



## Brief paper

A two-experiment approach to Wiener system identification<sup>☆</sup>Giulio Bottegal<sup>\*</sup>, Ricardo Castro-Garcia, Johan A.K. Suykens

ESAT-Stadius, KU Leuven, Kastelpark Arenberg 10, B-3001 Leuven (Heverlee), Belgium



## ARTICLE INFO

## Article history:

Received 22 February 2017

Received in revised form 17 November 2017

Accepted 15 January 2018

## Keywords:

System identification

Wiener systems

Experiment design

Least squares support vector machines

## ABSTRACT

We propose a new methodology for identifying Wiener systems using the data acquired from two separate experiments. In the first experiment, we feed the system with a sinusoid at a prescribed frequency and use the steady state response of the system to estimate the static nonlinearity. In the second experiment, the estimated nonlinearity is used to identify a model of the linear block, feeding the system with a persistently exciting input. We discuss both parametric and nonparametric approaches to estimate the static nonlinearity. In the parametric case, we show that modeling the static nonlinearity as a polynomial results into a fast least-squares based estimation procedure. In the nonparametric case, least squares support vector machines (LS-SVM) are employed to obtain a flexible model. The effectiveness of the method is demonstrated through numerical experiments.

© 2018 Elsevier Ltd. All rights reserved.

## 1. Introduction

Block-oriented system identification aims at representing systems as interconnected linear and nonlinear blocks. The Wiener system is one of such representations, where a linear time-invariant (LTI) block representing the dynamics of the process is followed by a static nonlinear function. Wiener models have proved to be useful to represent nonlinear systems in many application areas, e.g., chemical processes (Kalafatis, Arifin, Wang, & Cluett, 1995; Zhu, 1999), and biological systems (Hunter & Korenberg, 1986). Identification of Wiener systems has been object of intensive research for many years; an overview of previous works can be found in Giri and Bai (2010).

Maximum likelihood/prediction error techniques are discussed in Hagenblad, Ljung, and Wills (2008). The main issue with maximum likelihood is that estimation of the parameters requires the solution of a nonlinear optimization problem. A possible approach to reduce the dimensionality of the problem is to use separable least squares (Bruls, Chou, Haverkamp, & Verhaegen, 1999), or build recursive identification schemes (Wigren, 1993). In Westwick and Verhaegen (1996), a subspace-based method is proposed. Nonparametric methods based on a weighted kernel regression are discussed in several contributions (Greblicki, 1997; Mzyk, 2007),

where, however, it is required that the input is an i.i.d. sequence. Semi-parametric techniques relying on a Bayesian nonparametric model of the static nonlinearity and a parametric model of the LTI block are proposed in Lindsten, Schön, and Jordan (2013) and Pilonetto (2013). Other approaches based on instrumental variables and the Wiener G-functionals are found in Janczak (2007) and Tiels and Schoukens (2011), respectively. Experiment design techniques specifically tailored for Wiener system identification are discussed in Mahata, Schoukens, and De Cock (2016).

In this paper, we discuss a novel method for Wiener systems that separates the estimation of the nonlinearity from the identification of the LTI block, facilitating the identification process and reducing the computational burden of maximum likelihood/prediction error techniques. To do so, it is required that the user has the freedom to design the input to the system. Under this setup, a direct approach to identify the Wiener structure is to design a Gaussian input and exploit Bussgang's theorem to separate the linear and the nonlinear blocks (see e.g. Enqvist & Ljung, 2005). This strategy however, being based on computing second order moments, may require collecting long data sets. Furthermore, it is well known that even nonlinearities lead to system estimates equal to zero (Schoukens & Tiels, 2017).

Our approach is based on designing two separate experiments, each consisting of feeding the system with a specific input. In the first experiment, the system is driven by a simple sinusoidal signal with prefixed frequency and phase. Using this signal, we show that we can easily reconstruct the static nonlinearity as a function of the unknown phase delay introduced by the LTI block. We discuss three possible modeling approaches for the nonlinearity. Depending on the adopted approach, we show how to fully recover the nonlinear function (up to a scaling factor), that is, how to

<sup>☆</sup> The material in this paper was partially presented at the 56th IEEE Conference on Decision and Control, December 12–15, 2017, Melbourne, Australia. This paper was recommended for publication in revised form by Associate Editor Er-Wei Bai under the direction of Editor Torsten Söderström.

