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Brief paper

Suppressing phase damping decoherence by periodical imperfect projective measurements☆

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ABSTRACT

We explore how to overcome phase damping decoherence when imperfect projective measurements are available. It is demonstrated that a "relaxed" control goal may be achieved by combining simple open-loop coherent control with periodic imperfect projective measurements. Inspired by the idea of soft optimization, we propose to control the state of a qubit staying near a reference pure state with high probability for a sufficiently long time. This "relaxed" control goal is expressed in terms of three-parameter performance indexes, and it is in remarked contrast to the "hard" requirement that the state of the controlled qubit always stays at the reference pure state. We not only establish necessary conditions for relaxed robust problems, but also present sufficient conditions for the existence of a solution to the relaxed robust problems. With the help of main results, one can estimate how the maximal total time depends on the target state and the perturbation bounds. Furthermore, it is demonstrated that imperfect realization of projective measurements will worsen the three-parameter performance indexes.

1. Introduction

As an important precondition for constructing quantum information systems (Nielsen & Chuang, 2004), decoherence suppressing is one of the key problems in the study of quantum coherent control (Shapiro & Brumer, 2003) and quantum feedback control (Jacobs, 2014; Wiseman & Milburn, 2010). In previous research, various methods have been developed against decoherence, e.g., quantum error-avoiding codes (Duan & Guo, 1997, 1998; Lidar, Chuang, & Whaley, 1998; Zanardi & Rasetti, 1997), quantum error-correction codes (Cirac, Pellizzari, & Zoller, 1996; Knill & Laflamme, 1997; Shor, 1995; Steane, 1995; Zurek & Laflamme, 1996), BangBang control (Pan, Xi, & Gong, 2011; Viola, Knill, & Lloyd, 1999; Viola & Lloyd, 1998; Viola, Lloyd, & Knill, 1999), open loop coherent control (Lidar & Schneider, 2005; Zhang, Wu, Li, Tarn, & Wu, 2007), quantum feedback control (Fortunato, Raimond, Tombesi, & Vitali, 1999; Goetsch, Tombesi, & Vitali, 1996; Tombesi & Vitali, 1995; Vitali, Tombesi, & Milburn, 1997, 1998) and Zeno control scheme (Facchi & Pascazio, 2001, 2002).

The interaction with environment not only causes decoherence of the target quantum system, but also leads perturbations to the sensor or the controller. Such perturbations bring uncertainties into the control systems and heavily impact their performance. For instance, quantum tracking control will be not feasible if there exist uncertainties in the knowledge of initial state (Zhang, Dai, Xi, Xie, & Hu, 2007). To deal with the uncertainties in quantum systems, quantum robust control has been studied by many researchers. For example, Bierzychudek et al. have designed H-infinity controllers for cryogenic current comparators (Bierzychudek, Goetz, Sanchez-Pena, Iuzzolino, & Drung, 2017; Bierzychudek, Sanchez-Pena, & Tonina, 2013). D. Dong and his coauthors have worked on robust control of quantum systems against several uncertainties (Dong, Chen, Qi, Petersen, & Nori, 2015; Dong, Lam, & Petersen, 2010; Dong, Mabrok et al., 2015; Dong & Petersen, 2012; Dong et al., 2016). Maalouf et al. have investigated the control for linear quantum systems by solving the quantum and the equivalent classical stochastic differential equations (Maalouf & Petersen, 2011a, b, 2012, 2014). A robust adaptive measurement scheme (Tanaka & Yamamoto, 2012) has also been proposed for qubit-state preparation against the uncertainty of the unitary evolution. For certain environmental perturbations and specific system structures, a variety of robust control schemes have been given in aforementioned investigations. However, how to overcome the

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uncertainty in quantum measurements, to our knowledge, has not been explored carefully.

