Automatica 93 (2018) 527–534

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Envelope-constrained H_∞ filtering for nonlinear systems with quantization effects: The finite horizon case☆

Lifeng Ma [a], Zidong Wang [b,c,*], Qing-Long Han [d], Hak-Keung Lam [e]

[a] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China
[b] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China
[c] Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, UK
[d] School of Software and Electrical Engineering, Swinburne University of Technology, Melbourne, VIC 3122, Australia
[e] Department of Informatics, School of Natural & Mathematical Science, King's College London, WC2R 2LS, UK

ARTICLE INFO

Article history:
Received 1 October 2016
Received in revised form 20 September 2017
Accepted 1 February 2018

Keywords:
Nonlinear systems
H_∞ filtering
Envelope constraint
Quantization effects
Finite-horizon filtering

ABSTRACT

This paper is concerned with the envelope-constrained H_∞ filtering problem for a class of discrete nonlinear stochastic systems subject to quantization effects over a finite horizon. The system under investigation involves both deterministic and stochastic nonlinearities. The stochastic nonlinearity described by statistical means is quite general that includes several well-studied nonlinearities as its special cases. The output measurements are quantized by a logarithmic quantizer. Two performance indices, namely, the finite-horizon H_∞ specification and the envelope constraint criterion, are proposed to quantify the transient dynamics of the filtering errors over the specified time interval. The aim of the proposed problem is to construct a filter such that both the prespecified H_∞ requirement and the envelope constraint are guaranteed simultaneously over a finite horizon. By resorting to the recursive matrix inequality approach, sufficient conditions are established for the existence of the desired filters. A numerical example is finally proposed to demonstrate the effectiveness of the developed filtering scheme.

1. Introduction

Due to the significance in control and signal processing, the nonlinear filtering problem has been attracting constant research interest in the past several decades. A number of approaches have been developed to deal with the filtering problem for nonlinear stochastic systems, among which some of the most widely used include but are not limited to Bayes filtering, particle filtering, extended Kalman filtering (EKF) and unscented Kalman filtering (UKF). The Bayes filter aims to, in a recursive fashion, estimate the hidden state by using the available measurements and the process model (Garcia, Hausotte, & Amthor, 2013). Based on the Bayesian theory in combination with the concept of sequential importance sampling, particle filtering is particularly useful in coping with nonlinear and/or non-Gaussian problems (Djuric et al., 2003). However, the high computational complexity largely hinders the utilization of particle filters. Another recursive filter that should be mentioned is the celebrated Kalman filter (Kalman, 1960), which is in fact a linear version of Bayes filter for systems subject to Gaussian noises. As for nonlinear stochastic Gaussian systems, several invariants based on Kalman filters have been developed among which the most widely applied are EKF and UKF. EKF provides an approximation of an optimal estimate by linearizing the nonlinear system at the state estimates, which has found wide applications in both theoretical research and engineering practice (Einicke & White, 1999). However, it is no longer applicable when the process/measurement models are highly nonlinear, which gives rise to the so-called unscented Kalman filtering. The UKF uses a deterministic sampling technique known as the unscented transform to pick a minimal set of sample points around the mean value and could give more accurate estimates than EKF especially for those highly nonlinear systems (Sarkka, 2007).

The past several decades have seen a surge of research interest on the H_∞ filtering problems for nonlinear systems and several effective approaches have been exploited to deal with filtering problems with the requested disturbance attenuation level, see e.g. Dong, Wang, Ding, and Gao (2016), Einicke and White (1999), Shaked and Berman (1995), Shen and Deng (1997), Shen, Wang, Shu, and Wei (2010) and Takaba and Katayama (1996). On another research frontier, networked control systems (NCSs) have attracted

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Xing Zheng under the direction of Editor Torsten Söderström.