<sup>\*</sup> Corresponding author.

E-mail addresses: [g.bottegal@tue.nl](mailto:g.bottegal@tue.nl) (G. Bottegal), [ricardo.castro@esat.kuleuven.be](mailto:ricardo.castro@esat.kuleuven.be) (R. Castro-Garcia), [johan.suykens@esat.kuleuven.be](mailto:johan.suykens@esat.kuleuven.be) (J.A.K. Suykens).

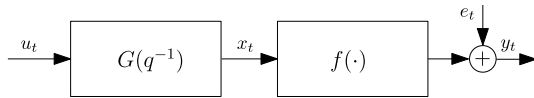


Fig. 1. Block scheme representation of a Wiener system.

remove the ambiguity introduced by the unknown phase delay. The first modeling approach relies on a parametric description of the nonlinearity as a linear combination of a number of basis functions. Here, the phase delay is recovered using a special instance of separable least squares. The second approach is a special case of the first, where the basis functions are monomials. In this case, the function can be fully estimated via a simple procedure involving least-squares estimation. The third approach is a nonparametric one; it relies on the least squares support vector machines (LS-SVM) framework (Suykens, Van Gestel, De Brabanter, De Moor, & Vandewalle, 2002), under the assumption that the nonlinearity is a smooth function. In this case, the phase delay is estimated along with the hyperparameters characterizing the kernel used in the LS-SVM estimation procedure.

We note that the idea of feeding a Wiener system with a sine signal was also explored in previous work. In Bai (2003), the phase delay introduced by the LTI block is estimated by comparing the frequency content of the output and the input. In Giri, Rochdi, and Chaoui (2009), the phase delay is estimated using a geometric approach. In this paper we use different approaches to phase estimation, depending on the model adopted for the static nonlinearity.

Using the estimated model of the static nonlinearity, we perform a second experiment where the system is fed with a persistently exciting input. In this way, we can identify the LTI block by means of a modified version of the standard prediction error method (PEM) for linear output-error (OE) systems (Ljung, 1999). The computational burden of this second step reduces essentially to the one of PEM for OE systems.

## 2. Wiener system identification using a two-experiment approach

We consider the following SISO system, also called a Wiener system (see Fig. 1 for a schematic representation):

$$\begin{aligned} x_t &= G(q^{-1})u_t \\ y_t &= f(x_t) + e_t \end{aligned} \quad (1)$$

In the former equation,  $G(q^{-1})$  represents the transfer function of a causal, asymptotically stable, LTI subsystem, driven by the input  $u_t$ , where  $q^{-1}$  denotes the time shift operator, namely  $q^{-1}u_t = u_{t-1}$ . In the latter equation,  $y_t$  is the result of a static nonlinear transformation, denoted by  $f(\cdot)$ , of the signal  $x_t$ , and  $e_t$  is white noise with (finite) unknown variance  $\sigma^2$  and finite higher-order moments. Furthermore, we assume that the noise is independent of the input and that  $f \in C^0$ , the set of continuous and pointwise defined functions. The problem under study is to estimate the LTI subsystem and the nonlinear function from a set of input and output measurements.

We assume that the user has the freedom to design the input signal  $u_t$ . In particular, we assume that the user has the possibility to run two separate experiments, each having a particular signal  $u_t$  as an input. The goal of this paper is to describe an identification technique for the system (1) that is linked to a particular choice of these experiments. It consists of the two following steps:

- (1) Feed the system with a sinusoid at a prescribed frequency and use the steady-state data to estimate the nonlinear function  $f(\cdot)$ ;

- (2) Feed the system with a persistently exciting input signal and identify the LTI subsystem using the information gathered on the first step regarding the static nonlinearity.