In this paper, we explore whether or not dephasing suppression is feasible when perturbed projective measurements are available. Here we consider the dephasing suppressing problem for a qubit subject to Markovian environment. In contrast to the strict fidelity requirement that the controlled qubit should always stay at the reference pure state, we borrow the idea about soft optimization for hard problems (Ho, Zhao, & Jia, 2007) and try to make the controlled qubit stay near a reference pure state with high probability for sufficiently long time (Zhang, Dai et al., 2007). Aiming at this relaxed control goal, we develop an experimental imperfection tolerating scheme by combining simple open loop controls with periodic imperfect projective measurements. Since the projective measurements are imperfect, the fidelity function and the measurement probability are uncertain. Fortunately, both of their lower bound functions can be exactly decided by the target state of the system, control parameter and perturbation bounds. Therefore the measurements period can be designed based on the lower bound functions. Furthermore, it is revealed that more uncertainties or perturbations lead to worse control performance, and our scheme may ensure longer performance preserving time with smaller control parameter.

The rest of this paper are organized as follows. The relaxed robust control problem is presented in Section 2. We deduce our main results in Section 3, and further give some comments about our control scheme. We end the paper with a concluding remark and a short discussion of further work in Section 4.

2. Problem description

In this paper, we explore how to suppress dephasing decoherence for a qubit Q subject to Markovian dynamics when projective measurements are perturbed. We consider a model of a two-state system coupling to a bosonic heat bath (Leggett et al., 1987). This open system model is widely used in the study of quantum superconducting circuits (Forn-Diaz et al., 2017; Kato, Golubov, & Nakamura, 2007; Makhlin, Schon, & Shnirman, 2004) and cavity quantum electrodynamics (Albert, Scholes, & Brumer, 2011; Nevado & Porras, 2013; Xue, Zhong, Li, & Sun, 2007), and is familiar in the implementation of quantum information processing (Hao, Tong, & Zhu, 2013; Teixeira, Kapale, Paternostro, & Semiao, 2016; Ye, Shalashilin, & Serafini, 2012). The dynamics of this model can be described by following spin-boson master equation (Paz & Zurek, 2001; Weiss, 2012)

$$\begin{aligned}\frac{\partial\rho(t)}{\partial t} &= -\frac{i}{\hbar}[H_R,\rho]-\frac{i}{\hbar}z^*(t)\sigma_x\rho(t)+\frac{i}{\hbar}z(t)\rho(t)\sigma_x\\ &-\frac{1}{\hbar}\tilde{D}(t)[\sigma_z,[\sigma_z,\rho]]-\frac{1}{\hbar}z(t)\sigma_z\rho\sigma_y-\frac{1}{\hbar}z^*(t)\sigma_y\rho\sigma_z,\end{aligned} \tag{1}$$

with $\sigma_\alpha, \alpha\in\{x,y,z\}$, the Pauli matrices. Here

$$H_R=\frac{1}{2}\hbar\Delta\sigma_x \tag{2}$$

is the Hamiltonian of the spin qubit,

$$\begin{aligned}\tilde{D}(t)&=\int_0^t \mathrm{d}s\nu(s)\cos\Delta s,\\ z(t)&=\int_0^t \mathrm{d}s\,(\nu(s)+i\eta(s))\sin\Delta s\end{aligned} \tag{3}$$

are coefficients of dephasing and of system decay, respectively. The bare frequency Δ denotes the tunneling amplitude between two states of the spin system. By employing $J(\omega)$ for the environmental spectral density and $N(\omega)$ for the mean occupation number of the environmental oscillators, the noise kernel $\nu(t)$ and the dissipation kernel $\eta(t)$ can be respectively expressed as

$$\nu(t)=\int_0^\infty \mathrm{d}\omega J(\omega)\cos\omega t(1+2N(\omega)), \tag{4}$$

$$\eta(t)=\int_0^\infty \mathrm{d}\omega J(\omega)\sin\omega t. \tag{5}$$

In this paper, we focus on suppressing the influence of pure dephasing on spin qubit Q, and hence assume that $\tilde{D}(t)=\hbar\gamma/2>0$ and $z(t)=0$. Thus the controlled spin-boson system can be described by following master equation

$$\frac{\partial\rho(t)}{\partial t}=-\frac{i}{\hbar}[\hat{H},\rho]+\gamma(\sigma_z\rho\sigma_z-\rho), \tag{6}$$

where the control Hamiltonian $\hat{H}$ is the original Hamiltonian H_R modulated by control fields. It can be generally described as a composition of Pauli matrices

$$\hat{H}=\frac{\hbar}{2}\left((\omega_0(t)+\Delta)\,\sigma_x+\omega_1(t)\sigma_y+\omega_2(t)\sigma_z\right). \tag{7}$$

For the sake of further discussions, we denote this control system as $Mod(\Delta;\gamma;\Omega(t))$, where $\Omega(t)=(\omega_0(t),\omega_1(t),\omega_2(t))$.

