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Brief paper

# Global robust output tracking control for a class of uncertain cascaded nonlinear systems☆

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ABSTRACT

This paper focuses on the global robust output tracking control for a class of uncertain cascaded nonlinear systems. Using only the measured output, we present a dynamic output feedback $\lambda$-tracking control scheme in a recursive method. Without the assistance of some kind of high-gain observers, we employ a reduced-order observer to rebuild the unmeasured states. The dynamic adaptation switching mechanism plays a key role in achieving the output $\lambda$-tracking. It is shown that the designed $\lambda$-tracker guarantees the system output track any desired reference signal with prescribed accuracy, and keep all signals in the closed-loop system globally bounded. A chaos control application in the fourth-order generalized Lorenz systems demonstrates the efficacy of the proposed control strategy.


## 1. Introduction

The robust output tracking control of uncertain nonlinear systems is a central problem in control theory (Byrnes & Isidori, 2000). The asymptotic tracking aims to design a feedback law, such that the tracking error between the output of a controlled plant and a prescribed smooth reference signal converges to zero as time approaches infinity. The problem of asymptotic tracking has a long-standing history and has been thoroughly investigated over the last three decades; see, for instance, Andrieu, Praly, and Astolfi (2009), Freeman and Kokotović (1996), Krishnamurthy, Khorrami, and Jiang (2006), Krstić, Kanellakopoulos, and Kokotović (1995), Li and Yang (2016), Zhang and Lin (2012), etc. From an application point of view, the asymptotic tracking objective is either not achievable or too demanding. In such a case, the practical tracking is proposed, which is to determine a feedback strategy guaranteeing the tracking error is ultimately bounded by a parameter $\lambda$, and hence practical tracking is also known as $\lambda$-tracking (Ilchmann & Ryan, 1994). Mainly because of weaker conditions and less information on reference signals, $\lambda$-tracking has received a lot of attention during the recent years, see the state feedback case like Ilchmann and Ryan (1994), Lin and Pongvuthithum (2003), Qian and Lin (2002), Yan and Liu (2010), Ye and Ding (2001), and the output feedback case BenAbdallah, Khalifa, and Mabrouk (2015), Bullinger and Allgower (2005), Gong and Qian (2007), Jia, Xu, Ma, Qi, and Zou (2016), Zhang and Xia (2015), etc.

The robust tracking control via output feedback is a more challenging problem than state feedback. As shown in Mazenc, Praly, and Dayawansa (1994) and Teel and Praly (1995), it is even unsolvable if the nonlinear vector fields grow too fast with respect to the unmeasurable states. In literatures, the output feedback practical tracking control has been investigated with some restrictive growth condition. For example, in Gong and Qian (2007), allowing some kind of higher-order growth of unmeasurable states, the practical tracking problem was addressed for a class of nonlinear systems by dynamic output feedback control. Recently, adaptive output feedback practical tracking control was addressed in BenAbdallah et al. (2015) with relaxed conditions of unmeasured states, whereas, the upper bound of the nonlinearities is required to be a polynomial function of the output. Without any kind of polynomial bounds, Zhang and Xia (2015) obtained a semi-global practical tracking control result for a class of stochastic nonlinear systems with dynamic uncertainties and unmeasured states. Using a high-gain observer involving a high-gain exponent, Jia et al. (2016) investigated the global practical tracking problem for nonlinear time-varying delay systems. However, it limits the linear growth of the unmeasurable states and the time-delay terms.

In this paper, we will investigate the robust output tracking control problem via output feedback for a class of uncertain cascaded

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nonlinear systems. Our main contributions consist of the following aspects:

(i) The global robust output tracking control problem is solved for the uncertain cascaded nonlinear system in the presence of non-vanishing disturbances. Technically, the gain adaption switching mechanism is invoked to prevent the parameter drift instability. It extends the results reported in Jiang, Mareels, Hill, and Huang (2004) and Wu, Yu, and Zhao (2011).

(ii) Different from the state feedback case in Lin and Pongvuthithum (2003), we skillfully design two parameters $\varGamma, \varUpsilon$ to scale the Lyapunov function in closed-loop system. The parameters $\varGamma$ and $\varUpsilon$, only serve as a tool for the stability analysis of the closed-loop system. This feature makes the designed $\lambda$-tracker independent on the parameters $\varGamma$ and $\varUpsilon$.

(iii) The proposed control scheme could be applied to the chaos control for the generalized Lorenz system in the presence of unknown parameters. It does not invoke the internal model and thereby could achieve the output tracking control for a much larger class of reference signals than the work in Xu and Huang (2010).

**Notations**: $R^n$ denotes the real $n$-dimensional space; $\|X\|$ denotes the Euclidean norm of a vector $X$; $|x|$ denotes its absolute value when $x \in R$. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class-$K$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class-$K_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$. $\text{dist}\big(x, [-\lambda, \lambda]\big) = \max\{0, |x| - \lambda\}$ for $x \in R$ and $\lambda > 0$.

## 2. Problem statement and preliminary

In this paper, we focus on the following class of uncertain nonlinear systems described by

$$\begin{aligned}\dot{\eta} &= q(\eta, y)\\ \dot{x}_i &= x_{i+1} + g_i(\eta, y),\ i = 1, \ldots, n-1\\ \dot{x}_n &= u + g_n(\eta, y)\\ y &= x_1\end{aligned} \tag{1}$$

where $u, y \in R$ are the input and output, and $x = (x_1, \ldots, x_n)^T \in R^n$ is part of the states with only $x_1$ observable while $\eta \in R^r$ represents cascaded dynamic uncertainty. For existence and uniqueness, it is further assumed that the uncertain functions $q(\cdot)$ and $g_i(\cdot)$ are locally Lipschitz.

