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Brief paper

Complexity of deciding detectability in discrete event systems ☆

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ABSTRACT

Detectability of discrete event systems (DESs) is a question whether the current and subsequent states can be determined based on observations. Shu and Lin designed a polynomial-time algorithm to check strong (periodic) detectability and an exponential-time (polynomial-space) algorithm to check weak (periodic) detectability. Zhang showed that checking weak (periodic) detectability is PSpace-complete. This intractable complexity opens a question whether there are structurally simpler DESs for which the problem is tractable. In this paper, we show that it is not the case by considering DESs represented as deterministic finite automata without non-trivial cycles, which are structurally the simplest deadlock-free DESs. We show that even for such very simple DESs, checking weak (periodic) detectability remains intractable. On the contrary, we show that strong (periodic) detectability of DESs can be efficiently verified on a parallel computer.

1. Introduction

The detectability problem of discrete event systems (DESs) is a question whether the current and subsequent states of a DES can be determined based on observations. The problem was introduced and studied by Shu and Lin (2011), Shu, Lin, and Ying (2007). Detectability generalizes other notions studied in the literature (Caines, Greiner, & Wang, 1988; Ramadge, 1986), such as stability of Ozveren and Willsky (1990). Shu et al. further argue that many practical problems can be formulated as the detectability problem for DESs.

Four variants of detectability have been defined: strong and weak detectability and strong and weak periodic detectability (Shu et al., 2007). Shu et al. (2007) investigated detectability for deterministic DESs. A deterministic DES is modeled as a deterministic finite automaton with a set of initial states rather than a single initial state. The motivation for more initial states comes from the observation that it is often not known which state the system is initially in. They designed exponential algorithms to decide detectability of DESs based on the computation of the observer. Later, to be able to handle more problems, they extended their study to nondeterministic DESs and improved the algorithms for strong (periodic) detectability of nondeterministic DESs to polynomial time (Shu & Lin, 2011). Concerning the complexity of deciding weak detectability, Zhang (2017) showed that the problem is PSpace-complete and that PSpace-hardness holds even for deterministic DESs with all events observable. Shu and Lin (2013) further extended strong detectability to delayed DESs and developed a polynomial-time algorithm to check strong detectability for delayed DESs. Yin and Lafortune (2017) extended weak and strong detectability to modular DESs and showed that checking both strong modular detectability and weak modular detectability is PSpace-hard. The complexities of these problems have recently been resolved (Masopust & Yin, 2017).

Zhang's intractable complexity of deciding weak (periodic) detectability opens the question whether there are structurally simpler DESs for which tractability can be achieved. To tackle this question, we consider structurally the simplest deadlock-free DESs modeled as deterministic finite automata without non-trivial cycles, that is, every cycle is in the form of a self-loop in a state of the DES. We show that even for these very simple DESs, checking weak (periodic) detectability remains PSpace-complete, and hence the problem is intractable for all practical cases.

On the other hand, we show that deciding strong (periodic) detectability of DESs is NL-complete. Since NL is the class of problems that can be efficiently parallelized (Arora & Barak, 2009), we obtain that strong (periodic) detectability can be efficiently verified on a parallel computer.

2. Preliminaries and definitions

For a set A, $|A|$ denotes the cardinality of A and 2^A its power set. An *alphabet* Σ is a finite nonempty set with elements called *events*. A *word* over Σ is a sequence of events of Σ. Let Σ^* denote the set of all finite words over Σ; the *empty word* is denoted by ε. For a word

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$u \in \Sigma^*$, $|u|$ denotes its length. As usual, the notation Σ^+ stands for $\Sigma^* \setminus \{\varepsilon\}$.

A *nondeterministic finite automaton* (NFA) over an alphabet Σ is a structure $A = (Q, \Sigma, \delta, I, F)$, where Q is a finite nonempty set of states, $I \subseteq Q$ is a nonempty set of initial states, $F \subseteq Q$ is a set of marked states, and $\delta : Q \times \Sigma \to 2^Q$ is a transition function that can be extended to the domain $2^Q \times \Sigma^*$ by induction. The *language recognized by* A is the set $L(A) = \{w \in \Sigma^* \mid \delta(I, w) \cap F \neq \emptyset\}$. Equivalently, the transition function is a relation $\delta \subseteq Q \times \Sigma \times Q$, where $\delta(q, a) = \{s, t\}$ denotes two transitions (q, a, s) and (q, a, t).

