Automatica 93 (2018) 352–362

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Synchronization control for reaction–diffusion FitzHugh–Nagumo systems with spatial sampled-data ☆

Sheng Chen [a,*], Cheng-Chew Lim [b], Peng Shi [b], Zhenyu Lu [c]

[a] School of Information and Control, B-DAT, CICAAEET, Nanjing University of Information Science and Technology, Nanjing, 210044, China
[b] School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia
[c] School of Electronics and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, 210044, China

ARTICLE INFO

Article history:
Received 11 November 2016
Received in revised form 4 December 2017
Accepted 3 March 2018
Available online 2 June 2018

Keywords:
Networked control systems
Synchronization
Data-based control
Spatial means
General algebraic connectivity

ABSTRACT

To gain a better understanding the synchronization mechanism of networked neurons, this paper studies the synchronization control for a class of reaction–diffusion FitzHugh–Nagumo systems, associated with a digraph containing at least one directed spanning tree. A novel control method adopting spatial sampling strategies is proposed, in which the control inputs are constructed directly on the spatial means of system state variables on sampling subsets. After discussing the existence and uniqueness of classical solutions, we show analytically that the synchronization of the controlled systems is equivalent to that of the corresponding FitzHugh–Nagumo systems under the corresponding control inputs. Based on the study on general algebraic connectivity, a sufficient condition for the system synchronization is given, together with case simulations to illustrate the effectiveness and potential of the new control method.

1. Introduction

It is well-known that the cerebral cortex in human brain is a highly distributed system in which numerous areas operate in parallel, exchange their results through reciprocal connections, and create coherent states via self-organizing dynamics. In past years, fruitful works were devoted to this operation mechanism. See Brette (2012), Tuckwell and Rodriguez (1998) and the references therein for some of them. It is universally accepted that neural synchronization coordinates different areas by neural signal transmission, playing an important role in the whole system. One fundamental task is to understand the synchronization mechanism through certain quantitative models for neurons, such as Hodgkin–Huxley models and FitzHugh–Nagumo systems.

Recent years witnessed some works on synchronization control for neural networks, where the neural dynamics is described by a linear reaction–diffusion (RD for short) equation, and a neural network is made up of neurons coupled by activation functions with possible transmission time-delays. For synchronizing a response network to a master network, Hu, Yu, and Teng (2012) introduced an intermittent control method, and Chen, Luo, and Zheng (2016) and Liu, Zhang, and Xie (2016) showed impulsive control methods. To guarantee a group of linearly-coupled networks synchronize to a reference network, Wang, Wu, and Huang (2015a) presented passivity-based methods, and Wang, Wu, Huang, and Ren (2016) showed pinning control strategies. To guarantee a swarm of linearly-coupled networks achieve synchronization without reference, Wang, Wu, and Guo (2014) adopted adaptive control strategies, and Wang, Wu, and Huang (2015b) employed passivity-based methods. In these works, note that all activation functions are globally Lipschitz, hence, the studied systems are a class of RD systems of which the reaction terms are globally Lipschitz with possible time-delays.

Recall that RD FitzHugh–Nagumo systems are a class of common quantitative models for real neural systems, in which the cubic functions are not globally Lipschitz. To ensure a swarm of RD FitzHugh–Nagumo systems achieve synchronization, Ambrosio and Aziz-Alaoui (2012, 2013) showed the control inputs designed directly with the difference of system state variables, followed by the synchronization control (Ambrosio, Aziz-Alaoui, & Phan, 2015) for RD systems of FitzHugh–Nagumo type.

Motivated by the work (Ambrosio & Aziz-Alaoui, 2012), in this paper we study the synchronization control for a swarm of RD FitzHugh–Nagumo systems whose solutions evolve toward homogeneous solutions, and the communication topology of the systems is described by a digraph which contains at least one directed

☆ This work was partially supported by National Natural Science Foundation of China (61503191, 61773131, 61473334, 71503136), Natural Science Foundation of Chinese Jiangsu Province (BK20150933), Joint Key Grant of National Natural Science Foundation of China and Zhejiang Province (U1509217), Australian Research Council (DP170102644), and 111 Project (B17048, B17017). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonis Papachristodoulou under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: 002186@nuist.edu.cn (S. Chen), cheng.lim@adelaide.edu.au (C.-C. Lim), peng.shi@adelaide.edu.au (P. Shi), zhenyulu@nuist.edu.cn (Z. Lu).

https://doi.org/10.1016/j.automatica.2018.03.043
0005-1098/

spanning tree. A novel control method is proposed for the systems, in which the control inputs are designed directly on the spatial means of system state variables on sampling subsets. After the existence and uniqueness of classical solutions is discussed, our analysis shows that the synchronization of the systems under our proposed control inputs is equivalent to that of their corresponding FitzHugh–Nagumo systems under the corresponding control inputs. Based on the investigation on general algebraic connectivity, a sufficient condition for the system synchronization is shown together with case simulations.

