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Brief paper

Adaptive control of a class of strict-feedback time-varying nonlinear systems with unknown control coefficients ☆

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ABSTRACT

In this paper, robust adaptive control of a class of strict-feedback nonlinear systems with unknown control directions is investigated. A novel Nussbaum-type function is developed and a key theorem is drawn which involves quantifying the addition of multiple Nussbaum functions with different control directions in a single inequality. Global stability of the closed-loop system and asymptotic stabilization of system output are proved. A simulation example is given to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Adaptive control of strict-feedback nonlinear systems has received a lot of attention since the appearance of recursive backstepping design (Krstic, Kanellakopoulos, & Kokotovic, 1995) and a great deal of work has been done for this class of systems in the past decades, for example, see Ge and Wang (2003), Huang, Song, Wang, and Wen (2017), Huang, Wen, Wang, and Song (2016), Krstic and Kokotovic (1995), Liu and Tong (2017), Wang, Huang, Wen, and Fan (2014), Wen, Zhou, Liu, and Su (2011), Ye (1999, 2011), Ye and Jiang (1998) and many references therein. In this brief, we consider the stabilization control of a class of single-input-single-output (SISO) uncertain nonlinear systems with time-varying disturbances and unknown control directions in strict feedback form

$$\begin{aligned} \dot{x}_i &= b_i\beta_i(\bar{x}_i, t)x_{i+1} + \theta_i^T\phi_i(\bar{x}_i) + d_i(t), i = 1, \ldots, n-1 \\ \dot{x}_n &= b_n\beta_n(x, t)u + \theta_n^T\phi_i(x) + d_n(t) \\ y &= x_1 \end{aligned} \quad (1)$$

where $x = [x_1, \ldots, x_n]^T \in \Re^n$ is the state vector, $\bar{x}_i = [x_1, \ldots, x_i]^T \in \Re^i$, $u \in R$ is the control input, $b_i = 1$ or $b_i = -1$ represents the control directions of (virtual) control coefficients, which are unknown, $\beta_i(\bar{x}_i, t) \neq 0$ are known smooth functions, $\theta_i \in \Re^{\gamma_i}$ is a vector of unknown parameters, $d_i(t)$ is a bounded time-varying disturbance, $\phi_i(\bar{x}_i)$ is a known smooth nonlinear function with compatible dimension.

When the signs of virtual control coefficients or high-frequency gain are unknown, the adaptive control problems are quite involved and Nussbaum-type functions (Chen, Li, Ren, & Wen, 2014; Nussbaum, 1983; Oliveira, Peixoto, & Liu, 2010; Oliveira, Peixoto, & Nunes, 2016; Wen et al., 2011; Ye & Jiang, 1998) are normally adopted. In Oliveira et al. (2010) an output-feedback tracking sliding mode control strategy is proposed for a class of uncertain nonlinear systems with unknown high-frequency gain matrix. In Oliveira et al. (2016) an adaptive output-feedback controller for uncertain linear systems without *a priori* knowledge of the plant high-frequency-gain sign is proposed. Adaptive control for a class of strict feedback nonlinear systems with unknown parameters was addressed in Ding (2000) and Ye and Jiang (1998). In Ge and Wang (2003) and Ye (1999), the adaptive control for a class of time-varying uncertain nonlinear systems with time-varying control coefficients was considered, where the Nussbaum function $e^{x^2}\cos(x)$ was used. However, the time-varying control coefficients in Ge and Wang (2003) and Ye (1999) are assumed to take value in a bounded interval. When this condition is not satisfied, for example, the control coefficients are time-varying functions of the states, such as $\beta_i(\bar{x}_i, t)$ in (1), with unknown signs, the problem could not

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be solved with existing control schemes. The main reason is that to avoid the addition of multiple Nussbaum functions in a single Lyapunov inequality, the stability analysis in Ge and Wang (2003), Liu and Tong (2017), Ye (1999) and Ye and Jiang (1998) etc. is established step by step. In the presence of $\beta_i(\bar{x}_i, t)$, the stability could not be proved following the current way. As far as we know, the adaptive control of (1) remains unsolved.