The NFA A is *deterministic* (DFA) if it has a unique initial state, i.e., $|I| = 1$, and no nondeterministic transitions, i.e., $|\delta(q, a)| \leq 1$ for every $q \in Q$ and $a \in \Sigma$. We say that a DFA A over Σ is *total* if its transition function is total, that is, $|\delta(q, a)| = 1$ for every $q \in Q$ and $a \in \Sigma$. For DFAs, we identify singletons with their elements and simply write p instead of $\{p\}$. Specifically, we write $\delta(q, a) = p$ instead of $\delta(q, a) = \{p\}$.

A *discrete event system* (DES) is modeled as an NFA G with all states marked. Hence we simply write $G = (Q, \Sigma, \delta, I)$ without specifying the set of marked states. The alphabet Σ is partitioned into two disjoint subsets Σ_o and $\Sigma_{uo} = \Sigma \setminus \Sigma_o$, where Σ_o is the set of *observable events* and Σ_{uo} the set of *unobservable events*.

The detectability problems are based on the observation of events, described by the projection $P : \Sigma^* \to \Sigma_o^*$. The *projection* $P : \Sigma^* \to \Sigma_o^*$ is a morphism defined by $P(a) = \varepsilon$ for $a \in \Sigma \setminus \Sigma_o$, and $P(a) = a$ for $a \in \Sigma_o$. The action of P on a word $w = \sigma_1\sigma_2 \cdots \sigma_n$ with $\sigma_i \in \Sigma$ for $1 \leq i \leq n$ is to erase all events from w that do not belong to Σ_o; namely, $P(\sigma_1\sigma_2 \cdots \sigma_n) = P(\sigma_1)P(\sigma_2) \cdots P(\sigma_n)$. The definition can readily be extended to infinite words and languages.

As usual when detectability is studied (Shu & Lin, 2011), we make the following assumptions on the DES $G = (Q, \Sigma, \delta, I)$: (1) G is *deadlock free*, that is, for every state of the system, at least one event can occur. Formally, for every $q \in Q$, there is $\sigma \in \Sigma$ such that $\delta(q, \sigma) \neq \emptyset$. (2) No loop in G consists solely of unobservable events: for every $q \in Q$ and every $w \in \Sigma_{uo}^+$, $q \notin \delta(q, w)$.

The set of infinite sequences of events generated by the DES G is denoted by $L^\omega(G)$. Given $Q' \subseteq Q$, the set of all possible states after observing a word $t \in \Sigma_o^*$ is denoted by $R(Q', t) = \cup_{w \in \Sigma^*, P(w) = t} \delta(Q', w)$. For $w \in L^\omega(G)$, we denote the set of all its finite prefixes by $Pr(w)$.

A *decision problem* is a yes–no question, such as "Is an NFA A deterministic?" A decision problem is *decidable* if there exists an algorithm solving the problem. Complexity theory classifies decidable problems to classes based on the time or space an algorithm needs to solve the problem. The complexity classes we consider in this paper are NL, P, NP, and PSpace denoting the classes of problems solvable by a nondeterministic logarithmic-space, deterministic polynomial-time, nondeterministic polynomial-time, and deterministic polynomial-space algorithm, respectively. The hierarchy of classes is NL $\subseteq$ P $\subseteq$ NP $\subseteq$ PSpace. Which of the inclusions are strict is an open problem. The widely accepted conjecture is that all are strict. A decision problem is NL-complete (resp. NP-complete, PSpace-complete) if (i) it belongs to NL (resp. NP, PSpace) and (ii) every problem from NL (resp. NP, PSpace) can be reduced to it by a deterministic logarithmic-space (resp. polynomial-time) algorithm. Condition (i) is called *membership* and condition (ii) *hardness*.

3. The detectability problems

In this section, we recall the definitions of the detectability problems (Shu & Lin, 2011). Let Σ be an alphabet, $\Sigma_o \subseteq \Sigma$ the set of observable events, and P the projection from Σ to Σ_o.