The originality in this paper mainly focuses on the proposed control method. Note that the control design employs spatial sampling strategies rather than the usual temporal sampling strategies. Under certain choices of sampling subsets, the control design could alleviate the work of collecting data for constructing the control inputs, compared with those defined on the system state variables in the full space. In fact, only the data of the state variables on the sampling subsets is required to be collected in the proposed method. In addition, the spatial sampling strategies adopted in the proposed method is in line with the potential-output structure of the neurons and their attached dendrites. Indeed, in the case when the sampling subsets are 3-dimensional and enough small, their sampled means can be viewed as the potentials at the contained 2-dimensional interfaces between neurons and their dendrites, to be delivered to other neurons through the dendrites, by the continuity of classical solutions.

The rest of the paper is organized as follows. In Section 2, the synchronization problem is stated with the proposed control, and the solution existence is discussed in its following section. Section 4 is devoted to the synchronization equivalence. In Section 5, general algebraic connectivity is studied, based on which the sufficient condition for system synchronization is given in Section 6. Before we conclude this paper, the case simulations are demonstrated.

2. Synchronization problem and control design

First of all, let us introduce some notations used throughout this paper. Let $\Omega \subset \mathbb{R}^m$ be a bounded (nonempty) open set with reasonably smooth boundary $\partial\Omega$, and $|\Omega|$ denote the volume of the set $\overline{\Omega} := \Omega \cup \partial\Omega$. The notation ∞ means $+\infty$. For any function $\xi : \overline{\Omega} \times (0, \infty) \to \mathbb{R}$, let $\nabla\xi(s, t) := (\partial\xi(s, t)/\partial s_1, \dots, \partial\xi(s, t)/\partial s_m)^\top$ denote its gradient function and $\triangle\xi(s, t) := \sum_{j=1}^{m} \partial^2\xi(s, t)/\partial s_j^2$ denote its Laplacian function if they exist, where $s := (s_1, \dots, s_m)$. Let $(L_k^2(\Omega), \langle\cdot,\cdot\rangle_{L_k^2})$ denote the Hilbert space of the equivalence classes of the measurable functions $\xi : \Omega \to \mathbb{R}^k$ such that $\langle\xi, \xi\rangle_{L_k^2} := \int_\Omega (\xi(s))^\top \xi(s) ds < \infty$, and define the corresponding norm $\|\xi\|_{L_k^2} := \langle\xi, \xi\rangle_{L_k^2}^{1/2}$ for any $\xi \in L_k^2(\Omega)$. As usual, we write $L_1^2(\Omega)$ as $L^2(\Omega)$, and $\|\cdot\|_{L_1^2}$ as $\|\cdot\|_{L^2}$. Let ∂_t denote the usual partial differential operator $\partial/\partial t$ with respect to time t. For any $n \in \mathbb{N}$, define the set notation $\overline{n} := \{1, 2, \dots, n\}$.

With the desire to further understand the neural synchronization mechanism, this paper explores the synchronization control problem of $n > 1$ RD FitzHugh–Nagumo systems having the form

$$\Sigma_i : \begin{cases} \epsilon\partial_t x_i(s, t) = d_1\triangle x_i(s, t) + c_{11}x_i(s, t) + c_{12}y_i(s, t) \\ \qquad\qquad + f(x_i(s, t)) + u_i; \\ \tau\partial_t y_i(s, t) = d_2\triangle y_i(s, t) + c_{21}x_i(s, t) + c_{22}y_i(s, t) \end{cases} \tag{1}$$

on the domain $\Omega \times (0, \infty)$ for any $i \in \overline{n}$ and satisfying the zero-flux Neumann boundary conditions

$$\partial x_i(s, t)/\partial\mathbf{n} = \partial y_i(s, t)/\partial\mathbf{n} = 0 \tag{2}$$

on the boundary $\partial\Omega$ for any $t \in (0, \infty)$ and $i \in \overline{n}$, where symbol $\mathbf{n}$ denotes the exterior normal vector on $\partial\Omega$. In system (1), $\epsilon, \tau, d_1, d_2 > 0$ and $c_{11}, c_{12}, c_{21}, c_{22} \in \mathbb{R}$ are the constant coefficients, $f : \mathbb{R} \to \mathbb{R}$ is a cubic function having the form $f(x) = -x^3 + bx^2 + cx$ with constants $b, c \in \mathbb{R}$, and each u_i is the control input to be designed.

