Automatica 93 (2018) 161–171

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Trajectory curvature guidance for Mars landings in hazardous terrains ☆

Pingyuan Cui, Tong Qin, Shengying Zhu, Yang Liu, Rui Xu *, Zhengshi Yu

Key Laboratory of Autonomous Navigation and Control for Deep Space Exploration, Ministry of Industry and Information Technology, School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China

ARTICLE INFO

Article history:
Received 12 April 2016
Received in revised form 4 January 2018
Accepted 26 February 2018

Keywords:
Powered descent guidance
Geometrically convex trajectories
Obstacle avoidance
Closed-loop analytical formulation

ABSTRACT

Focusing on the trajectory curvature, this paper presents an innovative and analytical guidance law for the construction of geometrically convex trajectories. Moving along such trajectories, the lander can increase the probability of a safe landing in hazardous terrains. Initially, the curvature theorems of the powered descent trajectories are developed. In these theorems, the inner relationship between trajectory curvature and lander states is revealed, and the state constraints for a geometrically convex trajectory are derived. Next, the trajectory curvature guidance is developed in an analytical formulation by satisfying the state constraints for a convex trajectory, and the selection of the key guidance parameters is investigated. Finally, the performance of the trajectory curvature guidance is analyzed in detail, illustrating its superior hazard avoidance and the camera's field of view.

1. Introduction

To date, all Mars landers have been targeted to large, flat areas based on the "safety first" principle, neglecting much of the scientific interest associated with other possible landing sites. Future Mars exploration missions would prefer a complex landing terrain to further the mission's scientific goals, such as detecting organics and signs of life (Grant et al., 2011). To achieve safe landing in regions with undulating terrain, powered descent guidance with an obstacle-avoidance function must be developed. In addition, the guided descent trajectory should keep the landing site in the field of view (FOV) of a navigation camera or hazard-detection lidar (Gerth & Mooij, 2014).

Motivated by the planetary landing missions, substantial attention has been paid to powered descent guidance in the last fifty years. The engineering-applied gravity turn and Apollo second-order polynomial guidance contributed substantially to the previous Mars landing missions because of their simplicity and reliability (Citron, Dunn, & Mesissinger, 1964; Klumpp, 1974; McInnes, 1996). However, these algorithms with simple formulation considered no performance index or constraint, degrading their overall performance and impeding applications in hazardous terrains. To obtain better performance, various descent-guidance laws have been proposed and developed, such as fuel-optimal guidance (Liu, Duan, & Teo, 2008; Scott, 1986; Topcu, Casoliva, & Mease, 2007), artificial potential function guidance (APFG) (Lopez & McInnes, 1995; McInnes, 1995), convex-programming guidance (CPG) (Açıkmeşe & Blackmore, 2011; Açıkmeşe & Ploen, 2007; Harris & Açıkmeşe, 2014), and waypoint-optimized guidance (Guo, Hawkins, & Wie, 2013). Each of them has a distinctive feature and occupies an important position in the field of powered descent guidance. In terms of their obstacle-avoidance ability, APFG and CPG deserve particular attention. Lyapunov stability theory is the basis of APFG. By establishing a potential function in which the obstacles are set to a high-potential area, the lander can achieve a soft landing without violating the terrain constraints. CPG exploits the concept of numerical trajectory optimization, which considers many complex constraints, including the glide slope constraint that ensures that the optimized trajectory is not too shallow and, thus, that the lander can avoid raised obstacles (Blackmore, Acikmese, & Scharf, 2010). Solving the classical optimal control problem with fuel-optimal performance index subjected to state and control constraints, CPG represents the state-of-the-art open-loop powered descent guidance by improving the optimization efficiency and ensuring the global optimality. Imperfectly, no analytical formula of the control acceleration can be obtained, causing a heavy burden for onboard computer. In fact, that is a common challenge to solve such a complex optimal control problem analytically. Throughout the history of powered descent guidance, performance trades were inevitable since analytical results could only be obtained by neglecting the nonlinearity and constraints,

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Kok Lay Teo under the direction of Editor Ian R. Petersen.

* Corresponding author.
E-mail addresses: cuipy@bit.edu.cn (P. Cui), phd.qintong@gmail.com (T. Qin), zhushengying@gmail.com (S. Zhu), liuyang5264@163.com (Y. Liu), xurui@bit.edu.cn (R. Xu), yuzhengshi@gmail.com (Z. Yu).

https://doi.org/10.1016/j.automatica.2018.03.049
0005-1098/

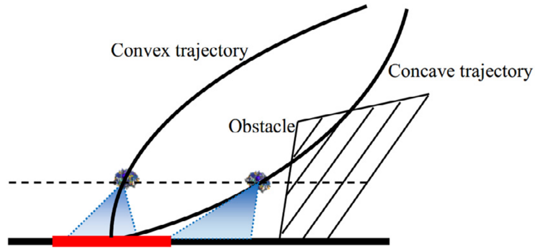

Fig. 1. Superiority of a convex trajectory in terms of hazard avoidance and the camera's FOV.

and numerical optimization was necessary when considering engineering constraints.

Unlike previous research studies, this paper focuses on the trajectory geometry and develops a closed-loop analytical guidance law to construct geometrically convex trajectories. Moving along a convex trajectory, the lander can avoid undulating obstacles and more likely maintain the visibility of the designed landing area (Fig. 1). That is how the proposed guidance law satisfies the state constraints. The control thrust saturations are fulfilled by adjusting parameters in the analytical guidance law. The fuel consumption relates to guidance parameters is also analyzed to obtain better parameters selection for less fuel consumption. Consequently, even though no classical optimal control problem under constraints is formulated or solved, the proposed guidance law considers both state and control constraints, and fuel consumption performance. Moreover, the control acceleration can be solved rapidly onboard because it does not rely on a numerical solver.

