Automatica 94 (2018) 458–464

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Event-triggered resilient filtering with measurement quantization and random sensor failures: Monotonicity and convergence☆

Qinyuan Liu [a,*], Zidong Wang [c], Xiao He [d], D.H. Zhou [b,d]

[a] Department of Computer Science and Technology, Tongji University, Shanghai 201804, China
[b] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China
[c] Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, UK
[d] Department of Automation, Beijing National Research Center for Information Science and Technology (BNRist), Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 15 September 2016
Received in revised form 26 September 2017
Accepted 1 February 2018

Keywords:
Recursive filter
Quantization effect
Resilient property
Sensor failure
Event-triggered communication

ABSTRACT

This paper is concerned with the remote state estimation problem for a class of discrete-time stochastic systems. An event-triggered scheme is exploited to regulate the sensor-to-estimator communication in order to preserve limited network resources. A situation is considered where the sensors are susceptible to possible failures and the signals are quantized before entering the network. Furthermore, the resilience issue for the filter design is taken into account in order to accommodate the possible gain variations in the course of filter implementation. In the simultaneous presence of measurement quantizations, sensor failures and gain variations, an event-triggered filter is designed to minimize certain upper bound of the covariance of the estimation error in terms of the solution to Riccati-like difference equations. Further analysis demonstrates the monotonicity of the minimized upper bound with respect to the value of thresholds. Subsequently, a sufficient condition is also established for the convergence of the steady-state filter. A numerical example is presented to verify the effectiveness of the proposed filtering algorithm.

1. Introduction

Over the past decades, the tremendous advances in information sensing and network communication technologies have provided unprecedented opportunities to the development of networked systems. Compared with the classical point-to-point connections, networked systems have inherent advantages including simple extension, high flexibility and reduced maintenance/installation costs, and have therefore attracted increasing attention. The state estimation or filtering problem has long been a fundamental research issue in control engineering and signal processing with tremendous application insights in almost all practical systems especially in guidance, navigation and vehicle control (Linares-Perez, Caballero-Águila, & Garcia-Garrido, 2014; Liu, Wang, He, Ghinea, & Alsaadi, 2017; Liu, Wang, He, & Zhou, 2017; Ma, Wang, Han, & Lam, 2017). Up to now, considerable research interest has been devoted to the networked state estimation problems, where the raw measurements are transmitted from sensors to a central unit for processing through a wired/wireless communication channel, see for instance Hespanha, Naghshtabrizi, and Xu (2007) and Liu, Wang, and Zhou (2017).

The emergence and popularity of network communications have been posing new challenges for engineers and scientists when designing optimal state estimators. For most practical applications in networked environments, the network bandwidth for communication is exhaustible. Unnecessarily transmissions of the measurements from the sensor to the estimator might result in some undesirable phenomena and thus deteriorate the communication quality, which would reduce estimation accuracy. To handle such a resource limit issue, event-triggered communication, has recently drawn much research attention with the objective of reducing the number of network transmissions while maintaining certain system performance (e.g. stability for a control problem), see for instance Heemels, Johansson, and Tabuada (2012), Lemmon (2010), Li and Lemmon (2011), Li, Hu, and Lemmon (2012), Li, Wang, and Lemmon (2013), Liu, Wang, He, and Zhou (2015), Meng and Chen

☆ This work was supported in part by the Research Fund for the Taishan Scholar Project of Shandong Province of China, the National Natural Science Foundation of China under Grants 61490701, 61525305, 61473163, 61522309, 61733009, the Royal Society of the UK, and the Alexander von Humboldt Foundation of Germany. The work of Xiao He was also partially supported by the Special Fund of Suzhou-Tsinghua Innovation Leading Action under Grant 2016SZ0202. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Dimos V. Dimarogonas under the direction of Editor Christos G. Cassandras.

* Corresponding author at: Department of Computer Science and Technology, Tongji University, Shanghai 201804, China.

