Automatica 93 (2018) 189–196

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Extremum seeking-based perfect adaptive tracking of non-PE references despite nonvanishing variance of perturbation ☆

Miloje S. Radenković [a,*], Miroslav Krstić [b]

[a] *Department of Electrical Engineering, University of Colorado Denver, Denver, CO 801217, USA*
[b] *Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA*

ARTICLE INFO

Article history:
Received 23 February 2017
Received in revised form 5 January 2018
Accepted 12 January 2018

Keywords:
Extremum seeking
Adaptive control
Parameter estimation
Control direction
Martingale convergence

ABSTRACT

This paper presents a novel algorithm for adaptive stabilization of unstable discrete time systems with unknown control directions. In a departure from commonly used recursive least squares and gradient type parameter estimators, the proposed algorithm is based on extremum seeking (ES) method. The perturbation signal is a martingale difference sequence (m.d.s) with a non-decaying (bounded from below) variance. In spite of a non-vanishing perturbation, somewhat surprisingly it is shown that globally, almost surely (a.s.) the tracking error converges to zero, input and output signals are uniformly bounded, and the parameter estimates are convergent sequences.

1. Introduction

1.1. Adaptive control of systems with unknown control directions

It is well known that all well behaved adaptive controllers require information about the system control direction. The control direction is determined by the sign of the system's high-frequency gain. When the sign of this gain is unknown, adaptive system can exhibit numerical instability due to the loss of stabilizability of the parameter estimates. Design of adaptive controllers in case of unknown control directions is a challenging problem. Inspired by the conjecture and questions raised by Morse (1983), Nussbaum (1983) proved that the knowledge of the sign of the high-frequency gain is not necessary requirement for adaptive stabilization. In this approach adaptive controller employs a mechanism for inverting the sign of the control signal if the system states start to grow without bound. Generalization of this result to higher order systems was presented in Mudgett and Morse (1985), and was further advanced in hundreds of subsequent publications (see for example a survey paper Ilchmann (1991)). Without relying on the Nussbaum approach, in this paper we develop a novel algorithm for reference tracking and adaptive stabilization of unstable systems with unknown control directions via perturbation-demodulation ES loop.

1.2. Extremum seeking for unstable plants

The perturbation-demodulation based ES scheme is convenient for finding and in real-time maintaining the optimizing value of an unknown input/output map. In this approach the estimate of the gradient of the nonlinear map is obtained by adding a perturbation signal to the optimizer estimates, and subsequently demodulating the observed output. The original intent of this method was to find optimal operating point of stable plants by automated tuning of system parameters. In the 1950s and 1960s this line of research went by names of extremal control, extremum regulation, hill-climbing regulation, etc. (Meerkov, 1967; Morosanov, 1957; Ostrovskii, 1957; Roberts, 1965). The absence of rigorous theory and difficulties in implementing ES controllers, in the 1970s lead to a decline in interest for this topic. The first stability analysis of an ES algorithm was published in 2000, and it is based on the method of averaging and perturbation theory (Krstic, 2000; Krstic & Wang, 2000; Wang & Krstic, 2000). Afterwards a large number of results covering various ES control topics have appeared (see for example the survey paper Tan, Moase, Manzie, Nešić, and Mareels (2010)). Since in this paper we are considering adaptive control of unstable systems, we will not review results related to ES optimization methods for stable plants. We only comment on contributions to ES based stabilization of unstable systems. A first use of ES loop to unstable plants is considered in Zhang, Siranosian,

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Warren E. Dixon under the direction of Editor Daniel Liberzon.

