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Cooperative global robust output regulation for a class of nonlinear multi-agent systems by distributed event-triggered control[☆]

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ABSTRACT

This paper studies the event-triggered cooperative global robust output regulation problem for a class of nonlinear multi-agent systems via a distributed internal model design. We show that our problem can be solved practically in the sense that the ultimate bound of the tracking error can be made arbitrarily small by adjusting a design parameter in the proposed event-triggered mechanism. Our result offers a few new features. First, our control law is robust against both external disturbances and parameter uncertainties, which are allowed to belong to some arbitrarily large prescribed compact sets. Second, the nonlinear functions in our system do not need to satisfy the global Lipschitz condition. Thus our systems are general enough to include some benchmark nonlinear systems that cannot be handled by existing approaches. Finally, our control law is a specific distributed output-based event-triggered control law, which lends itself to a direct digital implementation.

1. Introduction

Over the past decade, various cooperative control problems for multi-agent systems have been widely studied, and the recent progress can be found in the survey paper (Zhu, Xie, Han, Meng, & Teo, 2017). The cooperative robust output regulation problem is one of the fundamental and important cooperative control problems, which aims to make all followers asymptotically track some reference inputs and reject some external disturbances, where the reference inputs and the external disturbances are both generated by an exosystem called the leader system. The problem has been first studied for linear uncertain multi-agent systems in Su, Hong, and Huang (2013), Wang, Hong, Huang, and Jiang (2010), and then for nonlinear uncertain multi-agent systems in Ding (2013), Dong and Huang (2014) and Su and Huang (2015). In this paper, we will further study the event-triggered cooperative global robust practical output regulation problem for a class of nonlinear multi-agent systems in normal form with unity relative degree.

Our study is motivated by the need of implementing continuous-time control laws in digital platforms. Compared with the traditional sampled-data implementation (Astrom & Wittenmark, 1977; Franklin, Powel, & Emami-Naeini, 2010), where the data-sampling is performed periodically, the event-triggered control approach generates the samplings and control actuation depending on the system state or output, and is more efficient in reducing the number of control task executions while maintaining the control performance (Arzen, 1999; Astrom & Bernhardsson, 1999; Heemels, Johansson, & Tabuada, 2012). So far, extensive efforts have been made for event-triggered control of both single linear systems and single nonlinear systems. For example, Ref. Heemels et al. (2012) gave an introduction to the event-triggered control and studied the stabilization problem for a class of linear systems by a state-feedback event-triggered control law. Ref. Donkers and Heemels (2012) analyzed the closed-loop stability and the $\mathcal{L}_\infty$-performance for a class of linear systems by an output-based event-triggered control law. In Tabuada (2007), the stabilization problem for a class of nonlinear systems was solved by a state-based event-triggered control law. Ref. Girard (2015) further proposed a dynamic event-triggered mechanism to solve the stabilization problem for the same class of nonlinear systems as that in Tabuada (2007). In Liu and Jiang (2015), a state-based event-triggered control law was designed to solve the robust stabilization problem for a class of nonlinear systems subject to external disturbances by applying the cyclic small gain theorem. Ref. Xing, Wen, Liu, Su, and Cai (2017) solved the tracking problem for a class of high-order uncertain nonlinear systems by an event-triggered adaptive control law. In Liu and Huang (2017b), the global robust output regulation problem for a class of nonlinear systems was further solved by an output-based event-triggered

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control law. Other relevant contributions can also be found in Abdelrahim, Postoyan, Daafouz, and Nešić (2016), Dolk, Borgers, and Heemels (2017), Postoyan, Tabuada, Nesic, and Anta (2015) and Tallapragada and Chopra (2013) etc.

The event-triggered control approach has also been applied to the cooperative control problems for multi-agent systems. For example, the event-triggered consensus problem was first studied for single-integrator multi-agent systems in Dimarogonas, Frazzoli, and Johansson (2012), Fan, Feng, Wang, and Song (2013) and Seyboth, Dimarogonas, and Johansson (2013), double-integrator multi-agent systems in Li, Liao, Huang, and Zhu (2015), Mu, Liao, and Huang (2015), and general linear multi-agent systems in Cheng and Ugrinovskii (2016), Zhang, Feng, Yan, and Chen (2014) and Zhu, Jiang, and Feng (2014). In Hu and Liu (2017), based on the feedforward design, the cooperative output regulation problem for a class of exactly known linear multi-agent systems was studied by a distributed event-triggered control law. Ref. Liu and Huang (2017a) further designed a distributed output-based event-triggered control law to solve the cooperative robust output regulation problem for a class of minimum-phase linear uncertain multi-agent systems based on the internal model approach. More recently, the event-triggered cooperative control problems were further studied for nonlinear multi-agent systems. For example, Ref. Xie, Xuan, Chu, and Zou (2015) studied the event-triggered average consensus problem for both first-order nonlinear systems and second-order nonlinear systems by centralized event-triggered strategy and distributed event-triggered strategy. In Li, Chen, and Xiao (2016), the event-triggered leaderless consensus problem for nonlinear multi-agent systems was studied by a distributed state-based event-triggered control law. Under the assumption that the state information of the virtual leader system is available for each follower, Ref. Zhao, Peng, He, and Song (2017) studied the event-triggered leader-following consensus problem for second-order nonlinear multi-agent systems. Note that, in Li et al. (2016), Xie et al. (2015) and Zhao et al. (2017), the nonlinear multi-agent systems are assumed to satisfy the global Lipschitz condition and contain no parameter uncertainties or external disturbances.

