Automatica 94 (2018) 416–425

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Nonparametric identification of linear dynamic errors-in-variables systems☆

Erliang Zhang[a,*], Rik Pintelon[b]

[a] School of Mechanical Engineering, Zhengzhou University, Science Avenue 100, 450001, Zhengzhou, China
[b] Department ELEC, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

ARTICLE INFO

Article history:
Received 13 September 2017
Received in revised form 24 January 2018
Accepted 3 April 2018

Keywords:
Errors-in-variables
Nonparametric identification
Transfer functions
Arbitrary input
Frequency domain estimation
Identifiability

ABSTRACT

The present work handles the nonparametric identification of linear dynamic systems within an errors-in-variables framework, where the input is arbitrary and both the input and output disturbing noises are white with unknown variances. Using the property that the frequency response function and the system leakage term can be locally approximated very well by a low-order degree polynomial, a frequency domain estimator is developed, which gives consistent estimates for the frequency response function and the input–output noise variances. The consistency and uniqueness of the estimator are theoretically analyzed under mild conditions, and uncertainty bounds are also provided. The proposed method is finally validated on a simulated linear dynamic system.

1. Introduction

Dynamic systems are parametrically and nonparametrically identified (see, e.g., textbooks like Ljung, 1999, Pintelon & Schoukens, 2012). Parametric identification methods model the system with finite number of parameters independent of the number of data points, while for nonparametric methods the number of parameters increases with the number of data points. The nonparametric identification is the primary stage of the identification process to gain insight into the system complexity and to guide the user in the model selection/validation of the parametric modeling step.

Nonparametric frequency response function (FRF) identification has been developed within an output error (OE) framework where the input is known, with the emphasis on the suppression of the leakage error which is caused by transforming a finite number of time domain samples into the frequency domain via the discrete Fourier transform (DFT). The classical spectral analysis reduces the leakage error using windowing techniques (Brillinger, 1981; Ljung, 1999). The leakage error is highly structured with a smooth frequency characteristic (McKelvey, 2002; Pintelon, Schoukens, & Vandersteen, 1997). This property is utilized for the development of the improved FRF-estimation methods, such as the local polynomial method (LPM) (Pintelon, Schoukens, Vandersteen, & Barbé, 2010a; Schoukens, Vandersteen, Barbé, & Pintelon, 2009), the local rational modeling method (McKelvey & Guérin, 2012), and the transient and impulse response modeling method (Hägg, Schoukens, Gevers, & Hjalmarsson, 2016). From an alternative point of view, regularization techniques are proposed for nonparametric impulse response (Pillonetto, Dinuzzo, Chen, De Nicolao, & Ljung, 2014) and FRF (Lataire & Chen, 2016) estimation.

Compared with the OE framework, the nonparametric FRF identification in the errors-in-variables (EIV) framework is more difficult to handle, as both input and output data are noisy. Classical estimators based on spectral analysis (e.g., H_1 and H_2 Bendat & Piersol, 2010) are all biased. Early attempts to minimize the bias error lead to the H_s estimator and the special case of this estimator called H_v (White & Collis, 1998; White, Tan, & Hammond, 2006). The H_s estimator is shown to be a special case of a maximum likelihood estimator when the ratio of the input and output noise spectra is known (White et al., 2006). However, the knowledge of this noise spectra ratio is hardly available in practice. Nonlinear averaging techniques have been developed to reduce the bias error for frequency response function measurements (Guillaume, Pintelon, & Schoukens, 1992; Schoukens & Pintelon, 1990). Unbiased estimation of the FRF is realized by considering specific excitations,

☆ This work was supported in part by the National Science Foundation of China (U1504618), Science Foundation for The Excellent Youth Scholars of Zhengzhou University (16F0001132), the Flemish Government (Methusalem Fund METH1), the Belgian Federal Government (Interuniversity Attraction Poles programme IAP VII, DYSCO), and the Education Bureau of Henan Province, China (14A460019). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonio Vicino under the direction of Editor Torsten Söderström.

