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Brief paper

Sensitivity of linear systems to input orientation and novelty☆

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ABSTRACT

This paper introduces a framework for quantitative characterization of the sensitivity of time-varying linear systems (or networks) in terms of input orientation. The motivation for such an approach comes from the study of biophysical sensory networks in the brain, wherein responsiveness to both energy and salience (in terms of input orientation and novelty) is presumably critical for mediating behavior and function. Here, we use an inner product to define the angular separation of the current input with respect to past inputs. Then, by constraining input energy, we define an optimal control problem to obtain the minimally novel input –the one that has the smallest relative angle –that effects a given state transfer. We provide analytical conditions for existence and uniqueness for the solution in both continuous and discrete-time. A closed-form expression for the minimally novel input is derived and from this solution, a fundamental relationship between control energy and input orientation sensitivity is highlighted. We provide an example that demonstrates the utility of the developed sensitivity analysis.
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1. Introduction

Understanding the sensitivity of dynamical systems to their afferent inputs is a fundamental problem in control theory. Classically, sensitivity analysis is performed in the frequency domain and characterizes, in essence, the response of a system to harmonic inputs of fixed energy. In this paper, we are interested in understanding the sensitivity of systems not in terms of frequency or energy, but rather in terms of the geometry, i.e., the orientation, of inputs in Euclidean space.

More concretely, we are motivated by a need for more complete sensitivity characterizations of biological and neuronal systems. As an example, consider a simple, prototypical 'feedforward-type' layered model of a sensory networks, wherein sensory neurons are tuned/oriented to a high dimensional feature space (e.g., different sounds, tastes, or colors) (Dayan & Abbott, 2001). One may put forth a supposition that the sensitivity of such a network, with respect to its afferent inputs, is critical for facilitating perception and behavior (see Fig. 1).

However, in the study of the sensitivity of such a system, energy is but one salient property of the inputs. Also important is the input *orientation*, i.e., the alignment of an input with certain features, and *novelty*, the difference in orientation of an input from past inputs (Pimentel, Clifton, Clifton, & Tarassenko, 2014). Indeed, the *novelty* of an input stimulus may be just as, if not more, important for perception than its energy (Downar, Crawley, Mikulis, & Davis, 2002). By means of analogy, a hot room feels hotter when entering it from the cold. The ability to assess the responsiveness of systems to novelty – at a particular moment in time, relative to past inputs – has immediate implications in the analysis and control of physiological neuronal network dynamics in several different regimes (Ching, Brown, & Kramer, 2012; Ching & Ritt, 2013; Lepage, Ching, & Kramer, 2013). Said somewhat more mathematically, suppose that an input to a system (network) is denoted $u(t) \in \mathbb{R}^m$. Then, whereas a conventional sensitivity analysis may focus on change in output per change in $\|u(t)\|$, we are concerned with the problem of change in output per change in $\sphericalangle u(t)$.

Specifically, in this paper, we seek a quantification of the sensitivity of linear time-varying systems in terms of input orientation/novelty (Kumar et al., 2015). In particular, we ask how responsive are the state trajectories to inputs that differ in orientation from those that have previously been applied. Fig. 2 illustrates this basic notion for a two-dimensional linear system with a three-dimensional input. A particular input drives the system from a point in the state space at $t = -2$ to an intermediate point at

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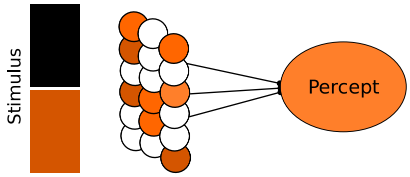

Fig. 1. In a sensory network, the input layer is tuned/oriented (i.e., sensitive) to features of the afferent input (stimulus), such as color. The sensitivity will determine the ultimate percept, or behavior. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

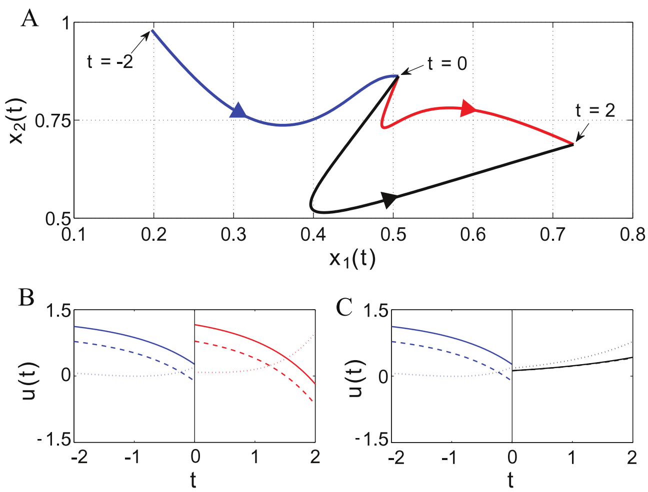

Fig. 2. Minimum novelty control vs. minimum energy control: (A) The trajectory (blue) brings the system from an initial state at $t = -2$ on intermediate state at $t = 0$. Subsequently, two trajectories are contrasted in the phase-plane for the minimum novelty control, which is the subject of this paper (red, B); and the minimum energy control (black, C). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$t = 0$; from this point emerge two trajectories, both of which reach a common endpoint at $t = 2$. The inputs that induce these trajectories are qualitatively different. The first one is not 'novel', noting the similarity between the input over $t \in [-2, 0]$ and that over $t \in [0, 2]$). In contrast, the second input is relatively more novel and, as well, uses less energy.

