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Nonsingular terminal sliding-mode control of nonlinear planar systems with global fixed-time stability guarantees ☆

Maria Letizia Corradini, Andrea Cristofaro *

School of Science and Technology, University of Camerino, Camerino (MC), Italy

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ABSTRACT

This paper proposes the use of a novel nonsingular Terminal Sliding surface for the finite-time robust stabilization of second order nonlinear plants with matched uncertainties. Mathematical characteristics of the proposed surface are such that a fixed bound naturally exists for the settling time of the state variable, once the surface has been reached. A simple redesign of the control input able to ensure the feature of fixed-time reaching of the sliding surface will be also presented, and fixed-time stability will be guaranteed by the proposed Terminal Sliding Mode Control design method. A careful simulation study has been performed using a benchmark system taken from the literature.

1. Introduction

The well known features characterizing Sliding Mode Control (SMC) (Utkin, 1992) of robustness against matched perturbations and computational simplicity have made this technique to be successfully applied in a number of different applicative contexts for the robust control of linear and nonlinear systems. The design procedure basically consists of two steps: first a surface is designed such that the reduced order system shows the desired properties, next a control law is determined ensuring the attainment of the sliding motion on the designed surface. Standard design methods rely on a linear sliding surface, therefore the associated sliding motion exhibits a linear behavior with convergence rate arbitrarily fast, tunable by means of the parameters of the siding surface itself. Though the sliding manifold can be reached in an arbitrary finite time, systems states cannot converge to zero in finite time. The so called Terminal Sliding Mode Control (TSMC) (Man & Yu, 1997) has been developed to fill this gap, and to achieve finite time convergence of the system dynamics. Generally speaking, finite-time stability and stabilization problems have been intensively studied for applications requiring severe time response constraints, see Bhat and Bernstein (2000), Moulay and Perruquetti (2006), Orlov (2009), and in observation problems when a finite-time convergence of the state estimate to the real values is required (Bejarano & Fridman, 2010; Menard, Moulay, & Perruquetti, 2010; Perruquetti, Floquet, & Moulay, 2008; Shen, Huang, & Gu, 2011). It is worth mentioning that standard TSMC shows singularity problems in some regions of the state space, thus hindering its adoption in real applications. Modified sliding surfaces have been proposed for second-order systems in Feng, Yu, and Man (2002) and Yang and Yang (2011) to avoid the singularity domain. A saturation function has been introduced in Feng, Yu, and Man (2013) for dealing with the singularity problem in the case of chained nonlinear systems with matched perturbations. A finite-time disturbance observer within TSMC design has been proposed in Yang, Li, Su, and Yu (2013) for dynamical systems with unmatched disturbances.

In addition to the property of finite-time stability, previously discussed, the extra requirement can be considered that a bound exists, for the finite settling time, independent of the initial condition. If this property holds, the system is said to be fixed-time stable (Polyakov & Fridman, 2014), since convergence to zero can be guaranteed with a predefined settling time, a priori known. The phenomenon of fixed-time stability was first discovered in the framework of differentiators design in the paper (Cruz-Zavala, Moreno, & Fridman, 2011), where a uniform exact differentiator is proposed for the first time and then extended in Angulo, Moreno, and Fridman (2013). Fixed-time stabilization has been studied in Polyakov (2012) with reference to uncertain linear plants, while the control design problem of the robust finite-time and fixed-time stabilization of a chain of integrators by the Implicit Lyapunov Function method has been solved in Polyakov, Efimov, and Perruquetti (2015). Two uniform sliding mode controllers for a second-order uncertain system have been proposed in Cruz-Zavala, Moreno, and Fridman (2012) providing convergence to an

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* Corresponding author.
E-mail addresses: letizia.corradini@unicam.it (M.L. Corradini), andrea.cristofaro@unicam.it (A. Cristofaro).

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arbitrarily small set centered at the origin in finite time, which can be bounded by some constant independent of the initial conditions and uncertainties.

In the recent paper by Zuo (2014), a previously presented framework addressing finite-time stability for second-order nonlinear plants has been extended to guarantee fixed-time stability in the presence of matched perturbations, using a variant of the standard sliding manifold for TSMC together with a continuous sinusoidal function introduced to eliminate the singularity.

In this paper, the problem of designing a fixed-time terminal sliding surface for second order nonlinear systems, possibly affected by matched perturbations, is approached starting from a different characterization. After the formal definition of the mathematical features of the surface needed for ensuring the attainment of a terminal sliding motion, hence finite-time stability, a proposal is made of a sliding surface drastically novel with respect to previous proposals available in the literature. A redesign of the control input is also proposed able to ensure the feature of fixed-time reaching of the sliding surface. The second order Single Inverted Pendulum (SIP) system, proposed in Zuo (2014), has been used as benchmark testbed, and a comparative analysis has been performed with previous available results on this system.

2. Problem statement

Consider the following continuous-time, time invariant, uncertain second order plant described by:

$$\dot{x}_1(t) = x_2(t) \tag{1}$$

$$\dot{x}_2(t) = f(t,\mathbf{x}) + g(t,\mathbf{x})(u(t) + d(t,\mathbf{x})) \tag{2}$$

where: $\mathbf{x}(t) = [x_1(t)\ \ x_2(t)]^T \in \mathbb{R}^2$ is the state vector (assumed available for measurement), $u(t) \in \mathbb{R}$ is the control input, $f(t,\mathbf{x}), g(t,\mathbf{x}) : \mathbb{R}\times\mathbb{R}^2 \to \mathbb{R}$ are sufficiently smooth vector fields satisfying $f(t,0) = 0$ and $g(t,\mathbf{x}) \neq 0$ over the domain of interest, and the uncertain term $d(t,\mathbf{x})$ summarizes matched parameter variations and/or external disturbances affecting the system.