* Corresponding author.
E-mail addresses: erliang.zhang@zzu.edu.cn (E. Zhang), rik.pintelon@vub.ac.be (R. Pintelon).

https://doi.org/10.1016/j.automatica.2018.04.039
0005-1098/

such as periodic inputs (Pintelon & Schoukens, 2001), cyclostationary inputs (Antoni, Wagstaff, & Henrio, 2004), nonstationary inputs (Shalvi & Weinstein, 1996), pulse-like excitations (Hostettler, Birk, & Nordenvaad, 2016), and use of a known reference signal via an indirect method (Pintelon, Schoukens, Vandersteen, & Barbé, 2010b).

To summarize, the consistent estimation of the nonparametric FRF of a linear dynamic system excited by arbitrary inputs remains an open question in the presence of the input–output noises with unknown power spectra. To generate uncertainty bounds on the FRF estimate one needs the disturbing noise power spectra. Therefore, estimating the input–output noise power spectra is as important as estimating the FRF itself. The present work contributes to it by handling nonparametric EIV modeling using arbitrary inputs from input–output data disturbed by noise sources with unknown variances. The proposed method is based on the local polynomial modeling of the FRF and the system leakage term since it is superior in discarding the leakage error while maintaining the full frequency resolution compared with the classical spectral analysis method (Pintelon et al., 2010a).

The main features of the proposed frequency domain nonparametric approach are as follows: (1) Arbitrary inputs are allowed, and critical prior knowledge on input–output noise variances is not needed, (2) The FRF and the input–output noise variances are consistently estimated for dynamical systems with unknown model complexity, and (3) Uncertainty bounds of all the estimates are provided along with their Cramér–Rao lower bounds. These properties are theoretically derived, and also demonstrated on a simulated example.

In this paper, the following notations are used. Vectors are column vectors. If $\mathbf{x}$ is a vector, $\|\mathbf{x}\|_2^2$ stands for the inner product of the vector (squared l^2 norm). The *ordo* $O(x)$ stands for an arbitrary function with the property $\lim_{x\to 0} |O(x)/x| < \infty$. $(\cdot)^T$ is the transpose operation, and $(\cdot)^\dagger$ denotes the conjugate transpose operator. The symbol $\mathbb{E}$ stands for the expectation, computed w.r.t. the measurements. For a complex variable X, $\mathrm{Re}(X)$ and $\mathrm{Im}(X)$ are the real and imaginary parts of X, respectively. $\mathrm{cond}(\mathbf{M})$ denotes the condition number of the matrix $\mathbf{M}$. $g(\mathbf{x})|_{\mathbf{x}^*}$ is the value of $g(\mathbf{x})$ at $\mathbf{x}^*$. $\mathbf{x}_0$ denotes the true (expected) value of the (random) variable $\mathbf{x}$.

2. Problem formulation

Consider the linear time-invariant system described in Fig. 1. Let $\{u(t)\}_{t=0}^{N-1}$ and $\{y(t)\}_{t=0}^{N-1}$ be a set of input and output observations at N equidistant points, whose corresponding Discrete Fourier Transforms (DFTs) are calculated as

$$X(k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} x(t) \exp\left(-\frac{j2\pi k}{N} t\right) \tag{1}$$

where $x = u,\ y$ and $X = U,\ Y$. $X(k)$ is scaled by $1/\sqrt{N}$ such that its power remains the same as N increases.

