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Switching and sweeping vibration absorbers: Theory and experimental validation[☆]

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ABSTRACT

This paper investigates the principal properties of time varying operation modes for tuneable vibration absorbers mounted on distributed structures to reduce vibrations produced by stationary broad frequency band disturbances. The study considers a practical application where an electro-mechanical tuneable vibration absorber, comprising a seismic coil-magnet linear transducer, is fixed on a thin walled circular duct flexible structure. The absorber is commanded to either periodically switch or continuously vary the stiffness and damping of the elastic suspension holding the moving magnet. As a result, the tuneable vibration absorber fundamental natural frequency and damping ratio are respectively switched or swept to cyclically reduce the resonant response of multiple flexural natural modes of the duct structure. The study analytically shows, and confirms with simulations and experiments, that the vibration control produced by a simple "blind sweep" operation mode of the tuneable vibration absorber does not differ significantly from the (sub)optimal performance attainable via a switching operation mode.

1. Introduction

Tuneable Vibration Absorbers (TVA) have been used for a long time to minimize vibrations of small to large scale mechanical systems and civil structures (Den Hartog, 1956; Mead, 1988). These devices comprise a seismic mass mounted on an elastic suspension. They can be operated to suppress the effects of either tonal or broad band disturbances (Sun, Jolly, & Norris, 1995). In the first case, to offer the hosting structure the highest possible impedance at the excitation frequency, the TVA should be characterized by a very small internal damping and its fundamental natural frequency should be tuned to the mode to be controlled. In the second case, which is considered in this paper, the TVA fundamental natural frequency should be tuned to the natural frequency of the mode of the hosting structure to be controlled. Also, its internal damping should be set to minimize the amplitude of the pair of resonances resulting from the dynamic interaction of the controlled mode of the hosting structure and the fundamental mode of the TVA. As summarized in Zilletti, Elliott, and Rustighi (2012), several tuning laws of the TVA fundamental natural frequency and damping ratio have been proposed over the years, considering simplified single degree of freedom models of the hosting structure. In general, for reasonable size of the TVA, all laws set the TVA fundamental natural frequency close to the natural frequency of the targeted natural mode of the hosting structure and the TVA damping ratio between 20% and 40%.

The response of typical flexible structures encountered in practical applications is characterized by the superposition of several modal contributions. Normally, in structural vibration studies, this aspect is quantified in terms of the so-called "modal overlap factor" (Fahy & Gardonio, 2007), which indicates the number of modes that determine the response of a structure at each frequency. For example, the modal overlap for the flexural response of a circular cylinder tends to rise with frequency. Thus, at low frequencies, the flexural response of the cylinder is characterized by individual modal contributions at each frequency. Above the cut off frequency, where the modal overlap becomes greater than two, the flexural response at each frequency is instead due to the superposition of multiple modal contributions. Thus, to control the response over a wide frequency band, a rather large number of TVA devices should be employed, each tuned to control a specific structural mode of the hosting structure that resonates within the targeted frequency band (Estève & Johnson, 2002). Recently Gardonio and Zilletti (2013, 2015) have proposed "switching" and

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"sweeping" operation modes, such that the TVAs can control the resonant responses due to multiple modes of a structure. In the switching operation mode, the fundamental natural frequency and internal damping of the TVAs are cyclically modified in such a way as to tune the TVAs to control the resonant response of target natural modes of the hosting structure. Alternatively, in the sweeping operation mode, the fundamental natural frequency and internal damping of the TVAs are continuously and uniformly varied within given ranges, so that the TVA periodically brings down the resonant response of the hosting structure natural modes that resonate in a target frequency band. Compared to conventional TVAs, a smaller number of *switching* or *sweeping* TVAs could be employed to reduce the vibration of the hosting structure in a wide frequency band up to the so called mid frequency range where the narrow frequency band response depends on clusters of modes (Langley & Fahy, 2004). This in turn reflects in lower material costs, lower installation time and costs, less added weight and thus lower operation costs. The *switching* TVA still requires a precise tuning of the fundamental natural frequency and damping ratio. However the proposed *sweeping* TVA overcomes also this requirement by blindly sweeping the TVA fundamental natural frequency and inherent damping over given ranges. Therefore, *sweeping* TVAs offer a robust solution that guarantees good vibration control performance even when the dynamic response, and thus the resonance frequencies, of the structure under control may vary because of changes in temperature or tensioning effects, for example.