* Corresponding author at: College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China.
E-mail addresses: malifeng@njust.edu.cn, lifengma1983@googlemail.com (L. Ma), Zidong.Wang@brunel.ac.uk (Z. Wang), qhan@swin.edu.au (Q. Han), hak-keung.lam@kcl.ac.uk, hak_keung_lam@yahoo.com.hk (H. Lam).

https://doi.org/10.1016/j.automatica.2018.03.038
0005-1098/

much attention owing to their clear application insights in a wide range of areas (Zhang, Han, & Jia, 2015; Zhang, Han, & Zhang, 2017). It has been recognized that, in the context of NCSs, the quantization effects stemming from analog-to-digital conversion processes are ubiquitous, which would probably lead to the deterioration of the system performance. In the NCS research, there are mainly two types of quantization models, namely, the uniform quantization (Tsai & Song, 2009) and the logarithmic quantization (Fu & Xie, 2005). In particular, a sector-bound technique has been presented in Fu and Xie (2005) that is capable of coping with the logarithmic quantization issues conveniently, and such an elegant paradigm has then been quickly followed in the area, see e.g. Liu, Liu, and Alsaadi (2016) and Shen et al. (2010).

The envelope-constrained filtering (ECF) algorithm has been stirring some research interest in the past few decades. The main idea of ECF algorithm is to confine the output of the filtering error (stimulated by a specified input) into a prescribed envelope. Such an envelope is determined by the desired output and tolerance band. The ECF technique has found successful applications in a variety of engineering branches ranging from signal processing to digital communications (Cantoni, Vo, & Teo, 2001; Ding, Wang, Shen, & Dong, 2015). Up to now, several methodologies have been utilized in the literature to deal with the envelope-constrained filtering problems, see, e.g. Tan, Soh, and Xie (2000) and Zang, Cantoni, and Teo (1999). It should be pointed out that almost all the results relevant to ECF have been concerned with the *linear time-invariant* systems. When it comes to *general nonlinear time-varying* systems, the corresponding envelope-constrained filtering problem has not been thoroughly investigated yet and this motivates us to shorten such a gap in the current study. It is, therefore, the main purpose of this paper to deal with the identified challenges by launching a major study on the so-called envelope-constrained H_∞ filtering problem.

The rest of this paper is organized as follows. Section 2 formulates the envelope-constrained H_∞ filtering problem for discrete-time nonlinear system subject to quantization effects. The main results are presented in Section 3 where sufficient conditions for solvability of the addressed filtering problem are given in terms of recursive linear matrix inequalities (RLMIs). Section 4 gives a numerical example and Section 5 outlines our conclusion.

2. Problem formulation

Consider the following nonlinear system defined on the horizon $[0, N]$:

$$\begin{cases} x_{k+1} = f(x_k) + g(x_k) + B_k w_k \\ y_k = h(x_k) + D_k v_k \\ z_k = L_k x_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$ and $z_k \in \mathbb{R}^{n_z}$ represent, respectively, the system state, the measurement output and the signal to be estimated. $w_k \in l_2([0, N]; \mathbb{R}^{n_w})$ and $v_k \in l_2([0, N]; \mathbb{R}^{n_v})$ are the disturbance inputs. B_k, D_k and L_k are known time-varying matrices with appropriate dimensions.

The deterministic nonlinearities $f(x_k)$ and $h(x_k)$ are known and analytic everywhere over the finite horizon $[0, N]$. On the other hand, the stochastic nonlinearity $g(x_k)$ is assumed to have the following first moment for all x_k:

$$\mathbb{E}\{g(x_k)|x_k\} = 0 \tag{2}$$

with the covariance given by

$$\begin{aligned} &\mathbb{E}\{g(x_k)g^T(x_j)|x_k\} = 0, \quad k \neq j \\ &\mathbb{E}\{g(x_k)g^T(x_k)|x_k\} = \sum_{l=1}^{q} \varrho_{l,k}\varrho_{l,k}^T \left(x_k^T \Upsilon_{l,k} x_k\right) \end{aligned} \tag{3}$$

where $\varrho_{l,k}$ and $\Upsilon_{l,k} \geq 0$ $(l = 1, 2, \ldots, q)$ are, respectively, known column vectors and matrices with compatible dimensions.

