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Robustness of critical bit rates for practical stabilization of networked control systems[☆]

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ABSTRACT

In this paper we address the question of robustness of critical bit rates for the stabilization of networked control systems over digital communication channels. For a deterministic nonlinear system, the smallest bit rate above which practical stabilization (in the sense of set-invariance) can be achieved is measured by the invariance entropy. Under the assumptions of chain controllability and uniform hyperbolicity on the set of interest, we prove that the invariance entropy varies continuously with respect to system parameters. Hence, in this case the critical bit rate is robust with respect to small perturbations.

1. Introduction

In networked control, the communication between sensors, controllers and actuators is accomplished through a shared digital communication network. There are several aspects of such networks which put severe constraints on the available data rates. In the first place, the digital nature of the communication channels puts a limit on the number of bits that can be transmitted reliably in one unit of time. This naturally leads to the problem of determining the smallest channel capacity or bit rate above which a certain control objective such as stabilization can be achieved. Numerous authors have studied this problem both in deterministic and stochastic setups, for a variety of control objectives, under different assumptions on the network topologies and on the coding and control policies, see, e.g., the papers (Colonius, 2012; Colonius & Kawan, 2009; Delvenne & Kawan, 2016; Liberzon & Hespanha, 2005; Matveev & Pogromsky, 2016; Nair, Evans, Mareels, & Moran, 2004), the monographs (Kawan, 2013; Matveev & Savkin, 2009; Yüksel & Başar, 2013) and the surveys (Franceschetti & Minero, 2014; Nair, Fagnani, Zampieri, & Evans, 2007). In many of these works, expressions or estimates of the critical capacities in terms of dynamical entropies or Lyapunov exponents have been obtained.

In practice, there are always unknown parameters in the system under consideration. Therefore, one important issue is the robustness of the critical bit rates under variation of system parameters. Since both entropy and Lyapunov exponents as functions of the dynamical system are known to have jump discontinuities, one cannot expect that critical bit rates behave robustly without appropriate assumptions on the system under consideration. To the best of our knowledge, this issue so far has only been addressed in Matveev and Pogromsky (2016) for state estimation objectives. In the paper at hand, we identify a setup in which the desired robustness property is satisfied for the problem of practical stabilization (i.e., set-invariance).

The paper (Nair et al., 2004) introduced the notion of topological feedback entropy as a measure for the smallest rate of information above which a compact subset of the state space can be rendered invariant by a controller which receives the state information via a noiseless discrete channel. An equivalent notion, called invariance entropy, was introduced in Colonius and Kawan (2009). The monograph (Kawan, 2013) presents the foundations of a theory which aims at a characterization of invariance entropy in terms of dynamical quantities such as Lyapunov exponents and escape rates. This works particularly well under the assumption that the subset to be stabilized has a uniformly hyperbolic structure.

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In fact, for a uniformly hyperbolic chain control set of a control-affine system, the paper (Da Silva & Kawan, 2016b) provides a closed expression of the invariance entropy in terms of Lyapunov exponents. Uniformly hyperbolic chain control sets are also known to vary continuously in the Hausdorff metric under variation of system parameters, cf. Colonius and Du (2001). We use these results to prove that the invariance entropy of a uniformly hyperbolic chain control set varies continuously with respect to parameters. Uniformly hyperbolic chain control sets arise around hyperbolic equilibrium points when the control range is sufficiently small and certain regularity assumptions are satisfied. In Colonius and Lettau (2016) an example of a stirred tank reactor is studied, where this happens. A large class of algebraic examples for uniformly hyperbolic chain control sets was identified in Da Silva and Kawan (2016a).

The paper is organized as follows. In Section 2, we give a review of control-affine systems and invariance entropy. In Section 3, we provide a new justification that the invariance entropy is a measure for the smallest data rate above which a set can be rendered invariant by a symbolic coding and control scheme. Section 4 contains the proof of the main result about the continuity of the invariance entropy on a uniformly hyperbolic chain control set. The proof uses semicontinuity properties of spectral sets for additive and subadditive cocycles over control flows of parametrized control-affine systems, which are derived in Section 4.1. The actual proof is given in Section 4.2. Section 5 discusses an example, where the nominal system is linear and Section 6 outlines some future directions. Some proofs and reviews of technical concepts are presented in Appendices A–C.