In the case when input $u_i \equiv 0$, system Σ_i in (1) generalizes the seminal system (Nagumo, Arimoto, & Yoshizawa, 1962), and provides a general form covering some RD systems studied in past years, such as those in the works (Ambrosio & Aziz-Alaoui, 2012; Chen, Jimbo, & Morita, 2015; He, Ai, & Liu, 2013). Note that the function f in (1) covers some common cubic functions, such as $f_1(x) = 3x - x^3$ and $f_2(x) = x(x - 1)(a - x)$ with a certain constant $a \in \mathbb{R}$. By the well-known inequality $-z^2 + 2dz \le d^2$ for any $d, z \in \mathbb{R}$, the cubic function f satisfies the following lemma.

Lemma 1. *Consider the function f in (1). For any $x, y \in \mathbb{R}$,*
(i) the derivative $f'(x) \le M_1$, and $(x - y)(f(x) - f(y)) \le M_1(x - y)^2$ holds with the constant $M_1 := b^2/3 + c$;
(ii) given any constant $a \in \mathbb{R}$, there exists a constant $K > 0$ such that the inequality $ax^2 + xf(x) \le K$ holds.

Proof. (i) Note that $f'(x) = 3(-x^2 + 2bx/3) + c \le b^2/3 + c = M_1$. Following Taylor's formula, $(x - y)(f(x) - f(y)) \le (x - y)^2(f'(y) + (\frac{1}{4}f''(y))^2) = (x - y)^2(-\frac{3}{4}y^2 + \frac{b}{2}y + \frac{b^2}{4} + c)$. It follows that $(x - y)(f(x) - f(y)) \le M_1(x - y)^2$.
(ii) Following $-\frac{1}{2}x^4 + bx^3 \le \frac{b^2}{2}x^2$, we have $xf(x) + ax^2 \le -\frac{1}{2}x^4 + (\frac{b^2}{2} + c + a)x^2 \le K$, where $K = (b^2 + 2c + 2a)^2/8$. □

Throughout this paper, all systems in (1) are assumed to satisfy Assumption 2 and to have the unique classical solutions $x_1, y_1, \dots, x_n, y_n$ on the domain $\overline{\Omega} \times [0, \infty)$. See the next section for a discussion on the solution existence and uniqueness. For the conditions in Assumption 2, condition (i) ensures that in the space $(L^2(\Omega), \|\cdot\|_{L^2})$ there exists a time-invariant set for all state variables $x_i(\cdot, t)$ and $y_i(\cdot, t)$ of the systems in (1), and condition (ii) ensures that all $x_i(\cdot, t)$ and $y_i(\cdot, t)$ evolve toward homogeneous solutions in the space, according to Propositions 4 and 7 in the next sections. Note that there are at least three independent cases in which condition (ii) can be satisfied. In the first two cases, the constant d_1 is large enough and the constant M is small enough or nonpositive. In the third case, the set Ω is convex and its diameter diam(Ω) is small enough, by the formula $\lambda_\triangle \ge (\pi/\text{diam}(\Omega))^2$ from Payne and Weinberger (1960).

Assumption 2. System (1) satisfies the following conditions:
(i) there exist certain constants $M_2 \in \mathbb{R}$ and $\delta > 0$ such that the inequality $z^\top\overline{C}z \le z^\top \text{diag}(M_2, -\delta)z$ holds for any $z \in \mathbb{R}^2$, where $\overline{C} := (c_{ij}) \in \mathbb{R}^{2\times 2}$ is the matrix defined by all the constants c_{ij} in (1).
(ii) the inequality $d_1\lambda_\triangle > M := M_1 + M_2$ holds, where M_1 is given in Lemma 1, M_2 is listed in (i), and $\lambda_\triangle$ is the smallest positive eigenvalue of Neumann operator $-\triangle$ on the set Ω satisfying the zero-flux Neumann boundary condition.

For the systems $\Sigma_1, \dots, \Sigma_n$ in (1), the control objective is to make all of them achieve synchronization in the sense that, given arbitrary admissible initial values to the systems, $x_1(s, t) = \dots = x_n(s, t)$ and $y_1(s, t) = \dots = y_n(s, t)$ hold for any $s \in \Omega$ when $t \to \infty$. Following the continuity of classical solutions, note that $x_1(s, t) = \dots = x_n(s, t)$ and $y_1(s, t) = \dots = y_n(s, t)$ hold for any $s \in \partial\Omega$ when $t \to \infty$, provided that the control objective is achieved. To achieve synchronization, the coordination between the variable states of the systems is usually necessary, in view of the arbitrariness of the initial values. The next paragraph recalls some concepts from graph theory for describing the communication topology of the systems.

Let $Q := (V_Q, E_Q, A_Q)$ denote a simple weighted digraph, where $V_Q = \{v_i : i \in \overline{n}\}$ is its vertex set, $E_Q \subseteq V_Q \times V_Q$ is its directed