In this paper, the quantization effects are taken into consideration. Denote the quantizer as

$$\sigma(\cdot) \triangleq \begin{bmatrix} \sigma_1(\cdot) & \sigma_2(\cdot) & \cdots & \sigma_{n_y}(\cdot) \end{bmatrix}$$

which is symmetric, i.e., $\sigma_j(-y) = -\sigma_j(y)$ $(j = 1, 2, \ldots, n_y)$. The quantizer is assumed to be logarithmic type and the process of the quantization is described by

$$\sigma(y_k) = \begin{bmatrix} \sigma_1(y_k^{(1)}) & \sigma_2(y_k^{(2)}) & \cdots & \sigma_{n_y}(y_k^{(n_y)}) \end{bmatrix}^T \tag{4}$$

where $y_k^{(j)}$ $(j = 1, 2, \ldots, n_y)$ denotes the jth entry of the vector y_k. For each $\sigma(\cdot)$, the set of quantization levels is described by

$$\begin{aligned} \mathscr{U}_j = &\left\{\pm\hat{\mu}_i^{(j)},\ \hat{\mu}_i^{(j)} = \chi_j^i \hat{\mu}_0^{(j)},\ i = 0, \pm1, \pm2, \ldots\right\} \cup \{0\}, \\ &0 < \chi_j < 1,\ \hat{\mu}_0^{(j)} > 0. \end{aligned} \tag{5}$$

where χ_j $(j = 1, 2, \ldots, n_y)$ is the quantization density. Each of the quantization level corresponds to a segment such that the quantizer maps the whole segment to this quantization level. According to Fu and Xie (2005), the associated quantizer is defined as follows:

$$\sigma(y_k^{(j)}) = \begin{cases} \hat{\mu}_i^{(j)}, & \dfrac{1+\chi_j}{2}\hat{\mu}_i^{(j)} \leq y_k^{(j)} \leq \dfrac{1+\chi_j}{2\chi_j}\hat{\mu}_i^{(j)} \\ 0, & y_k^{(j)} = 0 \\ -\sigma(-y_k^{(j)}), & y_k^{(j)} < 0. \end{cases} \tag{6}$$

Consequently, it can be easily seen from the above definition (6) that the following inequality holds:

$$\left(\sigma(y_k) - G_1 y_k\right)^T \left(\sigma(y_k) - G_2 y_k\right) \leq 0 \tag{7}$$

where $G_1 \triangleq \mathrm{diag}_{n_y}\{2\chi_j/(1+\chi_j)\}$ and $G_2 \triangleq \mathrm{diag}_{n_y}\{2/(1+\chi_j)\}$. Since $0 < \chi_j < 1$, it is obvious that $0 \leq G_1 < I \leq G_2$. Then, $\sigma(y_k)$ can be decomposed as follows:

$$\sigma(y_k) = G_1 y_k + \varphi(y_k) \tag{8}$$

where $\varphi(y_k)$ is a nonlinear vector-valued function which, from (7), satisfies

$$\varphi^T(y_k)\left(\varphi(y_k) - G y_k\right) \leq 0 \tag{9}$$

with G being defined as $G \triangleq G_2 - G_1$.

Definition 1 (*Durieu, Walter, & Polyak, 2001*). A bounded ellipsoid $\mathscr{E}(c, P, n)$ of $\mathbb{R}^n$ with a nonempty interior in the mean square sense can be defined by

$$\mathscr{E}(c, P, n) \triangleq \{x \in \mathbb{R}^n : \mathbb{E}\{(x-c)^T P^{-1}(x-c)\} \leq 1\}$$

where $c \in \mathbb{R}^n$ is the center of $\mathscr{E}(c, P, n)$ and $P > 0$ is a positive definite matrix.

In this paper, the filter to be designed is of the following form:

$$\hat{x}_{k+1} = F_k \hat{x}_k + H_k \sigma(y_k), \quad \hat{x}_0 = 0. \tag{10}$$

Denote $e_k \triangleq x_k - \hat{x}_k$ and $\tilde{z}_k \triangleq z_k - \hat{z}_k$. Subtracting (10) from (1) and taking (8) into account, we obtain the following filtering error system:

$$\begin{cases} e_{k+1} = f(x_k) + g(x_k) + B_k w_k - F_k \hat{x}_k \\ \qquad\quad - H_k G_1 h(x_k) - H_k \varphi(y_k) - H_k G_1 D_k v_k \\ \tilde{z}_k = L_k e_k. \end{cases} \tag{11}$$

By defining

$$\Phi_k \triangleq \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}_k}, \quad \Psi_k \triangleq \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_k},$$