2. Preliminaries

Notation. All manifolds considered in this paper are smooth, i.e., equipped with a C^∞ differentiable structure. If $f : M \to N$ is a differentiable map between smooth manifolds M and N, then $\mathrm{d}f(x) : T_xM \to T_{f(x)}N$ denotes its derivative at $x \in M$. On a Riemannian manifold M, we always write $d(x, y)$ for the geodesic distance of two points $x, y \in M$. The norm on each tangent space T_xM is simply denoted by $|\cdot|$. We write $\mathrm{int}A$ and $\mathrm{cl}A$ for the interior and closure of a set A, respectively. By $\mathrm{dist}(x, A)$ we denote $\inf_{y\in A} d(x, y)$. If $u_1 : [0, \tau_1] \to U$ and $u_2 : [0, \tau_2] \to U$ are functions, we write $u_1u_2 : [0, \tau_1+\tau_2] \to U$ for their concatenation, i.e., $(u_1u_2)(t) = u_1(t)$ for $t \in [0, \tau_1]$ and $(u_1u_2)(t) = u_2(t - \tau_1)$ for $t \in (\tau_1, \tau_1 + \tau_2]$. We also write u^n for the concatenation of n copies of u.

2.1. Control-affine systems

A *control-affine system* is given by

$$\dot{x} = f_0(x) + \sum_{i=1}^{m} u_i(t)f_i(x), \quad u \in \mathcal{U}, \tag{1}$$

where $f_0, f_1, \ldots, f_m$ are smooth vector fields on a Riemannian manifold M and $\mathcal{U} = L^\infty(\mathbb{R}, U)$ for a compact and convex set $U \subset \mathbb{R}^m$ with $0 \in \mathrm{int}U$.

By $\varphi(t, x, u)$ we denote the unique solution of (1) for the control function $u \in \mathcal{U}$, satisfying the initial condition $\varphi(0, x, u) = x$. For simplicity, we assume that all solutions are defined on $\mathbb{R}$, which yields a *transition map*

$$\varphi : \mathbb{R} \times M \times \mathcal{U} \to M, \quad (t, x, u) \mapsto \varphi(t, x, u).$$

We also write $\varphi_{t,u}(x) = \varphi(t, x, u)$. The set $\mathcal{U}$ of control functions becomes a compact metrizable space with the weak*-topology of $L^\infty(\mathbb{R}, \mathbb{R}^m) = L^1(\mathbb{R}, \mathbb{R}^m)^*$ and φ can be extended to a continuous skew-product flow

$$\Phi : \mathbb{R} \times (\mathcal{U} \times M) \to \mathcal{U} \times M, \quad \Phi_t(u, x) = (\theta_t u, \varphi(t, x, u)),$$

called the *control flow* of the control system (1). Here $\theta : \mathbb{R} \times \mathcal{U} \to \mathcal{U}$, $\theta_t u = u(t + \cdot)$, is the *shift flow* on $\mathcal{U}$. These general facts can be found in Colonius and Kliemann (2000).

The *set of points reachable from x up to time T* is

$$\mathcal{O}^+_{\leq T}(x) := \bigcup_{u\in\mathcal{U}, t\in[0,T]} \{\varphi(t, x, u)\}$$

and $\mathcal{O}^+(x) := \bigcup_{T>0}\mathcal{O}^+_{\leq T}(x)$ is the *positive orbit of* x. With $\mathcal{O}^-_{\leq T}(x)$ and $\mathcal{O}^-(x)$ we denote the corresponding sets for the time-reversed system. We say that system (1) is *locally accessible from x* if $\mathrm{int}\mathcal{O}^\pm_{\leq T}(x) \neq \emptyset$ for all $T > 0$. A sufficient condition for local accessibility is the *Lie algebra rank condition, briefly LARC*. This condition is satisfied at $x \in M$ if the Lie algebra $\mathcal{L}$ generated by the vector fields $f_0, f_1, \ldots, f_m$ satisfies $\mathcal{L}(x) = \{f(x) : f \in \mathcal{L}\} = T_xM$.