In quantum physics, measurement is described by a collection of measurement operators $\{M_m\}_{m\in\{1,2,\ldots,l\}}$. The index m refers to measurement outcomes that may occur in the experiment. If the state prior to the measurement is ρ, then the probability that result m occurs is

$$p(m)=tr(M_m^\dagger M_m\rho)=tr(M_m\rho M_m^\dagger), \tag{8}$$

and the state will collapse to

$$\rho_m=\frac{M_m\rho M_m^\dagger}{tr(M_m\rho M_m^\dagger)} \tag{9}$$

after the measurement. We describe the uncertainties of perturbed projective measurement by two geometric parameter bounds θ_ϵ and ϕ_ϵ. The operators of imperfect projective measurement $\tilde{M}_\theta^{\theta_\epsilon,\phi_\epsilon}$ $=\{\tilde{P}_+,\tilde{P}_-\}$ in our dephasing model are

$$\tilde{P}_+=|\tilde{\psi}_\theta\rangle\langle\tilde{\psi}_\theta|,\quad \tilde{P}_-=|\tilde{\psi}_\theta^\perp\rangle\langle\tilde{\psi}_\theta^\perp|, \tag{10}$$

where $|\tilde{\psi}_\theta\rangle=\cos\frac{\tilde{\theta}}{2}|0\rangle+e^{i\tilde{\phi}}\sin\frac{\tilde{\theta}}{2}|1\rangle$, $|\tilde{\psi}_\theta^\perp\rangle=\sin\frac{\tilde{\theta}}{2}|0\rangle-e^{i\tilde{\phi}}\cos\frac{\tilde{\theta}}{2}|1\rangle$, with $\theta-\theta_\epsilon\le\tilde{\theta}\le\theta+\theta_\epsilon$ and $-\phi_\epsilon\le\tilde{\phi}\le\phi_\epsilon$.

It is also assumed that the initial state of Q is known with some uncertainties originating from imperfect measurements. With geometric parameter bounds θ_ϵ and ϕ_ϵ, perturbed initial states can be described by

$$\begin{aligned}&|\hat{\psi}_\theta\rangle=\cos\frac{\hat{\theta}}{2}|0\rangle+e^{i\hat{\phi}}\sin\frac{\hat{\theta}}{2}|1\rangle,\\ &\theta-\theta_\epsilon\le\hat{\theta}\le\theta+\theta_\epsilon,-\phi_\epsilon\le\hat{\phi}\le\phi_\epsilon,\end{aligned} \tag{11}$$

and we define $\hat{I}_\theta^{\theta_\epsilon,\phi_\epsilon}=\{|\hat{\psi}_\theta\rangle\}$ as the set of possible initial states.

We can present the relaxed robust control problem $R_{\gamma,\theta}^{\theta_\epsilon,\phi_\epsilon}(T_0,\epsilon_P,\epsilon_F)$ as follows:

For the system $Mod(\Delta;\gamma;\Omega(t))$ initially at the pure state $|\hat{\psi}_\theta\rangle\in\hat{I}_\theta^{\theta_\epsilon,\phi_\epsilon}$, when the perturbed projective measurements $\tilde{M}_\theta^{\theta_\epsilon,\phi_\epsilon}=\{\tilde{P}_+,\tilde{P}_-\}$ defined in Eq. (10) are available, find a control law so that the fidelity

$$F(\rho(t),|\psi_\theta\rangle)\ge1-\epsilon_F,\qquad(0<\epsilon_F<1) \tag{12}$$

with the probability

$$P_r(t)\ge1-\epsilon_P,\qquad(0<\epsilon_P<1) \tag{13}$$

during the time interval $t\in[0,T_0]$.