Assumption 2.1. The uncertain term $d(t,\mathbf{x})$ is bounded by a known function $\rho(\mathbf{x})$:

$$|d(t,\mathbf{x})| \le \rho(\mathbf{x}) \quad \forall t \ge 0,\ \forall \mathbf{x} \in \mathbb{R}^2.$$

The finite-time stabilization of the plant (1) will be addressed in this note, with the extra requirement that the settling time is upper bounded by an a priori value not dependent of the initial condition $\mathbf{x}(0)$. A novel terminal sliding surface will be designed to avoid the known singularity problems in the control input, previously discussed, often arising with TSM control as a consequence of the standard form of the terminal sliding surface, traditionally chosen as (Zuo, 2014): $s(t) = x_2(t) + \alpha x_1(t)^{m/n} + \beta x_1(t)^{p/q}$ with proper constraints on the positive integers m, n, p, q.

Problem 2.1. The problem here considered consists in designing a TSM controller such that:

- the origin of (1) is globally finite-time stable;
- an upper bound of the finite settling time is available, and is independent of the initial state;
- the control input is nonsingular.

3. Motivating example

Consider a second order plant of the form (1) with $n = 2$, and define the following sliding surface

$$s(\mathbf{x}) = x_2(t) + 2\beta\sqrt{|\arctan(x_1(t))|}(1 + x_1^2(t))\sigma(x_1(t)) \tag{3}$$

with $\sigma(x_1(t)) = \mathrm{sign}(x_1(t))$, having redefined the sign function in order to satisfy $\mathrm{sign}(0) = 0$. Once a sliding motion is established on the surface $s(\mathbf{x}) = 0$, the dynamics of the variable $x_1(t)$ are governed by:

$$\dot{x}_1(t) = -2\beta\sqrt{|\arctan(x_1(t))|}(1 + x_1^2(t))\sigma(x_1(t)) \tag{4}$$

In the case $x_1(t) \ge 0 \Leftrightarrow \arctan(x_1(t)) \ge 0$, it holds

$$\frac{dx_1(t)}{\sqrt{\arctan(x_1(t))}(1 + x_1^2(t))} = -2\beta dt \tag{5}$$

and the solution of (5) satisfies $\sqrt{\arctan(x_1(t))} - \sqrt{\arctan(x_1(0))} = -\beta t$, this proving that a settling time can be found, independently of the initial condition, given by

$$T_s(x_1(t)) = \frac{1}{\beta}\sqrt{\arctan(x_1(0))} \le \frac{1}{\beta}\sqrt{\frac{\pi}{2}} \tag{6}$$

such that $x_1(t) = 0$ for $t > \frac{1}{\beta}\sqrt{\frac{\pi}{2}}$ independently of $x_1(0)$. Analogous considerations can be derived in the converse case $x_1(t) < 0$.

A further feature of the sliding surface (3) is relative to the equivalent control needed for ensuring the achievement of the sliding motion. The condition $\dot{s}_2(\mathbf{x}) = 0$, in the nominal case, provides (the case $x_1(t) > 0$ is considered without loss of generality):

$$f(t,\mathbf{x}) + g(t,\mathbf{x})u(t) + \beta\frac{1 + 4x_1(t)\arctan(x_1(t))}{\sqrt{\arctan(x_1(t))}}x_2(t) = 0 \tag{7}$$

Due to (4) it follows that, after the (arbitrary) reaching time interval T_r needed for the sliding motion to be established by the standard condition $s(\mathbf{x})\dot{s}_2(\mathbf{x}) < -\eta|s(\mathbf{x})|$ with $\eta \in \mathbb{R}^+$, the control input becomes

$$u(t) = -g(t,\mathbf{x})^{-1}\big(f(t,\mathbf{x}) - 4\beta^2\cdot (1 + x_1(t)^2)(1/2 + 2x_1(t)\arctan(x_1(t)))\big) \tag{8}$$

and $\lim_{x_1\to 0}u(t) = \gamma \in \mathbb{R},\quad t > T_r$. The reported discussion proves the following result.

Theorem 3.1. *With reference to the planar uncertain system (1) satisfying Assumption 2.1, the achievement of a sliding motion on the surface $s(\mathbf{x}) = 0$ guarantees the robust finite-time stabilization of the plant. Once the sliding surface is attained ($t > T_r$), the states reach the origin within a fixed time:*

$$T_s \le \frac{1}{\beta}\sqrt{\frac{\pi}{2}} \tag{9}$$

and the control input shows no singularity as the solution approaches the origin.

4. A class of finite-time terminal sliding surfaces

To generalize the previous example, consider a real valued function $h(z) : \mathbb{R} \to \mathbb{R}$, sufficiently smooth almost everywhere. Define $h'(z) \overset{\text{def}}{=} \frac{\partial h(z)}{\partial z}$; $h''(z) \overset{\text{def}}{=} \frac{\partial h'(z)}{\partial z}$, and consider the following sliding surface

$$s(\mathbf{x}) = x_2 + \beta(h'(x_1))^{-1}; \quad \beta > 0 \tag{10}$$

As exploited in the next definitions and the successive lemmas, a proper choice of the function $h(z)$ might provide some interesting features for sliding motions onto the surface $s(\mathbf{x}) = 0$.

Definition 4.1. With reference to the plant (1), the sliding surface (10) is referred to as a *finite-time terminal sliding surface* if the following conditions hold:

(i) $h'(z) \neq 0$ for all $z \in \mathbb{R}\setminus\{0\}$;
(ii) $h(0) = 0$;

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