* Corresponding author.
E-mail addresses: Miloje.Radenkovic@ucdenver.edu (M.S. Radenković), krstic@ucsd.edu (M. Krstić).

https://doi.org/10.1016/j.automatica.2018.03.068
0005-1098/

and Krstic (2007). The results reported in Moase and Manzie (2012) allow for the input dynamics to be unstable. In Haghi and Ariyur (2011) authors consider a model reference control via ES in case of unknown control directions and partial knowledge of system parameters. It is demonstrated that the tracking error converges globally to an $\mathcal{O}(1/\omega)$ neighborhood of the origin, where ω is the frequency of the perturbation signal. A finite-time horizon ES based control for unknown and unstable linear discrete time systems is considered in Frihauf, Krstic, and Basar (2013). It is proved that a certain quadratic cost function converges locally, exponentially to some neighborhood of its optimal value. The first systematic design of ES control for unstable plants, along with a novel method to handle unknown control directions is presented in Scheinker and Krstic (2013). It is shown that the proposed controller provides semi-global exponential practical stability, while the system states converge to perturbation frequency dependent $\mathcal{O}(1/\omega)$ proximity of the origin. Compared to Mudgett and Morse (1985), the above controller is robust to external disturbances and sign changes of the high-frequency gain. Our present work is related to Radenkovic and Krstic (2017), where the authors consider the problem of adaptive stabilization of possibly unstable linear discrete time systems with unknown control directions via ES. There it is assumed that: (i) the reference signal is persistently exciting (PE); (ii) the open loop system is irreducible; (iii) perturbation and demodulation signal are martingale difference sequences(m.d.s) whose variances tend to zero, and (iv) the employed parameter estimator has a vanishing gain sequence. It is proved that (a.s.) the tracking error converges to zero. Conditions (i)–(iii) are crucial for obtaining this result.

1.3. Contribution of this paper

This paper solves the problem of discrete time adaptive stabilization and optimal tracking in case of unknown control directions, by devising ES based algorithm with a non-vanishing perturbation. It is well known that the energy of perturbation signal affects the proximity within which the extremum point can be reached (Frihauf et al., 2013; Haghi & Ariyur, 2011; Krstic, 2000; Krstic & Wang, 2000; Moase & Manzie, 2012; Scheinker & Krstic, 2013). This proximity is on the order of the perturbation energy (see for example Theorem 2 and Corollary 2 in Frihauf et al., 2013). In this paper it is proved that (a.s.) the output tracking error converges to zero despite the non-vanishing (bounded from below) variance/energy of the perturbation signal. Similar problem has been considered in Radenkovic and Krstic (2017) under different assumptions and by using a different algorithm. The algorithm in Radenkovic and Krstic (2017) is a long memory algorithm whose gain converges to zero as time tends to infinity. The algorithm proposed in this paper has a non-vanishing gain, hence a short memory algorithm. In contrast to Radenkovic and Krstic (2017) in this paper: (1) we do not assume that the reference signal is persistently exciting; (2) open loop system is not required to be irreducible, and (3) the proposed controller enables to employ probing and demodulation sequences with non-vanishing variances. The results in Radenkovic and Krstic (2017) require that these variances converge to zero, as a consequence of which the resulting scheme is a long memory algorithm. Because in this paper the variance of demodulation signal does not tend to zero, the resulting parameter estimator has a non-vanishing gain. In addition to obtaining asymptotically zero tracking error, we show that (a.s.) input and output signals remain bounded, while the parameter estimates are convergent sequences. This result is global in the sense that it holds for all initial conditions.

1.4. Notation

The subscript T denotes the transpose of the matrix; $\mathbb{R}$ is the set of real numbers; $\mathbb{R}^n$ is the set of n-dimensional vectors with real entries; $\|x\|$ denotes the Euclidean norm of the vector x; $sgn(b)$ is the sign function of a $b \in \mathbb{R}$; l_2 denotes the normed vector space of sequences $\{e(k)\}$, $k \geq 0$ that are square summable, i.e., $\sum_{k=0}^{\infty} e(k)^2 < \infty$; the l_∞ sequence space is defined as $l_\infty = \{\{e(k)\} \in \mathbb{R} : \sup_{k\geq 0}|e(k)| < \infty\}$.