E-mail addresses: liuqy@tongji.edu.cn (Q. Liu), Zidong.Wang@brunel.ac.uk (Z. Wang), hexiao@tsinghua.edu.cn (X. He), zdh@mail.tsinghua.edu.cn (D.H. Zhou).

https://doi.org/10.1016/j.automatica.2018.03.031
0005-1098/

(2014), Sijs and Lazar (2012) and Trimpe and D'Andrea (2014) and the references therein. Specifically, in Li et al. (2012), the authors have presented the minimum bit-rates required to realize the event-triggered control system that is resilient to transient unexpected disturbances. As for event-triggered state estimation problems, in Trimpe and D'Andrea (2014), the authors have proposed a variance-based event-triggering state estimation strategy whose idea is to decide whether an event should be triggered by calculating the error variance.

A critical assumption behind the conventional filter algorithm is that the filter has access to the accurate measurements of the plant. This assumption is, however, not true in certain situations. Note that sensors may sometimes experience abnormal working conditions or even failures, which gives rise to inaccurate measurements suffering from random degradations. The research on filtering problems with degraded/missing measurements dates back as early as in Nahi (1969), where the optimal recursive filter has been designed for systems with missing measurements. On the other hand, for state estimation problem over communication networks, the raw measurements are usually quantized first before being transmitted. The gap between the quantized measurement and the raw one makes the conventional filter algorithm no longer applicable. A great number of results have been available in recent literature on the topic of the networked control/estimation problems with discrete quantized measurements, see, e.g., Fu and de Souza (2009), Leung, Seneviratne, and Xu (2015) and Zurkowski, Yuksel, and Linder (2016).

It should be noted that another problem that we might encounter is the fluctuations or inaccuracies in the realization of filter/controller algorithms. According to Keel and Bhattacharyya (1997), such inaccuracies arise from a variety of factors including, but are not limited to, finite-resolution measuring instruments, finite word length, round-off errors in numerical computation, and safe-tuning margin requirements. If not properly handled, the filter/controller gain variations might seriously deteriorate the system performance. As a result, particular research efforts have been paid on the design of *resilient* filters/controllers that are insensitive to the aforementioned inaccuracies (Hounkpevi & Yaz, 2007; Liu, Wang, He, Ghinea, et al., 2017).

Motivated by the above discussion, the aim of this paper is to develop event-triggered resilient filters for a class of networked systems subject to both quantization effects and random sensor failures. Generally speaking, the primary technical challenges we are encountering can be highlighted as threefold: *(1) due to the existence of the errors induced by quantization and event-triggered communication, the analytical expression of the error covariance is intractable and, as opposed to the traditional Kalman filtering case, there is no basis now for the subsequent minimum variance estimation; (2) although it is acknowledged that the increase of both the measurement inaccuracies and the event-triggering thresholds would inevitably reduce the filtering accuracy, there is a lack of rigorous mathematical proof as to the monotonicity which turns out to be a difficult task; and (3) the establishment of the convergence conditions for the designed steady-state optimal filter is a non-trivial objective as well.* In this paper, we endeavor to tackle the identified three challenges. Particularly, a recursive Riccati-like difference equation is first constructed whose solution is a tight upper bound of the actual covariance, and the filter gain is then designed to minimize the upper bound as an alternative. By matrix transformations and inductive methods, we demonstrate that the minimized upper bound is a monotonically increasing function with respect to the thresholds. Furthermore, a sufficient condition for the convergence of the steady-state filter is established in terms of a matrix inequality.

Notations. $\mathbb{R}_+^{n\times m}$ denotes the set of $n\times m$ nonnegative definite matrices and X' represents the transpose of a matrix X. $\|.\|$ represents the Euclidean norm for a vector or the spectral radius for a matrix. $\mathrm{vec}\{x_1, x_2, \ldots, x_m\}$ denotes the column vector $[x_1', x_2', \ldots, x_m']'$. $\mathrm{diag}\{x_1, x_2, \ldots, x_n\}$ is a block diagonal matrix with the ith block being x_i and all other entries being zero.

