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Brief paper

Output formation-containment of interacted heterogeneous linear systems by distributed hybrid active control☆

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ABSTRACT

This paper investigates the output formation-containment problem of interacted heterogeneous linear systems, where each heterogeneous system, whether the leader or the follower, has different dimensions and dynamics. Different from existing literature, discrete-time communication manner is deployed to reduce the communication consumption. By the impulsive control method, a distributed hybrid active controller is designed using the discrete-time information of neighbors. It achieves the output formation-containment of heterogeneous systems if two related conditions are satisfied, namely, local linear matrix inequalities (LMIs) and a bounded constraint on the average interacted interval. Moreover, the controller parameter design is further simplified by replacing the LMI condition with a Hurwitz condition, which can be easily guaranteed by solving a Riccati equation. Finally, a numerical example is provided to demonstrate the effectiveness of the theoretical result.

1. Introduction

Recently, containment control problem for multi-agent systems has drawn much interest, where it aims to guard followers by driving them into the convex hull spanned by the leaders (Dong, Meng, Shi, Lu, & Zhong, 2014; Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Li, Xie, & Zhang, 2015; Li, Yu, Liu, & Huang, 2016). Most existing works focus on homogeneous systems, however, neglect the individual difference in a group.

As research on the consensus and formation cooperation of heterogeneous multi-agent systems emerges (Kim, Shim, & Jin, 2011; Liu, Wang, Xiao, & Yang, 2017; Meng, Yang, Dimarogonas, & Johansson, 2014; Su & Huang, 2012; Wieland, Sepulchre, & Allgöwer, 2011), few works discuss the containment control problem of heterogeneous systems. Zheng and Wang (2014) studied the containment of heterogeneous integrators, where the dynamics of leaders and followers were governed by either the single integrator or the double integrator. Chu, Gao, and Zhang (2016) and Haghshenas, Badamchizadeh, and Baradarannia (2015) investigated the containment problem of heterogeneous general linear systems by the distributed state feedback controller and the distributed dynamic adaptive output feedback controller, respectively. However, in the above works, there are two aspects worth further investigating. Firstly, they all assume leaders are identically dynamical and neutrally stable to ensure there exists an accessible convex hull spanned by leaders. In other words, when leaders are heterogeneous, the existing methods may fail to achieve the containment. To this case, a cooperation is required within leaders, helping themselves to achieve a reliable formation. Note that a cooperation in leaders makes a lot of advances, such as reduce overall navigational error in a pigeon group (Simons, 2004) or reduce the risk of obstacle avoidance in robots (Li, Zhang, Su, & Yang, 2014). In this way, output formation-containment control problem of heterogeneous systems naturally arises, where leaders cooperatively achieve a desired convex-form formation and the followers drive into the convex hull spanned by the leaders. Secondly, in existing results, communication of heterogeneous systems is required on the whole time horizon, which brings a heavy communication burden. Intermittent control provides one way to reduce communication consumption (Li, Phillips, & Sanfelice, 2016; Phillips, Li, & Sanfelice, 2016; Wen, Duan, Ren, & Chen, 2014). Our previous work (Wang, Liu, Xiao, & Lin, 2017) proposed an intermittent communication scheme to achieve the formation-containment of heterogeneous systems. However, intermittent

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communication still requires a continuous-time interaction on separated time intervals. To further reduce the communication consumption, we utilize the impulsive control method in this paper, where each system only obtains its neighbors' information at discrete time instants. He et al. (2015) designed a distributed impulsive control framework to achieve the quasi-synchronization of heterogeneous dynamic networks. However, the impulsive control is directly injected on the internal state of the system, where it fails to work on the majority of physical systems which cannot afford a sudden state jump in practice, such as the position and attitude control of an unmanned automatic vehicle. Hence, it unavoidably brings a challenge when implementing existing impulsive techniques (Guan, Liu, Feng, & Jian, 2012; Lu, Kurths, Cao, Mahdavi, & Huang, 2012; Tang, Xing, Karimi, Kocarev, & Kurths, 2016; Wang, Duan, & Cao, 2012) into the formation-containment problem.

In this paper, the output formation-containment problem of heterogeneous systems is addressed by the distributed hybrid active controller. The proposed controller is designed by introducing a local auxiliary state and injecting the impulses on it at discrete time instants. Based on the stability analysis method of hybrid systems, output formation-containment problem of heterogeneous systems is solved if two bounded conditions are satisfied. Furthermore, a detailed parameter design is presented. The key contributions of this paper can be summarized as follows: (1) Output formation-containment problem of heterogeneous systems has been formulated and addressed via a distributed hybrid active controller using the discrete-time information of neighbors, where it does not require leaders to be identically dynamical and avoids the continuous-time communication as needed in existing literature; (2) By injecting the impulse on an auxiliary state, the proposed hybrid active controller provides an interface that enables impulsive techniques applied on physical systems, of which some states cannot afford a suddenly jump in practice.

The preliminaries and problem statement are given in Section 2. The distributed hybrid active state feedback controllers are designed in Section 3, where problem transformation and proof analysis are given. The illustrative numerical example is given in Section 4 and the conclusions are drawn in Section 5.

2. Preliminaries and problem statement

In this section, some preliminaries of algebra graph theory are given, as well as description and definition of the output formation-containment for heterogeneous linear systems.