2. Problem statement

Consider the following discrete-time system

$$A(q^{-1})y(k+1) = b_0 B(q^{-1})u(k), \quad b_0 \neq 0 \tag{1}$$

where $\{y(k)\} \in \mathbb{R}$ and $\{u(k)\} \in \mathbb{R}$ are output and input sequences, respectively, $k = 0, 1, 2, \ldots,$ is the discrete time index, and q^{-1} is a unit delay operator. Polynomials $A(q^{-1})$ and $B(q^{-1})$ are given by

$$A(q^{-1}) = 1 + \sum_{i=1}^{L} a_i q^{-i}, \; B(q^{-1}) = 1 + \sum_{i=1}^{L} b_i q^{-i}, \; L > 0. \tag{2}$$

It is assumed that the parameters $b_0, a_i, b_i, i = 1, \ldots, L$, are unknown. The parameter b_0 is often referred to as the high frequency-gain, or the 'control direction'. In the above, L represents an upper bound (known to a designer) on the unknown actual system order $L^* \leq L$. Therefore in (2) we have $a_i = 0, b_i = 0$ for $i = L^*+1, \ldots, L$. The aim of this paper is to find the input sequence $\{u(k)\}$ so that for a given reference trajectory $\{y^*(k)\}$, the tracking error

$$e(k+1) = y(k+1) - y^*(k+1) \tag{3}$$

satisfies $\{e(k)\} \in l_2$. It is assumed that $y^*(k)$, $k \geq 0$, is generated by the following model

$$D(q^{-1})y^*(k+1) = 0, \quad y^*(0) \neq 0, \quad k \geq 0, \tag{4}$$

where $D(q^{-1})$ is known polynomial defined by

$$D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_N q^{-N}, \quad N > 0. \tag{5}$$

For example if $D(q^{-1}) = 1 - q^{-1}$, then $y^*(k) = y^*(0), \forall k \geq 0$. If $D(q^{-1}) = 1 + d_1 q^{-1} + q^{-2}$, $d_1 = -2\cos(\omega_0)$, then $\{y^*(k)\}$ is a cosine sequence with frequency ω_0. Observe that in the above examples the zeros of $D(q^{-1})$ satisfy $|q| = 1$. To motivate the development of the control algorithm, we define the following signals

$$e_f(k) := D(q^{-1})e(k), \quad u_f(k) := D(q^{-1})u(k). \tag{6}$$

Then from (1) and (3) it follows that

$$A(q^{-1})e_f(k+1) = b_0 B(q^{-1})u_f(k), \tag{7}$$

or

$$e_f(k+1) = -q\left[A(q^{-1}) - 1\right]e_f(k) + b_0 B(q^{-1})u_f(k) \tag{8}$$

where q is the forward shift operator, e.g., $qe_f(k) := e_f(k+1)$. Similarly from (5) and (6) we can write

$$e_f(k+1) = e(k+1) + D_1(q^{-1})e(k), \tag{9}$$

where $D_1(q^{-1}) = q\left[D(q^{-1}) - 1\right]$. After substituting (9) on the LHS of (8) one can obtain

$$e(k+1) = b_0\theta^T\varphi(k) + b_0 u_f(k), \tag{10}$$

where $\theta \in \mathbb{R}^{2L+1}$ is given by

$$\theta^T = \left[-1/b_0, -a_1', \ldots, -a_L', b_1, \ldots, b_L\right], \tag{11}$$

with $a_i' = a_i/b_0$, $1 \leq i \leq L$, and

$$\begin{aligned}\varphi^T(k) = [&D_1(q^{-1})e(k), e_f(k), \ldots, e_f(k-L+1), \\ &u_f(k-1), \ldots, u_f(k-L)],\end{aligned} \tag{12}$$

where $e_f(k)$ and $u_f(k)$ are defined by (6). From (10) it is obvious that the control law $u_f^0(k) = -\theta^T\varphi(k)$ gives $e(k+1) = 0, \forall k \geq L$,