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Further results on "Reduced order disturbance observer for discrete-time linear systems"☆

Jinya Su *, Wen-Hua Chen

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough LE113 TU, UK

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ABSTRACT

Reduced order Disturbance OBservers (DOB) have been proposed in Kim et al. (2010) and Kim and Rew (2013) for continuous-time and discrete-time linear systems, respectively. The existence condition of the promising algorithm has been established but is not straightforward to check. This work further improves the reduced order DOB design by formulating it as a functional observer design problem. By carefully designing the state functional matrix, a generic DOB is resulted with an easily-checked necessary and sufficient existence condition and an easily-adjusted convergence rate. It is also shown that both the reduced order DOB in Kim and Rew (2013) and the full order DOB in Chang (2006) are special cases of this new DOB.

1. Introduction

Motivated by the prosperous applications in disturbance rejection control and fault diagnosis (see, Su & Chen, 2017; Wei & Guo, 2010; Wei, Wu, & Karimi, 2016; Yang, Cui, Li, & Zolotas, 2018), a discrete-time Disturbance OBserver (DOB) was proposed in Chang (2006). To relax observer existence conditions and reduce observer order, a reduced order DOB was designed in Kim et al. (2010) to reconstruct disturbances/faults with a minimal observer order for continuous-time linear systems. Recently, this innovative work was further extended to the discrete-time case in Kim and Rew (2013), where the systems under consideration are

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_k + G d_k \\ y_k = C x_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $d_k \in \mathbb{R}^q$ and $y_k \in \mathbb{R}^l$ are the states, inputs, disturbances and measurements at kth step. Disturbance distribution matrix G has a full column-rank (i.e., $rank(G) = q$); otherwise, the redundant inputs can be removed (see, Su, Li, & Chen, 2015). The disturbances d_k are assumed to be unknown but slowly time-varying, i.e., the following assumption is assumed

$$d^i_{k+1} = d^i_k + \Delta d^i_{k+1}, \tag{2}$$

with $d_k := [d^1_k, \ldots, d^q_k]^T$, $|\Delta d^i_{k+1}| = |d^i_{k+1} - d^i_k| \le T\mu_i$ where T is the sampling time, μ_i is a small positive value.

Remark 1. The disturbance assumption in (2), in comparison with the ones in Gillijns and De Moor (2007) and Su et al. (2015) where no particular disturbance models are assumed, can relax the observer existence condition. However, this is usually at the expense of a degraded performance when the disturbance assumption is violated.

Following the same notations as in Kim and Rew (2013) (see, pp. 970), define $N_c := (I_n - C^+ C)$ with C^+ being the Moore–Penrose inverse of C and H_e as

$$H_e := \begin{bmatrix} K N_c \\ K(\Phi - I_n) N_c \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} V^T,$$

with K being a gain matrix to be designed, $H_1 \in \mathbb{R}^{q\times h}$, $H_2 \in \mathbb{R}^{q\times h}$ and $V^T \in \mathbb{R}^{h\times n}$ with $h = rank(H_e)$.

Defining $\eta_k := V^T x_k \in \mathbb{R}^h$, the reduced order DOB constructed in Kim and Rew (2013) is given by

$$\begin{cases} \xi_{k+1} = R\xi_k + S y_k + W_u u_k + W_d \hat{d}_k \\ \hat{\eta}_k = \xi_k + Q y_k \\ z_{k+1} = z_k + K\{(\Phi - I_n)C^+ y_k + \Gamma u_k\} \\ \qquad + KG\hat{d}_k + H_2 \hat{\eta}_k \\ \hat{d}_k = KC^+ y_k - z_k + H_1 \hat{\eta}_k, \end{cases} \tag{3}$$

where $z_k \in \mathbb{R}^q$ and $\xi_k \in \mathbb{R}^h$, $W_u = (V^T - QC)\Gamma$, $W_d = (V^T - QC)G$ with matrices S, Q, R satisfying

$$(V^T - QC)\Phi - R(V^T - QC) - SC = 0. \tag{4}$$

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* Corresponding author.
E-mail addresses: J.Su2@lboro.ac.uk (J. Su), w.chen@lboro.ac.uk (W.-H. Chen).

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Denote disturbance and state function estimation error as $e_k = d_k - \hat{d}_k$ and $\epsilon_k = \eta_k - \hat{\eta}_k$. The composite error dynamic is given by Kim and Rew (2013)

$$\begin{bmatrix} e_{k+1} \\ \epsilon_{k+1} \end{bmatrix} = A_e \begin{bmatrix} e_k \\ \epsilon_k \end{bmatrix} + \begin{bmatrix} \Delta d_{k+1} \\ O_{h\times 1} \end{bmatrix},$$

where the composite error matrix A_e is defined as

$$A_e = \begin{bmatrix} I_q - KG + H_1(V^T - QC)G & H_1R - H_1 - H_2 \\ (V^T - QC)G & R \end{bmatrix}. \quad (5)$$

As pointed by Kim and Rew (2013), the existence of a stable reduced order DOB (3) depends on whether there exists a gain matrix K and other design parameters so that: (i) the Sylvester Eq. (4) holds; (ii) matrix A_e in (5) is asymptotically stable (i.e., the amplitudes of its eigenvalues are less than 1). Condition (i) is implied by the matrix rank equality

$$rank(\begin{bmatrix} Z_1 \\ V^T\Phi \end{bmatrix}) = rank(Z_1), Z_1 := rank(\begin{bmatrix} C \\ C\Phi \\ V^T \end{bmatrix}). \quad (6)$$

However, condition (ii) is not straightforward to check. Its existence is related to a static output feedback problem. As pointed out in Kim and Rew (2013) *and* Kim et al. (2010) *, although numerical solutions exist, the general solvability of the static output feedback is not known.*

To further develop this promising approach, this paper improves the results in Kim and Rew (2013) by presenting a generic reduced order DOB with an easily-checked existence condition. We transform the reduced order DOB design problem into a State Functional Observer (SFO) design problem (see, Darouach, 2000; Fernando, Jennings, & Trinh, 2011 for SFO theory). This is achieved by first augmenting the disturbances with the states and then carefully designing the state functional matrix. Consequently, a generic DOB is resulted with its necessary and sufficient existence condition. There are two promising features in the newly developed DOB in comparison with the existing results:

(i) The existence condition is easy to check, where two easily-checked matrix rank equalities are required;
(ii) The observer convergence rate can be easily adjusted via the existing pole assignment techniques.