For the controlled system (1), our control task is to solve the global $\lambda$-tracking problem, i.e., given any $C^1$ reference signal $y_r(t)$, to design a dynamic output feedback controller to achieve the tracking of $y_r(t)$ in the sense that $\lim_{t\to\infty}\text{dist}\big(y(t) - y_r(t), [-\lambda, \lambda]\big) = 0$ with prespecified accuracy $\lambda > 0$, while keeping all signals in closed-loop bounded over $[0, \infty)$ from any initial conditions.

Next, we introduce the definition of input-to-state practically stable (ISpS) (Jiang, Mareels, & Wang, 1996).

**Definition 1.** A control system $\dot{x} = f(x, u)$ with $x \in R^n, u \in R^m$ is ISpS if there exist a smooth, proper, and positive-definite function $V$ such that

$$\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|), \tag{2}$$

$$\frac{\partial V}{\partial x}(x) f(x, u) \le -\alpha_0(\|x\|) + \gamma_0(\|u\|) + d_0, \tag{3}$$

with $\alpha_0, \alpha_1, \alpha_2, \gamma_0 \in K_\infty$, and the constant $d_0 \ge 0$.

**Remark 1.** If $d_0 = 0$, ISpS degenerates into ISS; moreover, if $\alpha_0$ is only a positive definite, continuous function, it becomes much weaker integral input-to-state stable (iISS), see Ito (2010) and Liberzon, Sontag, and Wang (1999).

We make the following **Assumptions**.

**(A1)** The cascaded system $\dot{\eta} = q(\eta, y)$ is ISS with an ISS-Lyapunov function $U_0(\eta)$ satisfying

$$\begin{aligned}&\underline{\alpha}(\|\eta\|) \le U_0(\eta) \le \overline{\alpha}(\|\eta\|),\\ &\frac{\partial U_0}{\partial \eta}(\eta) q(\eta, y) \le -\alpha(\|\eta\|) + \gamma(|y|),\end{aligned} \tag{4}$$

where $\underline{\alpha}(\cdot), \overline{\alpha}(\cdot), \alpha(\cdot), \gamma(\cdot) \in K_\infty$.

**(A2)** For each $1 \le i \le n$, there exist two unknown positive constants $p_{i1}$ and $p_{i2}$, and two known nonnegative smooth functions $\phi_{i1}(\cdot)$ and $\phi_{i2}(\cdot)$, such that

$$|g_i(\eta, y)| \le p_{i1}\phi_{i1}(\|\eta\|) + p_{i2}\phi_{i2}(|y|). \tag{5}$$

**(A3)** There exists unknown constant $M > 0$, such that

$$|y_r(t)| \le M,\ |\dot{y}_r(t)| \le M,\ \forall\, t \ge 0. \tag{6}$$

**Remark 2.** The investigated system described in Eqs. (1) represents a large class of single-input-single-output nonlinear systems, for example, the popular class of output feedback form systems can be transformed into a special member of uncertain systems (1), see Ding, Li, and Zheng (2012) and Jiang et al. (2004). Also, many practical systems such as the continuously stirred tank reactor (CSTR) (Yu & Wu, 2012), the jet engine compression system (Krstić et al., 1995), etc., can be written into the form defined in (1).

**Remark 3.** **Assumption (A1)** is a generalized version of ISS type condition in Jia et al. (2016), Lin and Pongvuthithum (2003) and Zhang and Xia (2015). This assumption of ISS is stronger than the iISS in Jiang et al. (2004) and Wu et al. (2011). Nonetheless, it is necessary in some sense in order to realize the global output tracking control for the investigated system (1). For example, consider the following nonlinear system

$$\begin{aligned}\dot{\eta} &= -\frac{1}{2}\eta + \eta y^2\\ \dot{x}_1 &= u + e^{\eta + \sqrt{y}}\\ y &= x_1.\end{aligned} \tag{7}$$

The $\eta$-subsystem is iISS but not ISS (see Yu & Wu, 2011). It can be verified that the output tracking control for system (7) is unsolvable when $y_r(t) = 1$ because of the weaker iISS cascaded subsystem.

**Remark 4.** As a relaxed restriction of **Condition (C2)** in Jiang et al. (2004) and Wu et al. (2011), the uncertain nonlinearities $g_i(\eta, y)(i = 1, \ldots, n)$ in **Assumption (A2)** can be nonvanishing or biased. For example, this assumption allows the form of $e^{\eta + \sqrt{y}}$ included in $g_i(\eta, y)$. In addition, it is not required to satisfy any kind of polynomial bounds. **Assumption (A3)** only requires that the reference signal and its derivative have bounds but may be unknown. As stated in Yan and Liu (2010), this is the weakest assumption on the reference signal.

Let $z_1 = y(t) - y_r(t)$, and we have the following Proposition 1, and its proof is given in Appendix A.

**Proposition 1.** *Choose the smooth nondecreasing positive function $\rho(\cdot)$, then, the positive definite function*

$$V_0(\eta) = \int_0^{U_0(\eta)} \rho(s) ds \tag{8}$$

*is a $C^1$ input-to-state practically stable (ISpS) Lyapunov function with input $z_1$ and state $\eta$ satisfying*

$$\dot{V}_0(\eta) \le -\frac{1}{2}\alpha(\|\eta\|)\rho\big(U_0(\eta)\big) + z_1^2\gamma_{z_1}(z_1) + c, \tag{9}$$