present a Lyapunov-based sufficient condition to guarantee the desired level of control performance. Sections 4–6 provide the details of the three self-triggered control strategies. We illustrate the results with a numerical example in Section 7. In Section 8, we give concluding remarks. The paper has appeared in a preliminary form in Akashi, Ishii, and Cetinkaya (2016), but contains the full proofs for the results as well as extended discussions on the methods and simulations.

2. Problem formulation

We introduce the system settings for the self-triggered control problem. Consider the networked control system depicted in Fig. 1. Here, the plant is a discrete-time linear time-invariant system with single input given by

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

where the state and the control input are denoted, respectively, by $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}$. Assume the pair (A, B) to be controllable.

This networked system can be described as follows: The sensor and the actuator on the plant side communicate with the remote controller over a network, which is free from latencies and packet losses. The objective is to reduce the number of transmissions over the network by means of self-triggered control in the discrete-time setting. Hence, though the sensor can measure the state at any time step $k \in \mathbb{Z}_+$, it is sent to the controller only at transmission times denoted by $k_i \in \mathbb{Z}_+$, $i \in \mathbb{Z}_+$, with $k_0 = 0$ and $k_i < k_{i+1}$. At time k_i, the controller broadcasts the control input together with the next transmission time k_{i+1} to the actuator and the sensor. The sensor will send the next measurement at time k_{i+1} while the same input is applied until then, that is,

$$u_k = u_{k_i} \text{ for } k = k_i, k_i + 1, \dots, k_{i+1} - 1. \tag{2}$$

To measure performance, we employ the cost function

$$J(x_0) := \sum_{k=0}^{\infty} (x_k' Q x_k + R u_k^2), \tag{3}$$

where the weight matrix $Q \in \mathbb{R}^{n\times n}$ is positive definite and $R > 0$. It is well known that with the state feedback $u_k = -Kx_k$, we achieve the optimal cost of $J_{\rm opt}(x_0) := x_0' P x_0$, where P is the unique positive-definite solution to the discrete-time algebraic Riccati equation

$$A'PA - P - A'PB(R + B'PB)^{-1} B'PA + Q = 0, \tag{4}$$

and the feedback gain is $K := B'PA/(R + B'PB)$. Let $V(x) := x'Px$ be the Lyapunov-like function.

With self-triggered control, we can reduce communication over channels, but in turn must relax the performance constraint. So, for a given $\epsilon > 0$, we design self-triggered control schemes to bound the cost as

$$J(x_0) \le (1 + \epsilon) J_{\rm opt}(x_0). \tag{5}$$

The problem of this paper is formulated as follows: For the networked control system in Fig. 1, design self-triggered control schemes that compute the next transmission time k_{i+1} at time k_i based on the information of past states and inputs available at the controller and achieve quadratic stability and the performance constraint (5) for the closed-loop system. Here, quadratic stability is meant to be with respect to $V(x)$, i.e., $V(x_k)$ is a decreasing function of time k.

In this work, we propose three self-triggered control schemes, where their difference lies in the necessary computational resources at the controller. Specifically, we will see that with more computation at the controller for computing the transmission times, better performance can be attained with respect to control and communication. We also note that there is a tradeoff between the achievable control performance determined by ϵ in (5) and the number of transmissions over the network.

3. A Lyapunov-based condition

Here, we derive a condition for the transmission times k_i, under which the networked control system is stable and the performance constraint (5) holds.

Let the Lyapunov difference be $\Delta V(x, u) := V(Ax + Bu) - V(x)$. From (4), this can be expressed as

$$\begin{aligned} \Delta V(x_k, u_k) &= V(x_{k+1}) - V(x_k) \\ &= x_k'(A'PA - P)x_k + 2x_k' A'PBu_k + B'PBu_k^2 \\ &= -x_k'\big[Q - K'(R + B'PB)K\big]x_k \\ &\qquad + 2u_k(R + B'PB)Kx_k + B'PBu_k^2 \\ &= -(x_k'Qx_k + Ru_k^2) + (R + B'PB)(u_k + Kx_k)^2. \end{aligned} \tag{6}$$

The lemma below provides a basic condition to be used in the self-triggered schemes.