A set $D \subset M$ is a *control set* of (1) if it is maximal w.r.t. set inclusion with the following properties:

(i) D is *controlled invariant*, i.e., for each $x \in D$ there is $u \in \mathcal{U}$ with $\varphi(\mathbb{R}_+, x, u) \subset D$.
(ii) *Approximate controllability* holds on D, i.e., $D \subset \mathrm{cl}\mathcal{O}^+(x)$ for all $x \in D$.

The *lift of a control set D to* $\mathcal{U} \times M$ is defined by

$$\mathcal{D} := \mathrm{cl}\,\{(u, x) \in \mathcal{U} \times M \;:\; \varphi(\mathbb{R}, x, u) \subset \mathrm{int}D\}\,.$$

Control sets with nonempty interior have the *no-return property*: If $x \in D$ and $\varphi(\tau, x, u) \in D$ for some $\tau > 0$ and $u \in \mathcal{U}$, then $\varphi(t, x, u) \in D$ for all $t \in [0, \tau]$.

A *chain control set* of (1) is a set $E \subset M$ which is maximal with the following properties:

(i) E is *all-time controlled invariant*, i.e., for each $x \in E$ there is $u \in \mathcal{U}$ with $\varphi(\mathbb{R}, x, u) \subset E$.
(ii) *Chain controllability* holds on E, i.e., for each two $x, y \in E$ and all $\varepsilon, T > 0$ there are $n \in \mathbb{N}$, controls $u_0, \ldots, u_{n-1} \in \mathcal{U}$, states $x_0 = x, x_1, \ldots, x_{n-1}, x_n = y$ and times $t_0, \ldots, t_{n-1} \geq T$ such that

$$d(\varphi(t_i, x_i, u_i), x_{i+1}) < \varepsilon, \quad i = 0, 1, \ldots, n-1.$$

The *lift of a chain control set E to* $\mathcal{U} \times M$ is defined by

$$\mathcal{E} := \{(u, x) \in \mathcal{U} \times M \;:\; \varphi(\mathbb{R}, x, u) \subset E\}\,.$$

The set $\mathcal{E}$ is a closed invariant set of the control flow and it is compact if E is compact. If D is a control set such that local accessibility holds on $\mathrm{int}D \neq \emptyset$, then D is contained in a unique chain control set (see Colonius & Kliemann, 2000, Ch. 4). Moreover, the lifts of the chain control sets are the maximal invariant chain transitive sets of the control flow (see Appendix B for the definition of chain transitivity).

A compact chain control set E is called *uniformly hyperbolic* if there exists a decomposition

$$T_xM = E^-_{u,x} \oplus E^+_{u,x} \quad \forall (u, x) \in \mathcal{E}$$

with subspaces $E^\pm_{u,x}$ satisfying

(H1) $\mathrm{d}\varphi_{t,u}(x)E^\pm_{u,x} = E^\pm_{\Phi_t(u,x)}$ for all $(u, x) \in \mathcal{E}$, $t \in \mathbb{R}$.
(H2) There exist constants $0 < c \leq 1$ and $\lambda > 0$ such that for all $(u, x) \in \mathcal{E}$,

$$\left|\mathrm{d}\varphi_{t,u}(x)v\right| \leq c^{-1}\mathrm{e}^{-\lambda t}|v| \quad \text{for all } t \geq 0,\ v \in E^-_{u,x},$$
$$\left|\mathrm{d}\varphi_{t,u}(x)v\right| \geq c\mathrm{e}^{\lambda t}|v| \quad \text{for all } t \geq 0,\ v \in E^+_{u,x}.$$

From (H1) and (H2) it follows that $E^\pm_{u,x}$ depend continuously on (u, x), cf. Kawan (2013, Ch. 6). We write

$$J^+\varphi_{t,u}(x) := \left|\det \mathrm{d}\varphi_{t,u}(x)_{|E^+_{u,x}} : E^+_{u,x} \to E^+_{\Phi_t(u,x)}\right|$$