The sweeping operation mode can be seen as a high frequency uniform and continuous switching process of the TVA fundamental natural frequency and damping ratio between limiting values that define the targeted frequency band of operation. Thus the analysis of the switching and sweeping TVA systems proposed in this paper is based on the theory of time-varying periodic linear systems and the theory of switching systems. For the sweeping operation mode, particularly important are the definition and characterization of input–output norms and entropy. Among the many contributions in these areas, the interested reader is referred to Colaneri (2005) and Peterst and Iglesias (1997) for the concept of γ-entropy of a time-varying and periodic system in the continuous-time domain. For the frequency domain characterization of the periodic TVA system, the harmonic transfer function discussed in Zhou (2009) is also considered. Finally, the general monograph (Bittanti & Colaneri, 2009) on the theory of periodic systems in discrete-time, with time-domain and frequency-domain characterization and lifted reformulations is used. The theory of switching linear systems is important in practical applications where hybrid configurations and parameters abrupt changes should be modelled with a compact and unified mathematical description. The switching parameters can be tuned so as to guarantee stability for every possible switching configuration (Blanchini, Miani, & Mesquine, 2009) and system performance optimization (e.g., minimization of $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norms) (Blanchini, Casagrande, Colaneri, Gardonio, & Miani, 2014; Blanchini, Casagrande, Gardonio, & Miani, 2012; Margaliot & Hespanha, 2008). In this study, the theory of linear switching systems is extended to the case of energy bounded stationary stochastic white noise input. To this end, the switching covariance equation is considered for energy attenuation ($\mathcal{H}_2$ norm) and the worst-case switching covariance-like equation is considered for frequency peak "γ-entropy" attenuation ($\mathcal{H}_\infty$ norm). The results are nontrivial extensions of previous works on so-called Lyapunov–Metzler and Riccati–Metzler "argmin" switching (Geromel & Colaneri, 2006; Geromel & Deaecto, 2010). For the former approach, an interesting interpretation of the attenuation performance in terms of Markov-Jump parameters is provided. Since the covariance matrix is positive definite (so defined in a cone), also interesting relations arise with the theory of both deterministic and stochastic positive switching systems (Blanchini, Colaneri, & Valcher, 2015; Bolzern & Colaneri, 2015). The work presented in this paper is restricted to purely open loop switching and sweeping strategies. These strategies are inherently robust to parameters mismatch and to input bounded disturbances. Possible extension to feedback control can be considered in the light of recent contributions on the theory of switched systems, like Zhang, Zhuang, and Braatz (2016) where state-feedback MPC stabilizing strategies with mode-dependent dwell time were studied and Geromel, Colaneri, and Bolzern (2008) where dynamic feedback control laws based on state-observer and Lyapunov–Metzler inequalities were developed.

The aim of this study is to show analytically and to confirm with simulations how, and under what conditions, the proposed time-varying switching and sweeping modes effectively operate the TVA to produce broadband vibration control effects. In particular, it presents a covariance formulation which is used to identify how the performance of the sweeping TVAs is influenced by three key parameters: the resolution of the sweep, the speed of the sweep and the damping range of the sweep. Moreover it presents an analytical study that contrasts the effects produced by the proposed open loop sweep operation mode with a model based operation mode, which shows that the open loop and closed loop operation modes produce similar vibration control effects.

The technical content of the paper is organized as follows. Section 2 briefly describes the laboratory setup (Zilletti & Gardonio, 2015). Section 3 revises the background theory and both simulation and experimental results are presented, which highlight the principal features of fixed tuning TVA systems. Section 4 presents a comprehensive theoretical study on the implementation of a time-varying switching TVA. The implementation of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ periodic switching tuning laws is first investigated analytically in Sections 4.1 to 4.4. The effectiveness of the proposed approaches is then assessed numerically with three examples. Also, an experimental result is presented considering a practical switching law. Finally Section 5 considers the implementation of a time-varying sweeping mode, where the TVA fundamental natural frequency and internal damping are swept in specific ranges such that the TVA blindly operates within a given frequency band. As for the switching operation mode, the implementation of the open loop periodic sweep is first investigated analytically. Its effectiveness is then assessed numerically with an example based on the identified system and with an experimental result considering a practical sweeping law.

2. Experimental setup and plant model

This study is based on the laboratory demonstrator shown in Fig. 1, which is composed of a thin walled circular duct flexible structure equipped with an electro-mechanical TVA. The duct is mounted on flexible flanges and is excited by a radial point force exerted by a shaker. The TVA comprises a coil-magnet transducer with the magnet elastically suspended on the casing base disc where the coil is fixed. The primary excitation force is measured by a force cell mounted on the shaker head. Also, the flexural transverse vibration of the duct is measured with an accelerometer, which for the tests on the plain duct is mounted directly on the duct wall at the TVA location and for the tests on the duct with the TVA is mounted on the top of the TVA casing.

As schematically shown in the magnified sketch in Fig. 1(b), the TVA casing and moving magnet are equipped with accelerometer sensors whose output signals are subtracted and integrated in order to implement both relative displacement and relative velocity feedback loops via the coil-magnet transducer. The two feedback loops produce a pair of reactive forces proportional to the relative displacement and relative velocity between the casing and the suspended mass of the TVA, which is equivalent to producing elastic