In this paper, we propose a new Nussbaum function which allows the addition of multiple Nussbaum functions in a single Lyapunov inequality by quantifying the interconnection of multiple Nussbaum functions, such that the controller design and stability analysis of (1) can follow traditional backstepping. It is proved that the proposed robust adaptive scheme can guarantee asymptotical stabilization of the output in the presence of external disturbances. The main contributions of this paper can be summarized as follows. (i) A novel Nussbaum function is proposed in this paper such that the interconnections of multiple Nussbaum-type functions with different control directions in a single inequality could be quantified, which gives great convenience to the controller design and stability analysis in backstepping. (ii) Asymptotic stabilization control is achieved in the presence of external disturbances and uncertain parameters.

2. A novel Nussbaum function

A Nussbaum-type function $\mathcal{N}(\cdot)$ is the one with the following properties (Wen et al., 2011):

$$\lim_{\chi\to\infty}\sup\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau=\infty$$
$$\lim_{\chi\to\infty}\inf\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau=-\infty. \tag{2}$$

Commonly used Nussbaum-type functions include $\chi^2\cos(\chi)$, $e^{\chi^2}\cos(\chi)$, $\chi^2\sin(\chi)$. For the existing Nussbaum functions, it is still unclear how to analyze the interactions of the coexisting Nussbaum functions in a single inequality, as shown later in (8) and a remark. To overcome this difficulty, we propose the following Nussbaum function:

$$\mathcal{N}_i(\chi_i)=\frac{\alpha^2\beta^{2i}+1}{\beta^i\sqrt{\alpha^2\beta^{2i}+1}}e^{\alpha|\chi_i|}\sin\left(\frac{\chi_i}{\beta^i}\right) \tag{3}$$

where $i=1,\dots,N$, α and β are positive constants.

Lemma 1. *$\mathcal{N}_i(\chi_i)$ is an odd function. Let $G_i(\chi_i)=\int_0^{\chi_i}\mathcal{N}_i(\tau)d\tau$, then it is obvious that $G_i(\chi_i)$ is an even function. By direct calculation, we have*

$$\int_0^{\chi_i}\mathcal{N}_i(\tau)d\tau=e^{\alpha\chi_i}\sin(\frac{\chi_i}{\beta^i}-\epsilon_i) \tag{4}$$

for $\chi_i>0$, where $\epsilon_i=\arccos(\frac{\alpha\beta^i}{\sqrt{\alpha^2\beta^{2i}+1}})$. □

In the rest of the paper, we only consider the case when $\chi_i>0$, and for $\chi_i<0$, it could be analyzed similarly. For clarity, let $G_i^{b_i}(\chi_i)=\int_0^{\chi_i}b_i\mathcal{N}_i(\tau)d\tau=b_ie^{\alpha\chi_i}\sin(\frac{\chi_i}{\beta^i}-\epsilon_i)$, where $sign(b_i)=1$ or $sign(b_i)=-1$, clearly $G_i^{-b_i}(\chi_i)=-G_i^{b_i}(\chi_i)$.

Lemma 2. *Let $\beta=\frac{1}{M}$, where $M>4$ is a positive integer, then there exist periodical intervals $[\underline{a}_j,\bar{a}_j], j=1,\dots,\infty$ such that $\forall x\in[\underline{a}_j,\bar{a}_j]$, $sign(b_i)\sin(\frac{x}{\beta^i}-\epsilon_i)<0$ and thus all $G_i^{b_i}(x)<0$, $i=1,\dots,N$.*

Proof. Without loss of generality, assuming $sign(b_i)=1$, then $G_i(x)$ is negative in $x\in[\frac{2n\pi+\pi+\epsilon_i}{M^i},\frac{2n\pi+2\pi+\epsilon_i}{M^i}]$. Meanwhile, $G_{i+1}(x)$ is negative in $x\in[\frac{2m\pi+\pi+\epsilon_i}{M^{i+1}},\frac{2m\pi+2\pi+\epsilon_i}{M^{i+1}}]$ if $sign(b_{i+1})=1$ or in