2.1. Notations

In this paper, $\mathbb{R}$ denotes the set of real number, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrix. I_n represents $n \times n$ identity matrix and 1_n is a n-dimension column with all elements being 1. $\|\cdot\|$ denotes the Euclidean norm for vectors or the induced 2-norm for matrices. For a given matrix P, denote $P > 0$ if P is a symmetric positive definite matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ represent the maximum eigenvalue and the minimum eigenvalue of P, respectively. $\otimes$ stands for Kronecker product. The notation $col(x_1, \dots, x_N)$ stands for a column vector by stacking them together. Notation $diag\{b_1, \dots, b_N\}$ denotes the diagonal matrix with diagonal elements $b_1, \dots, b_N$, and the notation $block\ diag\{P_1, \dots, P_N\}$ represents a matrix in block diagonal form with the ith diagonal block as P_i.

2.2. Graph theory

Denote $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ as a graph with a set of nodes $\mathcal{V} = \{1, 2, \dots, N\}$, a set of edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ and a weighted adjacency matrix $\mathcal{A} = \left(a_{ij}\right)_{N\times N}$. Node i represents the ith system and an edge e_{ji} in graph is denoted by the ordered pair nodes $\{j, i\}$. $\{j, i\} \in \mathcal{E}$ if and only if node i can obtain the information from node j. A path in graph $\mathcal{G}$ is an ordered sequence $v_1, v_2, \dots, v_k$ of systems such that any ordered pair of vertices appearing consecutively in the sequence is an edge of the graph, i.e., (v_i, v_{i+1}), for any $i = 1, 2, \dots, k-1$. Define the adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ associated with $\mathcal{G}$ such that $a_{ij} > 0$ if $\{j, i\} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. We assume there is no self-loop in the graph $\mathcal{G}$, i.e., $a_{ii} = 0$. Define $N_i = \{j|a_{ij} > 0\}$. The Laplacian matrix $\mathcal{L} = (l_{ij})_{N\times N}$ of graph $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}$ if $i \neq j$, otherwise $l_{ij} = \sum_{k=1,k\neq i}^{N} a_{ik}$.

2.3. Problem statement

Consider a set of N interacted heterogeneous linear systems, which consists of M followers labeled with $i \in \mathcal{I}_F = \{1, 2, \dots, M\}$ and $N - M$ leaders labeled with $i \in \mathcal{I}_R = \{M+1, M+2, \dots, N\}$. The dynamics of the system i is described by

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i, \\ y_i = C_i x_i, \quad i \in \mathcal{V}, \end{cases} \tag{1}$$

where $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times r_i}$, $C_i \in \mathbb{R}^{p\times n_i}$, and $x_i \in \mathbb{R}^{n_i}$ is the state of the system, $y_i \in \mathbb{R}^p$ and $u_i \in \mathbb{R}^{r_i}$ are the measurement output and the control input of the system, respectively. n_i represents the order of system i. The interaction of heterogeneous systems (1) is discrete-time and it is subject to a communication topology $\mathcal{G}$, of which the corresponding Laplacian matrix $\mathcal{L}$ is in the form of $\begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ \mathbf{0} & \mathcal{L}_3 \end{bmatrix}$, where $\mathcal{L}_1 \in \mathbb{R}^{M\times M}$, $\mathcal{L}_2 \in \mathbb{R}^{M\times(N-M)}$ and $\mathcal{L}_3 \in \mathbb{R}^{(N-M)\times(N-M)}$. Denote $\mathcal{G}_{\mathcal{R}}$ and $\mathcal{G}_{\mathcal{F}}$ as the interaction topologies of leaders and followers, respectively. Some relevant assumptions are introduced.

Assumption 1. Both $\mathcal{G}_{\mathcal{R}}$ and $\mathcal{G}_{\mathcal{F}}$ are undirected and connected.

Assumption 2. For each follower, there exists at least one leader that has a directed path to it.

Assumption 3. (A_i, B_i) is stabilizable for $i = 1, \dots, N$.

Assumptions 1 and 2 ensure that the graph topology $\mathcal{G}$ has a spanning tree, which is necessary for the cooperation. Assumption 3 is a common assumption on the stabilizability of the heterogeneous linear systems. In the following, a useful lemma about the topology is given.

Lemma 1 (Meng, Ren, & You, 2010). *If the interaction topology $\mathcal{G}$ satisfies Assumption 2, all the eigenvalues of $\mathcal{L}_1$ are positive, each entry of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is nonnegative, and each row of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ has a sum equal to one.*

To achieve the cooperation, virtual structure formation approach is employed by introducing a virtual leader to describe the trajectory of the whole group.

$$\begin{cases} \dot{x}_0(t) = S x_0(t), \\ y_0(t) = R x_0(t), \end{cases} \tag{2}$$

where $S \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{p\times n}$, and $x_0 \in \mathbb{R}^n$ is the state of the virtual leader system, $y_0 \in \mathbb{R}^p$ is the output state.

Assumption 4. The linear matrix equations

$$\begin{aligned} &\Pi_i S = A_i \Pi_i + B_i U_i, \\ &C_i \Pi_i = R, \end{aligned} \tag{3}$$

have solutions (Π_i, U_i) for all $i \in \mathcal{V}$.