Let us first briefly discuss the second step of the proposed procedure. Let the LTI subsystem be described through the parameterized transfer function

$$G(q^{-1}, \theta) = \frac{b_0 + b_1q^{-1} + \dots + b_mq^{-m}}{1 + a_1q^{-1} + \dots + a_nq^{-n}}, \quad (2)$$

so that its dynamics are completely characterized by the parameter vector  $\theta := [b_0 \ b_1 \ \dots \ b_m \ a_1 \ \dots \ a_n]$ . Then, assuming that an estimate of the nonlinearity say,  $\hat{f}(\cdot)$ , is available after the first step of the procedure, we can set up a PEM-based identification criterion as follows

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{N_2} \sum_{t=1}^{N_2} (y_t - \hat{f}(G(q^{-1}, \theta)u_t))^2, \quad (3)$$

where  $N_2$  is the number of samples collected during the second experiment. Note that this is a mild generalization of the standard PEM, requiring only to account, in the optimization process, for the nonlinear transformation induced by  $\hat{f}(\cdot)$ . This does not make the solution of (3) harder than a standard PEM applied to an output-error model, because in both cases we have to face a nonlinear and non-convex optimization problem. Solution of PEM-type problems for OE systems has been object of intensive research for decades, and the field is now at a mature stage, with several developed methodologies that ensure achieving the global minimum of (3) (see, e.g., Eckhard, Bazanella, Rojas, & Hjalmarsson, 2017 for pre-filtering techniques for OE identification, or Lacy, Erwin, and Bernstein (2001) and Wigren (1994) for Wiener system identification when the nonlinearity is known).

As opposed to the aforementioned second step, the first step can be more involved and requires a more thorough analysis. We shall focus on this step in the remainder of the paper.

## 3. Approaches to estimate the nonlinearity

In this section we discuss the first step of the procedure, proposing three estimation approaches for the static nonlinearity.

We consider the input signal  $u_t = \sin(\omega t + \phi_0)$ , where  $\omega$  is a user-prescribed frequency and  $\phi_0$  is a known phase delay. Without loss of generality, in the remainder of the paper we shall consider  $\phi_0 = 0$ . Then, after the transient effect of  $G(q^{-1})$  has vanished, we have that  $x_t = A_\omega \sin(\omega t + \phi_\omega)$ , where  $A_\omega$  and  $\phi_\omega$  are the gain and the phase delay of the LTI subsystem  $G(q^{-1})$  at the frequency  $\omega$  (Ljung, 1999, Ch. 2). Due to the structural non-identifiability of Wiener systems,  $A_\omega$  cannot be determined (see Remark 2). We thus drop it and define a new signal  $\bar{x}_t := \sin(\omega t + \phi_\omega)$ , which is parameterized by the unknown quantity  $\phi_\omega$ . Accordingly, we write the output of the system as

$$y_t = f(\sin(\omega t + \phi_\omega)) + e_t. \quad (4)$$

Then, the problem under study, that is to estimate  $f(\cdot)$ , is coupled with the problem of estimating  $\phi_\omega$ . In the following, we describe three approaches to this problem, assuming that the number of collected samples of  $y_t$  (at its steady state) is equal to  $N_1$ .

**Remark 1.** Since we are estimating the static nonlinearity using the signal  $\bar{x}_t$  instead of  $x_t$ , we are obtaining a scaled (in the  $x$ -axis) version of  $f(\cdot)$ , that is, we are estimating  $f(x/A_\omega)$  instead of  $f(x)$ . This scaling effect is compensated in the second phase of the method; in fact (3) will return the estimate  $A_\omega G(q^{-1})$  instead of  $G(q^{-1})$ . Then, we need additional information (e.g., on the LTI system gain, see Bai, 1998) to uniquely recover  $G(q^{-1})$  and  $f(\cdot)$ ; this

Download English Version:

<https://daneshyari.com/en/article/7108656>

Download Persian Version:

<https://daneshyari.com/article/7108656>

[Daneshyari.com](https://daneshyari.com)