To develop such a guidance law, this paper first expresses the trajectory curvature using the lander states and then derives the state constraints for convex trajectories. Next, the trajectory curvature theorems are developed and proved. These theorems clarify that the trajectory curvature is determined by the initial lander states and establish a basis for the proposed innovative guidance. Subsequently, the trajectory curvature guidance (TCG) is developed to satisfy the state constraints for convex trajectory by utilizing a constant acceleration phase with its proper magnitude and duration calculated analytically. Finally, the performance of the developed guidance algorithm is analyzed in terms of obstacle avoidance, constraint satisfaction, fuel consumption, and camera's FOV of the landing site.

2. Curvature theorems of powered descent trajectories

To develop the trajectory curvature guidance, the relationship between trajectory curvature and lander states should first be clarified. The state constraints required by a geometrically convex trajectory should also be derived. In this section, the trajectory curvature is expressed by the lander's position and velocity. Then, the advantage of the trajectory convexity is clarified. Furthermore, curvature theorems of the powered descent trajectories that reveal the influence of initial states on the shape of the powered descent trajectory are developed and proved. These analyses constitute the basis of trajectory curvature guidance.

2.1. Relationship between trajectory curvature and lander states

The powered descent phase is investigated in the target-fixed reference system. The origin is located at the landing target, z axis points to the zenith, x axis is perpendicular to z axis and located in the plane composed of z axis and the lander's initial position, with the positive direction towards the lander, and y axis follows the right-hand principle. The Oxz plane is defined as longitudinal motion plane upon which the lander's motion mainly concentrates. The powered descent trajectory and guidance law will mainly be discussed in this plane.

A planar curve is geometrically convex if and only if $\mathrm{d}^2z/\mathrm{d}x^2 < 0$. The following derivation demonstrates that the condition for negative curvature can be expressed by the lander's acceleration and velocity.

$$\frac{\mathrm{d}^2 z}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\Big/\frac{\mathrm{d}x}{\mathrm{d}t}\right)\Big/\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{v_z}{v_x}\right)\Big/ v_x = \frac{a_z v_x - a_x v_z}{v_x^3} < 0 \quad (1)$$

where x and z are position components of the lander in the longitudinal plane of target-fixed reference system, v_x and v_z are corresponding velocity component, a_x and a_z are corresponding acceleration components, and t is the flight time. If the lander's initial horizontal velocity is negative, the condition for a convex curve can be expressed as

$$a_z v_x - a_x v_z > 0 \quad (2)$$

Similarly, the conditions for a concave curve and a straight line are given, respectively, by

$$a_z v_x - a_x v_z < 0 \quad (3)$$

$$a_z v_x - a_x v_z = 0 \quad (4)$$

The optimal guidance (OPG) law considering only the constraints of the terminal states in the linear powered descent dynamics (D'Souza, 1997; Guo et al., 2013) is given by D'Souza (1997)

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} a_{cx} \\ a_{cy} \\ a_{cz} - g \end{bmatrix} = -\frac{4}{t_{go}}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} - \frac{6}{t_{go}^2}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (5)$$

where a_{cx}, a_{cy}, and a_{cz} represent the tri-axial control accelerations, g is the constant local gravity, and t_{go} is the time-to-go, which is the remaining time of engine-burning at any instant of the powered descent, and a synchronizing variable for ensuring simultaneous solutions of all-dimension control acceleration (Cherry, 1964). It is the positive root of the following equation:

$$\begin{aligned} &g^2 t_{go}^4/2 - 2(v_x^2 + v_y^2 + v_z^2)t_{go}^2 - 12(xv_x + yv_y + zv_z)t_{go} \\ &\quad -18(x^2 + y^2 + z^2) = 0 \end{aligned} \quad (6)$$

Focusing on the planar trajectory (located in the Oxz plane of the target-fixed reference system), Eq. (5) is simplified as

$$\begin{bmatrix} a_x \\ a_z \end{bmatrix} = -\frac{4}{t_{go}}\begin{bmatrix} v_x \\ v_z \end{bmatrix} - \frac{6}{t_{go}^2}\begin{bmatrix} x \\ z \end{bmatrix} \quad (7)$$

Defining $\alpha = -4/t_{go}$ and $\beta = -6/t_{go}^2$, Eq. (7) can be divided into the following two equations:

$$a_x = \alpha v_x + \beta x \quad (8)$$

$$a_z = \alpha v_z + \beta z \quad (9)$$

Substituting Eqs. (8)–(9) into Eqs. (2)–(4), conditions for a convex curve, concave curve, and straight line are derived, respectively, as

$$z v_x - x v_z < 0 \quad (10)$$

$$z v_x - x v_z > 0 \quad (11)$$

$$z v_x - x v_z = 0 \quad (12)$$

Eqs. (10)–(12) illustrate that the sign of the trajectory curvature can be expressed by the lander states. If the inequality constraint in Eq. (10) is always satisfied in the powered descent phase, the whole trajectory is geometrically convex. Note that these conclusions are obtained under the assumption $v_x < 0$. If the direction of the horizontal velocity is reversed, the inequality constraints for convex and concave trajectories are interchanged. Thus, the strict state constraints for a convex trajectory should be the combination