Applying the Discrete Fourier transform (1) to the ordinary differential equation of a dynamical system, the input–output relationship in the frequency domain for an arbitrary excitation reads,

$$Y(k) = G(\Omega_k)U_0(k) + T(\Omega_k) + N_Y(k), \tag{2}$$

$$U(k) = U_0(k) + N_U(k), \tag{3}$$

where $U(k)$ and $Y(k)$ are the DFTs of input and output, respectively, $U_0(k)$ is the DFT of the noise-free input, $G(\Omega_k)$ is the FRF, $T(\Omega_k)$ is the system leakage term, whose value vanishes at a rate of $O(N^{-1/2})$, $N_U(k)$ and $N_Y(k)$ are the DFTs of the input–output noises, respectively. Ω_k is the generalized frequency variable (for continuous-time systems $\Omega_k = j2\pi f_k$, for discrete-time system

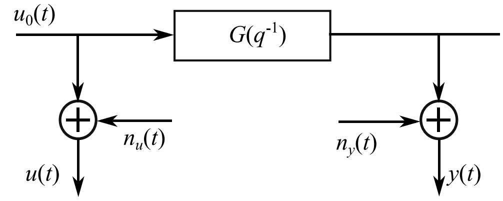

Fig. 1. Errors-in-variables measurement setup.

$\Omega_k = \exp(-j\omega_k T_s)$, with $f_k = f_s k/N$ and f_s the sampling frequency). The DC ($k = 0$) and Nyquist ($k = N/2$) components are excluded from the analysis, and $\mathcal{K}$ denotes the set of measured DFT lines ($k \in \mathcal{K}$).

$T(\Omega_k)$ is a system leakage term due to the difference between the beginning and ending effects of the finite measurement. Keep in mind that $T(\Omega_k)$ and $G(\Omega_k)$ are both highly structured with a smooth behavior.

Assumption 1. The plant's FRF and the system leakage term are infinitely differentiable at frequencies of interest.

This is a weaker condition than the standard assumption that the FRF and the leakage term admit parametric rational modeling. By consequence, in a local frequency window, the FRF and the system leakage term can be approximated arbitrarily well by a polynomial of sufficiently high order in least squares sense (Kreider, Kuller, Ostberg, & Perkins, 1966).

Assumption 2. The (band-limited) white measurement noise is independent and identically distributed at the sampling instances, the input–output noises are mutually uncorrelated and they are independent of the noise-free input.

Under Assumption 2, the DFT of the noise has zero mean and is asymptotically ($N \to \infty$) independent (over DFT line k) circular complex normally distributed. Moreover it is independent (over k), circular complex normally distributed for any N if the noise is Gaussian and white (Brillinger, 1981; Pintelon & Schoukens, 2012). i.e., $N_U(k) \in \mathcal{N}_c\left(0, \sigma_U^2\right)$, $N_Y(k) \in \mathcal{N}_c\left(0, \sigma_Y^2\right)$. $\mathcal{N}_c$ denotes the circular complex normal probability density function. Define $\boldsymbol{\sigma}^2$ as the vector of the input–output noise variances

$$\boldsymbol{\sigma}^2 = \begin{bmatrix} \sigma_Y^2 \\ \sigma_U^2 \end{bmatrix}. \tag{4}$$

Assumption 3. The amplitude of the system's FRF is frequency-dependent.

This assumption excludes all-pass systems which are commented to be not identifiable in the presence of white input–output noises (Castaldi & Soverini, 1996; Zhang & Pintelon, 2017).

Depending on the method we will discuss, the input is assumed to be an arbitrary DFT sequence that satisfies the property of a rough signal (Schoukens et al., 2009). The basic idea used is that the leakage error has a smooth DFT spectrum while this is not for the input. Formally, the signal $u_0(t)$ with the DFT $U_0(k)$ is called rough of order p at the DFT line k if it meets the following requirement,

$$\left|\mathrm{diff}^{(p)}\left(U_0(k)\right)\right| = O(N^0), \tag{5}$$

which means that $\left|\mathrm{diff}^{(p)}\left(U_0(k)\right)\right|$ does not disappear even if the record length $N \to \infty$, and where $\mathrm{diff}^{(1)}\left(U_0(k)\right) = U_0(k+1) - U_0(k)$ and $\mathrm{diff}^{(p)} = \mathrm{diff}^{(1)}\left(\mathrm{diff}^{(p-1)}\right)$. A special example of such rough