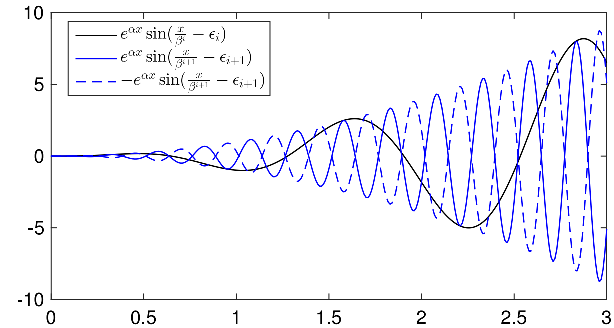

Fig. 1. Schematic figure of $e^{\alpha x}\sin(\frac{x}{\beta^i}-\epsilon_i)$ and $b_{i+1}e^{\alpha x}\sin(\frac{x}{\beta^{i+1}}-\epsilon_{i+1})$.

$x\in[\frac{2m\pi+\epsilon_i}{M^{i+1}},\frac{2m\pi+\pi+\epsilon_i}{M^{i+1}}]$ if $sign(b_{i+1})=-1$, as shown schematically in Fig. 1. Let $m=n*M+g$ where g is a positive integer that satisfies

$$(2g+1)\pi+\epsilon_{i+1}>M\pi+M\epsilon_i$$
$$(2g+2)\pi+\epsilon_{i+1}<2M\pi+M\epsilon_i \tag{5}$$

if $sign(b_{i+1})=1$ or

$$g\pi+\epsilon_{i+1}>M\pi+M\epsilon_i$$
$$(2g+1)\pi+\epsilon_{i+1}<2M\pi+M\epsilon_i \tag{6}$$

if $sign(b_{i+1})=-1$. If α is chosen such that for $M\epsilon_i<1$, $i=1,\dots,N$, i.e.

$$\alpha>\sqrt{\frac{M^{2i}\cos(1/M)^2}{1-\cos(1/M)^2}} \tag{7}$$

then it is easy to see that (5) or (6) could be satisfied, which means $G_i^{b_i}(x)$ and $G_{i+1}^{b_{i+1}}(x)$ are negative at the same intervals. Since $M>4$, it is easy to see that the intervals exist.

Take $\sin(\frac{x}{\beta^i}-\epsilon_i)<-0.5, i=1,\dots,N$ as an example. By simple calculation, the intervals satisfying $sign(b_i)\sin(\frac{x}{\beta^i}-\epsilon_i)<-0.5$, $i=1,\dots,N$ is $[\frac{2m\pi+0.5236+\epsilon_i}{M^N},\frac{2m\pi+2.6180+\epsilon_i}{M^N}]$ if $sign(b_N)=1$ or $[\frac{2m\pi+3.665+\epsilon_i}{M^N},\frac{2m\pi+5.759+\epsilon_i}{M^N}]$ is $sign(b_N)=-1$, where m is a positive integer depending on b_i, and the length of the intervals are $\mathcal{L}=\frac{2.094}{M^N}$. □

With Lemmas 1 and 2, we have the following main theorem.

Theorem 1. *If there exists a positive definite, radially unbounded function $V(t)$ satisfying the following inequality,*

$$V(t)\le\sum_{i=1}^{N}\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau-\sum_{i=1}^{N}\int_0^t\dot{\chi}_i(\tau)d\tau+c \tag{8}$$

where c is a constant, then by choosing

$$\alpha>\max\left\{\sqrt{\frac{M^{2i}\cos(1/M)^2}{1-\cos(1/M)^2}},\frac{\chi}{\delta}\ln(2N)\right\}$$

where δ is a positive constant satisfying $\delta<\mathcal{L}$, then $V(t)$, χ_i and $\sum_{i=1}^N\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ are bounded on $[0,\infty)$.

Proof. From (8) we obtain

$$V(t)\le\sum_{i=1}^{N}\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau-\sum_{i=1}^{N}\int_0^t\dot{\chi}_i(\tau)d\tau+c$$
$$=\sum_{i=1}^{N}G_i^{b_i}(\chi_i)-\sum_{i=1}^{N}\chi_i(t)d\tau+\bar{c} \tag{9}$$

where $\bar{c}=\sum_{i=1}^N\chi_i(0)+c-\sum_{i=1}^N G_i^{b_i}(0)$.