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Brief paper

Asynchronous sliding mode control of Markovian jump systems with time-varying delays and partly accessible mode detection probabilities[☆]

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ABSTRACT

In this work, the problem of *asynchronous* sliding mode control (SMC) is investigated for a class of uncertain Markovian jump systems (MJSs) with time-varying delays and stochastic perturbation. It is assumed that the system modes cannot be obtained synchronously by the controller, but instead there is a detector that provides estimated values of the system modes. This asynchronous phenomenon between the system modes and controller modes will be described in this work via a hidden Markov model with partly accessible mode detection probabilities. Based on a common sliding surface, an asynchronous SMC law depending on the detector mode is synthesized to ensure the mean square stability of the sliding mode dynamics and the reachability of the specified sliding surface simultaneously. Moreover, a design algorithm for obtaining the asynchronous SMC law is established. Finally, an application of the automotive electronic throttle system is provided to illustrate the effectiveness and advantages of the proposed asynchronous sliding mode control approach.

1. Introduction

Markovian jump systems (MJSs) have received considerable attention in the past decades, since many real-world systems, such as manufacturing systems, aerospace systems and networked control systems with abrupt variations in their structures can be effectively represented by MJSs, in which the abrupt variations may happen due to random failures or repairs of components, changing of subsystem interconnections, abrupt variations in the operating point, etc. (Costa, Fragoso, & Marques, 2005; Shi & Li, 2015; Shi & Yu, 2009). A variety of works have been published with respect to the stability and stabilization of MJSs, see Bolzern, Colaneri, and de Nicolao (2013), Feng, Lam, and Shu (2010), Fioravant, Gonçalves, and Geromel (2013), Zhang and Lam (2010) and the references therein. Especially, the MJSs subject to time delays has become an active research topic, since the time delays may happen inevitably in practical applications (Wei, Qiu, Karimi, & Wang, 2013; Zhang, Boukas, & Lam, 2008).

On the other hand, as an effective robust control approach for uncertain systems with parameter variations and external disturbances, significant progresses have been recently achieved on the sliding mode control (SMC) of MJSs (Basin, Panathula, & Shtessel, 2017; Basin & Rodríguez-Ramírez, 2014). Among them, the mode-dependent sliding surface was designed in Niu, Ho, and Wang (2007) to establish the connections among different sliding functions under Markovian jumping. A singular system approach in Ma and Boukas (2009) was proposed to design both mode-independent and mode-dependent sliding surfaces. Following these excellent results, the SMC problem for various MJSs has been widely studied including time delays (Karimi, 2012), unmeasured states (Wu, Gao, Liu, & Li, 2017; Yin, Yang, & Kaynak, 2017), incomplete transition information (Kao, Xie, Zhang, & Karimi, 2015; Zhang, Wang, & Shi, 2013), actuator degradation (Chen, Niu, & Zou, 2013), missing measurement (Chen, Niu, & Zou, 2014), etc.

However, it should be pointed out that, in all the aforementioned works, it was implicitly assumed that the information of system modes was fully accessible for the sliding mode controller all the time, in order to ensure the synchronization between the controller mode and the system mode. Thus, the *mode-dependent* sliding mode controller can be achieved. Unfortunately, the above

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ideal assumption is difficult to be satisfied in practical applications. For example, the mode information of plant cannot be completely accessible due to communication delays and missing measurement, which may bring the asynchronization phenomenon between sliding mode controller modes and system modes. Although the mode-independent SMC approach in Huang, Lin, and Lin (2012) may be feasible for the above case, it also brings more conservatism due to neglecting all the available modes information (Liu, Ho, & Sun, 2008). Hence, the importance of asynchronous control/filtering for MJSs has already begun receiving more attentions. For example, the l_2–l_∞ and H_2 asynchronous filtering for discrete-time MJSs were addressed in de Oliveira and Costa (2017) and Wu, Shi, Su, and Chu (2014), respectively. Based on the hidden Markov model, the H_2 and passive asynchronous control methods were further proposed in Costa, Fragoso, and Todorov (2015) and Wu, Shi, Shu, Su, and Lu (2017), respectively. However, to the authors' best knowledge, until now, the problem of *asynchronous* SMC for time-delay MJSs has not been investigated. Moreover, due to the special structure of SMC systems, the asynchronous characteristic between the controller modes and the system modes is also more complex and interesting, which motivates the present work.

In this work, we seek to investigate the asynchronous SMC problem for a class of discrete-time MJSs subjected to time-varying delays and stochastic perturbation. With a similar framework in Costa et al. (2015), it is assumed that the information of system modes to controller can be only estimated by a detector via a hidden Markov model with a mode detection probability matrix (MDPM). A key feature of this work is that the mode detection probabilities are further considered to be *partly accessible* to controller design, which shows more generalized than the existing works as in Costa et al. (2015), de Oliveira and Costa (2017) and Wu et al. (2017). Thus, in order to design asynchronous SMC under the above partly accessible assumption, we have to answer the following questions:

- **Q1:** How to design a suitable sliding function and SMC law just by using the detected modes?
- **Q2:** How to tackle the *partly* accessible mode detection probabilities in designing asynchronous SMC law?
- **Q3:** How to guarantee the mean square stability of the sliding mode dynamics and the reachability of the specified sliding surface subjected to *partly* accessible mode detection probabilities, time-varying delays and stochastic perturbation?