On this basis, we further investigate the relationship between the proposed DOB with the reduced order DOB in Kim and Rew (2013) and the full order DOB in Chang (2006) in terms of observer structure and existence conditions. Both of them are shown to be special cases of the newly developed DOB.

2. DOB design using SFO technique

SFO, firstly introduced in Luenberger (1966), received much attention in control engineering (see, Darouach, 2000) owning to its fine properties such as a lower observer order and a relaxed existence condition. Its existence condition has been rigorously established in Darouach (2000). However, little attention has been paid to its applications in disturbance/fault estimation. Consequently, this paper aims to exploit its potential in DOB design, which can provide a framework unifying the existing DOBs for discrete-time linear systems.

2.1. Observer design

Combining system (1) and disturbance (2), and defining $\bar{x}_k = [x_k^T, d_k^T]^T$, an augmented system is resulted

$$\begin{cases} \bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{\Gamma}u_k + \Delta\bar{d}_k \\ y_k = \bar{C}\bar{x}_k, \end{cases} \quad (7)$$

where the gain matrices and $\Delta\bar{d}_k$ are given as follows

$$\bar{A} = \begin{bmatrix} \Phi & G \\ O_{q\times n} & I_q \end{bmatrix}, \bar{\Gamma} = \begin{bmatrix} \Gamma \\ O_{q\times m} \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C & O_{l\times q} \end{bmatrix}, \Delta\bar{d}_k = \begin{bmatrix} O_{n\times 1} \\ \Delta d_k \end{bmatrix}.$$

Remark 2. For the case that measurement outputs are also subject to disturbances, i.e., $y_k = Cx_k + G_2d_k$, this approach is also applicable by choosing $\bar{C} = [C\ G_2]$. Then the remaining design procedures are the same, although the existence condition will be slightly different.

To obtain disturbance estimates, the state functional matrix L (see, Darouach, 2000 for its definition) is chosen with a special structure

$$L = \begin{bmatrix} L_0 & O_{\bar{h}\times q} \\ O_{q\times n} & I_q \end{bmatrix}, \quad (8)$$

where the design of sub-matrix $L_0 \in \mathbb{R}^{\bar{h}\times n}$ with full row-rank will be discussed in Section 2.3. Define

$$v_k = L\bar{x}_k, \text{ with } d_k = [O_{q\times n}, I_q]v_k, \quad (9)$$

which is the state function to be estimated.

Now the problem of DOB design can be transformed into the problem of state functional observer design for the augmented system (7) with state function v_k in (9) to be estimated.

According to SFO theory (see, Darouach, 2000; Fernando et al., 2011), the disturbance observer for $\hat{d}_k$ along with the state functional observer for v_k takes the following form

$$\begin{cases} w_{k+1} = Nw_k + Jy_k + Hu_k, \\ \hat{v}_k = Bw_k + Ey_k, \\ \hat{d}_k = [O_{q\times n}\ I_q]\hat{v}_k. \end{cases} \quad (10)$$

Defining an intermediate error $\chi_k = \bar{P}\bar{x}_k - w_k$ with $\bar{P}$ being an intermediate matrix, its dynamics is given by

$$\chi_{k+1} = N\chi_k + (\bar{P}\bar{A} - N\bar{P} - J\bar{C})\bar{x}_k + (\bar{P}\bar{\Gamma} - H)u_k + \bar{P}\Delta\bar{d}_k. \quad (11)$$

Moreover, the state function estimation error $e_k = v_k - \hat{v}_k$ can be written as

$$e_k = B\chi_k + (L - E\bar{C} - B\bar{P})\bar{x}_k, \quad (12)$$

from which one can obtain $e_k \to 0$ as $k \to \infty$ for any $\bar{x}_k$ if and only if the following two conditions hold concurrently:

(i) $\chi_k \to 0$ as $k \to \infty$;
(ii) $L - E\bar{C} - B\bar{P} = O$.

For any invertible matrix B, the aforementioned condition (ii) is implied by choosing $\bar{P}$ as

$$\bar{P} = B^{-1}L - B^{-1}E\bar{C}. \quad (13)$$

Ignoring the term $\bar{P}\Delta\bar{d}_k$ as it does not affect the analysis, Eq. (11) implies that $\chi_k \to 0$ as $k \to \infty$ if and only if the following conditions hold simultaneously

(a) $\bar{P}\bar{A} - N\bar{P} - J\bar{C} = O$ (Sylvester equation);
(b) $\bar{P}\bar{\Gamma} - H = O$;
(c) N is asymptotically stable (i.e., the amplitudes of its eigenvalues are less than 1).

Choosing $\bar{P}$ according to (13) with any invertible B and H according to Condition (b), then the existence condition reduces to Condition (c), i.e. N being asymptotically stable under the constraint Sylvester equation Condition (a). The existence condition