Compared with those results on event-triggered cooperative output regulation problem for linear multi-agent systems in Hu and Liu (2017) and Liu and Huang (2017a), the main technical challenges of this paper are that both our control law and event-triggered mechanism are nonlinear, and we have to resort to nonlinear techniques such as non-quadratic Lyapunov function, input-to-state stability, change supply pair technique to construct the control law and the event-triggered mechanism and to analyze the stability of the closed-loop system. Also, since the closed-loop system is nonlinear and hybrid, we need to pay special attention to rule out the Zeno behavior and finite escape time. Compared with the existing results on the event-triggered cooperative control problems for nonlinear multi-agent systems in Li et al. (2016), Xie et al. (2015) and Zhao et al. (2017), our problem offers at least three new features. First, our system contains both external disturbances and parameter uncertainties, which are allowed to belong to some arbitrarily large prescribed compact sets. Second, the nonlinear functions in our system do not need to satisfy the global Lipschitz condition. Thus our systems are general enough to include some benchmark nonlinear systems such as Lorenz systems, FitzHugh–Nagumo systems, which cannot be handled by the approaches in Li et al. (2016), Xie et al. (2015) and Zhao et al. (2017). Finally, our control law is a dynamic distributed output-based event-triggered control law, which is more complex than the static distributed state-based or static distributed output-based control law, since we need to sample not only the measurement output of each agent but also the state of the dynamic compensator. To overcome these challenges, we combine the distributed internal model approach with a distributed event-triggered mechanism. This event-triggered mechanism contains a design parameter which not only dictates the ultimate bound of the tracking error, but also affect the frequency of the triggering events. It is shown that this control law together with the event-triggered mechanism solves our problem in the sense that the steady-state tracking error of the closed-loop system can be made arbitrarily small. Besides, our method guarantees the existence of the minimal inter-execution time of the event-triggered mechanism, thus preventing the Zeno behavior from happening.

It is worth mentioning that the problem in this paper also contains the problem in Liu and Huang (2017b) as a special case by letting $N = 1$. Compared with (Liu & Huang, 2017b), the main challenge of this paper is that we need to design a specific event-triggered control law and a specific event-triggered mechanism satisfying some communication constraints for each subsystem, where the communication constraints mean that each subsystem can only make use of the information of its neighbors and itself for control. We call an event-triggered control law and an event-triggered mechanism satisfying such communication constraints as a distributed event-triggered control law and a distributed event-triggered mechanism, respectively.

Notation. For any column vectors a_i, $i = 1, \ldots, s$, denote $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$. $\|x\|$ denotes the Euclidean norm of vector x. $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm. $\mathbb{Z}^+$ denotes the set of nonnegative integers. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum eigenvalue and the minimum eigenvalue of a symmetric real matrix A, respectively. A matrix $M \in \mathbb{R}^{N\times N}$ is called an $\mathcal{M}$-matrix if all of its non-diagonal elements are non-positive and all of its eigenvalues have positive real parts.

2. Problem formulation and preliminaries

Consider a class of nonlinear multi-agent systems taken from Dong and Huang (2014) as follows:

$$\begin{aligned} \dot{z}_i &= f_i(z_i, y_i, v, w)\\ \dot{y}_i &= g_i(z_i, y_i, v, w) + b_i(w)u_i \qquad (1)\\ e_i &= y_i - q(v, w), \quad i = 1, \ldots, N \end{aligned}$$

where, for $i = 1, \ldots, N$, $(z_i, y_i) \in \mathbb{R}^n \times \mathbb{R}$ is the state, $e_i \in \mathbb{R}$ is the error output, $u_i \in \mathbb{R}$ is the input, $w \in \mathbb{R}^{n_w}$ is an uncertain constant vector, and $v(t) \in \mathbb{R}^{n_v}$ is an exogenous signal representing both reference input to be tracked and disturbance to be rejected. Here $v(t)$ is assumed to be generated by the following linear system:

$$\dot{v} = Sv, \quad y_0 = q(v, w). \qquad (2)$$

We assume that $b_i(w) > 0$ for all $w \in \mathbb{R}^{n_w}$, and the functions $f_i(\cdot)$, $g_i(\cdot)$ and $q(\cdot)$ are polynomials in z_i, y_i and v with coefficients depending on w, and satisfy $f_i(0, 0, 0, w) = 0$, $g_i(0, 0, 0, w) = 0$ and $q(0, w) = 0$ for all $w \in \mathbb{R}^{n_w}$.

System (1) is the so-called nonlinear multi-agent system in normal form with unity relative degree. Like in Dong and Huang (2014), the plant (1) and the exosystem (2) together can be viewed as a multi-agent system of $N + 1$ agents with (2) as the leader and the N subsystems of (1) as N followers. Given the plant (1) and the exosystem (2), we can define a digraph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ with 0 associated with the leader system and with $i = 1, \ldots, N$ associated with the N followers, respectively, and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. For each $j = 0, 1, \ldots, N$, $i = 1, \ldots, N$, and $i \neq j$, $(j, i) \in \bar{\mathcal{E}}$ if and only if the control u_i can make use of $y_i - y_j$ for feedback control. If the digraph $\bar{\mathcal{G}}$ contains a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})$, then node i_{k+1} is said to be reachable from node i_1. For $i = 1, \ldots, N$, let $\bar{\mathcal{N}}_i = \{j, (j, i) \in \bar{\mathcal{E}}\}$ denote the neighbor set of node i.