Definition 1 (*Strong Detectability*). A DES $G = (Q, \Sigma, \delta, I)$ is *strongly detectable* with respect to Σ_{uo} if we can determine, after a finite number of observations, the current and subsequent states of the system for all trajectories of the system, i.e., $(\exists n \in \mathbb{N})(\forall s \in L^\omega(G))(\forall t \in Pr(s))|P(t)| > n \Rightarrow |R(I, P(t))| = 1$.

Definition 2 (*Strong Periodic Detectability*). A DES $G = (Q, \Sigma, \delta, I)$ is *strongly periodically detectable* with respect to Σ_{uo} if we can periodically determine the current state of the system for all trajectories of the system, i.e., $(\exists n \in \mathbb{N})(\forall s \in L^\omega(G))(\forall t \in Pr(s))(\exists t' \in \Sigma^*)tt' \in Pr(s) \wedge |P(t')| < n \wedge |R(I, P(tt'))| = 1$.

Definition 3 (*Weak Detectability*). A DES $G = (Q, \Sigma, \delta, I)$ is *weakly detectable* with respect to Σ_{uo} if we can determine, after a finite number of observations, the current and subsequent states of the system for some trajectories of the system, i.e., $(\exists n \in \mathbb{N})(\exists s \in L^\omega(G))(\forall t \in Pr(s))|P(t)| > n \Rightarrow |R(I, P(t))| = 1$.

Definition 4 (*Weak Periodic Detectability*). A DES $G = (Q, \Sigma, \delta, I)$ is *weakly periodically detectable* with respect to Σ_{uo} if we can periodically determine the current state of the system for some trajectories of the system, i.e., $(\exists n \in \mathbb{N})(\exists s \in L^\omega(G))(\forall t \in Pr(s))(\exists t' \in \Sigma^*)tt' \in Pr(s) \wedge |P(t')| < n \wedge |R(I, P(tt'))| = 1$.

In what follows, we make often implicit use of the following lemma whose proof is obvious by definition.

Lemma 5. *Let $G = (Q, \Sigma, \delta, I)$ be a DES and P be the projection from Σ to Σ_o. Let $P(G) = (Q, \Sigma_o, \delta', I)$ denote the DES obtained from G by replacing every transition (p, a, q) by $(p, P(a), q)$, and by standard techniques eliminating ε-transitions. Then G is weak/strong (periodic) detectable with respect to Σ_{uo} if and only if $P(G)$ is weak/strong (periodic) detectable with respect to $\emptyset$.* □

4. Complexity of deciding weak detectability

To decide weak (periodic) detectability of a DES, Shu and Lin (2011) construct the observer and prove that the DES is weakly detectable if and only if there is a reachable cycle in the observer consisting of singleton DES state sets, and that the DES is weakly periodically detectable if and only if there is a reachable cycle in the observer containing a singleton DES state set. Because of the construction of the observer, the algorithms are exponential. However, as pointed out by Zhang (2017), the algorithms require only polynomial space.

Zhang (2017) further shows that deciding weak (periodic) detectability is PSpace-hard. His construction results in a deterministic DES with several initial states. Although the transitions are deterministic, the DES is not a DFA because of the non-unique initial state. We slightly improve Zhang's result.

Theorem 6. *Deciding whether a deterministic DES over a binary alphabet is weakly (periodically) detectable is PSpace-complete.*

Proof. Membership in PSpace is known (Zhang, 2017). To show hardness, we modify Zhang's construction reducing the finite automata intersection problem: given a sequence of total DFAs $A_1, \ldots, A_n$ over a common alphabet Σ, the problem asks whether $\cap_{i=1}^n L(A_i) \neq \emptyset$. Without loss of generality, we may assume that $\Sigma = \{0, 1\}$ (Kozen, 1977).

In every $A_i = (Q_i, \{0, 1\}, \delta_i, q_0^i, F_i)$, we replace every transition (p, x, q) by two transitions $(p, 0, p')$ and (p', x, q). Intuitively, we encode 0 as 00 and 1 as 01. Let $A_i' = (Q_i \cup Q_i', \{0, 1\}, \delta_i', q_0^i, F_i)$ denote the resulting DFA, where $Q_i' = \{p' \mid p \in Q_i\}$. Notice that no transition under event 1 is defined in states of Q_i of A_i'.