The major development in this paper centers on the following question: *how novel must an input be, relative to a preceding input, in order to induce a prescribed state transfer?* It turns out that an answer to this question can be obtained in closed form, through evaluation of a non-convex optimization problem. Formulation and solution of this problem forms the principal control-theoretic axis of this paper. More specifically, our major contributions are

(1) We define the notion of input novelty as the orientation between two inputs.
(2) We analytically derive, for both continuous and discrete, linear time-varying systems, the *minimum novelty control* that effects a desired state transfer. The problem seeks the smallest orientation associated with a given state transfer, relative to a prior input and constrained by a fixed average input energy.
(3) From the analytical results, we highlight a fundamental relationship between energy and input orientation sensitivity in linear systems.
(4) We present an example that highlights the utility of the proposed sensitivity analysis in the context of a decision model.

The remainder of the paper is organized as follows. In Section 2, we introduce the inner product-based input novelty measure for continuous-time, linear time-varying systems and formulate a non-convex optimal control problem that minimizes this novelty under the constraint of a fixed average input energy. We establish the existence and the uniqueness of a global optimal solution of the control problem and derive a closed-form expression for the minimally novel input. In Section 3, we derive analogous results for discrete time linear time-varying systems. Finally, in Section 4, we demonstrate utility of the developed approach.

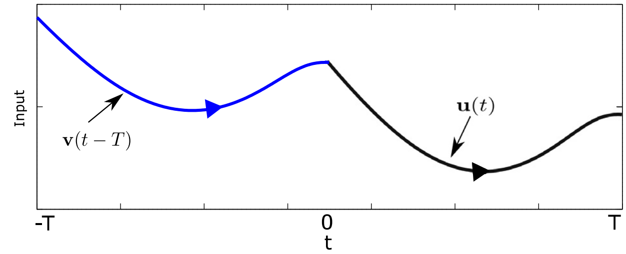

Fig. 3. A schematic of applied inputs to the system over the time interval $[-T, T]$. $\mathbf{v}(t-T)$ is the prior input which acts on the system from $t = -T$ to $t = 0$. The input $\mathbf{u}(t)$ acts on the system from $t = 0$ to $t = T$.

2. Continuous-time, linear dynamical systems

2.1. Input novelty

We consider a linear, time-varying system with dynamics of the form

$$\frac{d\mathbf{x}(t)}{dt} = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t). \tag{1}$$

Here $\mathbf{x}(t) \in \mathbb{R}^{n\times 1}$ represents the state of the system at time t, $A(t) \in \mathbb{R}^{n\times n}$ describes the time-varying dynamics, $B(t) \in \mathbb{R}^{n\times m}$ is the input matrix, and $\mathbf{u}(t) \in \mathbb{R}^{m\times 1}$ is the input to the system. We assume that the dynamical system (1) is controllable.

We consider an input $\mathbf{v}(t-T) \in \mathbb{R}^{m\times 1}$, $t \in [0, T]$, with total energy $T\gamma_v$, i.e.

$$\frac{1}{T}\int_0^T \|\mathbf{v}(t-T)\|_2^2 dt = \gamma_v \tag{2}$$

where $\|\mathbf{v}(t-T)\|_2$ is the Euclidean norm of the vector $\mathbf{v}(t-T)$. Further, we assume that $\mathbf{v}(t-T)$ is a prior input to the system that brings the state to $\mathbf{x}_0$ at $t = 0$. Thus, $\mathbf{v}(t-T)$ acts on the system from $t = -T$ to $t = 0$ whereas $\mathbf{u}(t)$ acts from $t = 0$ to $t = T$, as shown in Fig. 3. Here, $T > 0$ is a constant and $\gamma_v > 0$ is the average per-time energy of $v(t)$. We denote the state of the system at $t = 0$ as $\mathbf{x}(0) \equiv \mathbf{x}_0$, at $t = -T$ as $\mathbf{x}(-T) \equiv \mathbf{x}_r$ and at $t = T$ as $\mathbf{x}(T) \equiv \mathbf{x}_f$. Throughout the paper, we denote $(\cdot)'$ and $\|\cdot\|_2$ as the transpose and Euclidean norm of the underlying argument.

Definition 1 (*Input Orientation*)**.** The dot product of $\mathbf{v}(t-T)$ and $\mathbf{u}(t)$ is given by

$$\mathbf{v}'(t-T)\mathbf{u}(t) = \|\mathbf{v}(t-T)\|_2\|\mathbf{u}(t)\|_2 cos(\theta(t)). \tag{3}$$

Here, $\|\mathbf{v}(t-T)\|_2$ and $\|\mathbf{u}(t)\|_2$ are the Euclidean norm of inputs $\mathbf{v}(t-T)$ and $\mathbf{u}(t)$ respectively. We define $\theta(t)$ as the input orientation.

Definition 2 (*Input Novelty*)**.** We define a cost function $\mathbb{J}(T)$ associated with inputs $\mathbf{v}(t-T)$ and $\mathbf{u}(t)$ as

$$\mathbb{J}(T) = \frac{1}{T\sqrt{\gamma_v\gamma_u}}\int_0^T \mathbf{v}'(t-T)\mathbf{u}(t)dt, \tag{4}$$

where

$$\frac{1}{T}\int_0^T \|\mathbf{u}(t)\|_2^2 dt = \gamma_u, \tag{5}$$

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