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Distributed time synchronization for networks with random delays and measurement noise[☆]

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ABSTRACT

In this paper a new distributed asynchronous algorithm is proposed for time synchronization in networks with random communication delays, measurement noise and communication dropouts. Three different types of the drift correction algorithm are introduced, based on different kinds of local time increments. Under nonrestrictive conditions concerning network properties, it is proved that all the algorithm types provide convergence in the mean square sense and with probability one (w.p.1) of the corrected drifts of all the nodes to the same value (consensus). An estimate of the convergence rate of these algorithms is derived. For offset correction, a new algorithm is proposed containing a compensation parameter coping with the influence of random delays and special terms taking care of the influence of both linearly increasing time and drift correction. It is proved that the corrected offsets of all the nodes converge in the mean square sense and w.p.1. An efficient offset correction algorithm based on consensus on local compensation parameters is also proposed. It is shown that the overall time synchronization algorithm can also be implemented as a flooding algorithm with one reference node. It is proved that it is possible to achieve bounded error between local corrected clocks in the mean square sense and w.p.1. Simulation results provide an additional practical insight into the algorithm properties and show its advantage over the existing methods.

1. Introduction

Cyber–Physical Systems (CPS), Internet of Things (IoT) and Sensor Networks (SN) have emerged as research areas of paramount importance, with many conceptual and practical challenges and numerous applications (Akyildiz & Vuran, 2010; Holler et al., 2014; Kim & Kumar, 2012). One of the basic requirements in networked systems is, in general, *time synchronization*, *i.e.*, all the nodes have to share a common notion of time. The problem of time synchronization in networked systems has attracted a lot of attention, but still represents a challenge due to multi-hop communications, stochastic delays, communication and measurement noise, unpredictable packet losses and high probability of node failures, *e.g.*, Sundararaman, Buy, and Kshemkalyani (2005). There are numerous approaches to this problem, starting from different assumptions and using different methodologies, *e.g.*, Elson, Girod, and Estrin (2002), Sivrikaya and Yener (2004) and Sundararaman et al., (2005). An important class of time synchronization algorithms is based on full distribution of functions (Simeone, Spagnolini, Bar-Ness, & Strogatz, 2008; Solis, Borkar, & Kumar, 2006). Distributed schemes with the so-called *gradient property* have been proposed in Fan and Lynch (2006) and Sommer and Wattenhofer (2009). A class of *consensus based algorithms*, called CBTS (Consensus-Based Time Synchronization) algorithms, has attracted considerable attention, *e.g.*, He, Cheng, Chen, Shi, and Lu (2014a), He, Cheng, Shi, Chen, and Sun (2014b), Li and Rus (2006), Liao and Barooah (2013a), Schenato and Fiorentin (2011), Tian (2015) and Xiong and Kishore (2009). It has been treated in a unified way in a recent survey (Tian, Zong, & Cao, 2016), providing

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figure of merit of the principal approaches. In Carli, Chiuso, Zampieri, and Schenato (2008) and Yildirim, Carli, and Schenato (2015) a control-based approach to distributed time synchronization has been proposed. Fundamental and yet unsolved problems in all the existing approaches are connected with *communication delays and measurement noise*; see Freris, Graham, and Kumar (2011) for basic issues, and Chaudhari, Serpedin, and Qaraqe (2008), Choi, Liang, Shen, and Zhuang (2012), Garone, Gasparri, and Lamonaca (2015) and Xiong and Kishore (2009) for different aspects of delay influence.

In this paper we propose a new *asynchronous distributed* algorithm for time synchronization in lossy networks, characterized by *random communication delays*, *measurement noise* and *communication dropouts*. The algorithm is composed of two distributed recursions of *asynchronous stochastic approximation type* based on *broadcast gossip* and derived from predefined local error functions. The recursions are aimed at achieving asymptotic consensus on the *corrected drifts* and *corrected offsets* and, consequently, at obtaining *common virtual clock* for all the nodes in the network.

The proposed recursion for *drift synchronization* is based on noisy time increments, defined in three characteristic forms (a preliminary form has been presented in Stanković, Stanković, and Johansson (2016)). We prove *convergence to consensus* of the corrected drifts in the mean square sense and with probability one (w.p.1), under nonrestrictive conditions. Furthermore, we provide an estimate of the corresponding *asymptotic convergence rate*. It is shown that the proposed recursion with the increments of unbounded length with random boundaries provides convergence rate faster than $\frac{1}{t}$, what is essential for convergence to a common global virtual clock. Compared to the analogous existing algorithms (Schenato & Fiorentin, 2011; Tian, 2015), the proposed scheme is structurally different and simpler (not involving mutual drift estimation, typical for the CBTS algorithms) and, in addition, provides the best performance. Notice that the algorithm in Schenato and Fiorentin (2011) cannot handle communication delays and measurement noise, while the papers Tian (2015, 2017), derived from a particular form of increments of unbounded length, treat random delays, but not measurement noise and communication dropouts. Moreover, the algorithm proposed therein cannot provide convergence rate achievable by the proposed methodology. The approach in Garone et al. (2015) does not ensure consensus of corrected drifts in spite of additional pairwise inter-node communications.