This work will provide satisfying answers to the above three questions. *The main contributions of this work are highlighted as follows. (1) The asynchronous phenomenon between the system modes and the detector modes to controller is coped with a hidden Markov model with the partly accessible mode detection probabilities, which is more generalized than the one considered in* Costa et al. (2015), de Oliveira and Costa (2017) *and* Wu et al. (2017). *(2) A new common sliding function is constructed properly, and by just using the information of the detected modes, an asynchronous SMC law is designed to ensure the mean square stability of the sliding mode dynamics. The framework of the proposed asynchronous SMC covers the existing mode-dependent SMC approaches as in* Chen et al. (2014), Kao et al. (2015) *and* Karimi (2012) and Zhang et al. (2013) *and the mode-independent SMC approach as in* Huang et al. (2012) *as two special cases. (3) The reachability of a sliding region around the specified sliding surface is proven by using a stochastic Lyapunov method, and the design algorithm of the proposed asynchronous SMC law is derived by means of a necessary and sufficient condition.*

Notation. All matrices in this work are supposed to have compatible dimensions. $\mathbb{Z}_-$ denotes the set of negative integers. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to probability measure. For a real symmetric matrix M, $M > 0$ represents that M is a positive-definite matrix. The shorthand "diag{·}" denotes a block diagonal matrix. In symmetric block matrices, the symbol "$\star$" is used as an ellipsis for terms induced for symmetry. $\|\cdot\|$ denotes the Euclidean norm of a vector or its induced matrix norm.

2. Problem formulation

2.1. System description

Given the probability space $(\Omega, \mathcal{F}, \text{Prob}\{\cdot\})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of events, and $\text{Prob}\{\cdot\}$ is the probability measure defined on $\mathcal{F}$. The parameter $\{r(k) = i,\ k \geq 0\}$ is a Markov chain taking values on a finite set $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$ with transition probability matrix $\Pi \triangleq [\pi_{ij}]$, $i, j \in \mathcal{N}$, and having the following transition probability from mode i at sample time k to mode j at sample time $k + 1$:

$$\pi_{ij} = \text{Prob}\{r(k+1) = j \mid r(k) = i\}, \forall i, j \in \mathcal{N}, \tag{1}$$

where $\pi_{ij} \in [0, 1]$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. Now, we consider the following discrete-time MJSs with time-varying delays and stochastic perturbation:

$$\begin{cases} x(k+1) = [A(r(k)) + \Delta A(r(k), k)]\, x(k) \\ \quad + [A_d(r(k)) + \Delta A_d(r(k), k)]\, x(k - d(k)) \\ \quad + B(r(k))\, [u(k) + f(x(k), x(k - d(k)), r(k))] \\ \quad + [G(r(k))x(k) + G_d(r(k))x(k - d(k))]\, \varpi(k), \\ x(n) = \phi(n),\ \forall n \in \mathbb{Z}_-, \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input, and $\varpi(k) \in \mathbb{R}$ is a scalar Wiener process satisfying $\mathbf{E}\{\varpi(k)\} = 0$, $\mathbf{E}\{\varpi^2(k)\} = \zeta$, $\mathbf{E}\{\varpi(k_1)\varpi(k_2)\} = 0$ for $k_1 \neq k_2$. $\phi(n)$ is a given initial condition, $\forall n \in \mathbb{Z}_-$. The discrete time-varying delay satisfies $d_m \leq d(k) \leq d_M$, where d_m and d_M are known positive integers representing the lower and upper bounds of the time delay, respectively.

For each $r(k) = i$, the matrices $A_i \triangleq A(r(k))$, $A_{di} \triangleq A_d(r(k))$, $B_i \triangleq B(r(k))$, $G_i \triangleq G(r(k))$, $G_{di} \triangleq G_d(r(k))$ are known constant matrices, and the admissible uncertainties $\Delta A_i(k) \triangleq \Delta A(r(k), k)$, $\Delta A_{di}(k) \triangleq \Delta A_d(r(k), k)$ satisfy $\begin{bmatrix} \Delta A_i(k) & \Delta A_{di}(k) \end{bmatrix} = H_i \Phi_i(k) \begin{bmatrix} E_i & E_{di} \end{bmatrix}$, with E_i, E_{di} and H_i known constant matrices, and $\Phi_i(k)$ an unknown time-varying matrix satisfying $\Phi_i^{\mathrm{T}}(k)\Phi_i(k) \leq I$. Besides, we assume that

A1. For any $r(k) = i \in \mathcal{N}$, the nonlinear function $f_i(x(k), x(k - d(k))) \triangleq f(x(k), x(k - d(k)), r(k))$ possesses the following property:

$$\|f_i(x(k), x(k - d(k)))\| \leq \epsilon_i \|x(k)\| + \epsilon_{di} \|x(k - d(k))\|,$$

where $\epsilon_i > 0$ and $\epsilon_{di} > 0$ are two known scalars.

A2. The input matrix B_i is full column rank, that is, $\text{rank}(B_i) = m$.

Definition 1 (*Costa et al., 2005*)**.** Stochastic MJSs (2) with $u(k) \equiv 0$ is said to be mean square stable if, for any initial conditions $\{x(0),\ \phi(n)\} \in \mathbb{R}^n$, $r(0) \in \mathcal{N}$, the following condition holds:

$$\lim_{k \to \infty} \mathbf{E}\left\{\|x(k)\|^2\right\}\Big|_{x(0),\phi(n),r(0)} = 0. \tag{3}$$

2.2. Partly accessible mode detection probabilities

In practical applications, it is not always possible to directly measure the information of system modes $r(k)$, but instead there is a detector $\sigma(k)$ that provides estimated values of $r(k)$ with some probability (Costa et al., 2015). In this case, the emitted signal $\sigma(k)$ from detector to controller does not synchronize with the system mode $r(k)$. The hidden Markov model $(r(k), \sigma(k))$ as in Costa