2. Problem formulation

Consider the following discrete-time system:

$$x(k+1) = Ax(k) + \alpha(k)h(x(k)) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the plant, the process noise $w(k) \in \mathbb{R}^n$ is a Gaussian random vector with zero mean and covariance $Q > 0$, and $\alpha(k)$ is a scalar-valued random variable with zero mean and variance $\breve{\alpha}$. $h(x(k))$ is a nonlinear function to be defined later. The initial state $x(0)$ is a zero-mean Gaussian random vector with covariance $X_0 > 0$. The measurement equation subject to random sensor failures is given by

$$y_i(k) = \lambda_i(k)C_i x(k) + v_i(k), \ i = 1, 2, \ldots, m, \tag{2}$$

where $y_i(k) \in \mathbb{R}$ is the observation of ith sensor and the measurement noise $v_i(k) \in \mathbb{R}$ is a Gaussian random variable with zero mean and variance $R_i > 0$. $\lambda_i(k) \in \mathbb{R}$ is a random variable characterizing the random sensor failure, which takes values on the interval $[0, 1]$ according to certain probabilistic density functions with mean $\bar{\lambda}_i$ and variance $\breve{\lambda}_i$. Additionally, A and C_i are known matrices of appropriate dimensions.

Define $\Lambda(k) \triangleq \mathrm{diag}\{\lambda_1(k), \ldots, \lambda_m(k)\}$, $v(k) \triangleq \mathrm{vec}\{v_1(k), \ldots, v_m(k)\}$, $y(k) \triangleq \mathrm{vec}\{y_1(k), \ldots, y_m(k)\}$, $C \triangleq \mathrm{vec}\{C_1, \ldots, C_m\}$. As such, the measurement equation can be written in a more compact form as follows

$$y(k) = \Lambda(k)Cx(k) + v(k). \tag{3}$$

For presentation convenience, we denote the statistics of $\Lambda(k)$ as $\bar{\Lambda} \triangleq \mathbb{E}[\Lambda(k)] = \mathrm{diag}\{\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_m\}$, $\breve{\Lambda} \triangleq \mathrm{Var}[\Lambda(k)] = \mathrm{diag}\{\breve{\lambda}_1, \breve{\lambda}_2, \ldots, \breve{\lambda}_m\}$. The vector-valued nonlinear function $h(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is analytic everywhere and satisfies the following condition

$$(h(x) - h(y))'(h(x) - h(y)) \le \eta(x-y)'(x-y) \tag{4}$$

with $h(0) = 0$, where η is a positive scalar.

In this paper, the networked state estimation problems are taken into account, i.e., the measurements $y_i(k)$ from every sensor are sent to the remote estimation center via communication channels. For certain applications, the network resource might be highly scarce or constrained, and it is therefore vitally important to reduce the sensor-to-estimator communication rate. To tackle this issue, we introduce an event-triggered communication strategy, under which the signals are transmitted if and only if the specific conditions are satisfied. Specifically, let us denote a time sequence that records the triggering instants of ith sensor by $\{k_s^i\}_{s=1}^{\infty}$, where $0 = k_1^i < k_2^i < \cdots < k_s^i < \cdots$. Suppose that the latest triggering instant of ith sensor is k_s^i, then the forthcoming triggering instant can be determined iteratively on-line by checking the triggering condition as follows

$$k_{s+1}^i = \min_k\{k | k > k_s^i, \ |y_i(k) - y_i(k_s^i)| > \sigma_i\}, \tag{5}$$

where $\sigma_i > 0$ is a pre-assigned threshold, $y_i(k)$ and $y_i(k_s^i)$ represent the current and the latest transmitted measurements at the sampling step k, respectively.

From the event-triggered strategy, it can be seen that only when the inequality in (5) becomes true, the corresponding measurement will be transmitted to the remote estimator, otherwise it will be discarded. Between two successive event instants, the