We also propose a novel recursion for *offset synchronization*, which starts from a special error function, obtained from the difference between local times, with two important modifications aiming at: (1) eliminating the deteriorating effect of linearly increasing absolute time, and (2) coping with the influence of delays by introducing additional *delay compensation parameters*. It is proved that the proposed algorithm provides convergence in the mean square sense and w.p.1 to a set of bounded random variables. The algorithm for the offset correction proposed in Schenato and Fiorentin (2011) cannot handle these problems, while the algorithm in Tian (2015, 2017) allows unbounded corrected offsets and assume perfect clock readings. The approach in Yildirim et al. (2015) does not provide a rigorous insight into overall network stability. Attention is also paid to an improvement of the offset correction algorithm, based on linear consensus iterations, aiming at decreasing the dispersion of the offset convergence points. Special cases related to delay and noise are discussed in order to clarify potentials of the proposed algorithms.

The resulting time synchronization algorithm composed of the proposed drift and offset correction recursions ensures finite differences between local corrected clocks in the mean square sense and w.p.1. To the authors' knowledge, this is the first method with such a performance in the case of random delays, measurement noise and communication dropouts. It is also demonstrated that the proposed algorithm can be implemented as a *flooding algorithm*, with one preselected reference node.

Finally, some illustrative simulation results are presented, giving additional insights into the theoretically discussed issues.

2. Synchronization algorithms

2.1. Time and network models

Assume a network consisting of n nodes, formally represented by a *directed graph* $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ the set of arcs. Denote by $\mathcal{N}_i^+$ the out-neighborhood and by $\mathcal{N}_i^-$ the in-neighborhood of node i, $i = 1, \ldots, n$. Assume that each node has a local clock, whose output, defining the *local time*, is given for any *absolute time* $t \in \mathcal{R}$ by

$$\tau_i(t) = \alpha_i t + \beta_i + \xi_i(t), \tag{1}$$

where $\alpha_i \neq 0$ is the local *drift (gain)*, β_i is the local *offset*, while $\xi_i(t)$ is *measurement noise*, appearing due to equipment instabilities, round-off errors, thermal noise, etc. Liao and Barooah (2013a, b), Schenato and Fiorentin (2011) and Stanković, Stanković, and Johansson (2012). Each node i applies an affine transformation to $\tau_i(t)$, producing the *corrected local time*

$$\bar{\tau}_i(t) = a_i \tau_i(t) + b_i = g_i t + f_i + a_i \xi_i(t), \tag{2}$$

where a_i and b_i are the local *correction parameters*, $g_i = a_i \alpha_i$ is the *corrected drift* and $f_i = a_i \beta_i + b_i$ the *corrected offset*, $i = 1, \ldots, n$.

Distributed time synchronization is aimed at providing a *common virtual clock*, *i.e.*, *equal corrected drifts* g_i and *equal corrected offsets* f_i, $i = 1, \ldots, n$, by *distributed real-time estimation* of the parameters a_i and b_i. We assume that the nodes communicate according to the *broadcast gossip scheme*, *e.g.*, Aysal, Yildriz, Sarwate, and Scaglione (2009), Bolognani, Carli, Lovisari, and Zampieri (2012) and Nedić (2011), without global supervision or fusion center. Therefore, each node $j \in \mathcal{N}$ has its own *local communication clock* that ticks according to a Poisson process with rate μ_j, independently of the other nodes. At each tick of its communication clock (denoted by t_b^j, $b = 0, 1, 2, \ldots$), node j broadcasts its current state to its out-neighbors $i \in \mathcal{N}_j^+$. Each node $i \in \mathcal{N}_j^+$ hears the broadcast with probability $p_{ij} > 0$. Let $\{t_l^{j,i}\}$, $l = 0, 1, 2, \ldots$, be the sequence of absolute time instants corresponding to the messages heard by node i. The message sent at $t_l^{j,i}$ is received at node i at the time instant

$$\bar{t}_l^{j,i} = t_l^{j,i} + \delta_l^{j,i},$$

where $\delta_l^{j,i}$ represents the corresponding *communication delay* (for physical and technical sources of delays see Chaudhari et al. (2008), Choi et al. (2012), Freris et al. (2011), Leng and Wu (2011) and Xiong and Kishore (2009)). We assume in the sequel that

$$\delta_l^{j,i} = \bar{\delta}^{j,i} + \eta_i(\bar{t}_l^{j,i}), \tag{3}$$

where $\bar{\delta}^{j,i}$ is constant, while $\eta_i(\bar{t}_l^{j,i})$ represents a stochastically time-varying component with zero mean. After receiving a message from node j, node i reads its current local time, calculates its own current *corrected local time* and *updates the values of its correction parameters* a_i and b_i. The process is repeated after each tick of any node in the network; we assume, as usually, only one tick at a given time t (Nedić, 2011).