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Stability and stabilization of a class of stochastic switching systems with lower bound of sojourn time ☆

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ABSTRACT

This paper is concerned with the stability and stabilization issues for a family of discrete-time stochastic switching systems with bounded sojourn time. The stochastic switching systems are modeled by semi-Markov jump linear systems and the semi-Markov kernel approach is employed to handle the stability and stabilization problems. The sojourn time of each system mode is considered to have both upper and lower bounds, which is more general than the scenarios in previous literature that only consider the upper bound of sojourn time. The concept of σ-error mean square stability is put forward in a new form by taking into account the lower bounds of sojourn time for all system modes. By virtue of a Lyapunov function that not only depends on the current system mode but also on the elapsed time the system has been in the current mode, together with certain techniques eliminating powers of matrices, numerically testable stability and stabilization criteria in the sense of the proposed σ-error mean square stability are obtained for the closed-loop stochastic switching system. Finally, a numerical example and a practical example of a DC motor are utilized to demonstrate the effectiveness of the proposed control strategy and the superiority of allowing for the lower bound of sojourn time.

1. Introduction

Stochastic switchings are unavoidable in various practical systems, such as power electronics systems (Oliveira, Vargas, do Val, & Peres, 2014; Vargas, Costa, & do Val, 2013; Vargas, Sampaio, Acho, Zhang, & do Val, 2016), mechanical systems (Anulova, 2015; Iwankiewicz, Nielsen, & Larsen, 2005), and communication networks (Hespanha, 2004; Liu, Fridman, & Johansson, 2015; Zhao, Guo, & Ding, 2015), thus researches on stochastic switching systems are of great practical significance. The advances on stochastic switching systems can be seen in the literature (Basin & Calderon-Alvarez, 2009; Basin & Maldonado, 2014; Basin & Rodkina, 2008; Boukas, 2007; Liu, Zhao, Niu, Wang, & Xie, 2015; Luan, Liu, & Shi, 2011; Xiong, Lam, Shu, & Mao, 2014) and references therein. As an important class of stochastic switching systems, Markov jump systems (MJSs) have been widely utilized to model systems that have different system modes and may switch among them stochastically (Li, Lam, Gao, & Xiong, 2016). During the past decades, numerous studies on the issues of stability analysis and controller synthesis for MJSs have been launched, see, e.g., Aberkane (2011), Li, Chen, Lam, and Mao (2012), Niu, Ho, and Wang (2007), Shu, Lam, and Xiong (2010), Wu, Park, Su, and Chu (2012), Wu, Shi, Su, and Chu (2012) and Wu, Xie, Shi, and Xia (2009). It is worth emphasizing that these developed theories can be effectively applied only if the sojourn time between consecutive jumps should be subject to exponential distribution in the continuous-time domain (or geometric distribution in the discrete-time domain), which cannot cover all scenarios.

Another important class of stochastic switching systems, semi-Markov jump systems (S-MJSs) has been an important research area since it was first studied in Howard (1964), and recent years have witnessed a growing interest in the stability and control problems of this class of systems, see, e.g., Jiang, Xi, and Yin (2012), Li, Wu, Shi, and Lim (2015), Schioler, Simonsen, and Leth (2014), Wei, Qiu, and Fu (2015) and Yang, Zhang, and Yin (2016). In contrast with MJSs, the sojourn time in S-MJSs is not necessarily subject to either exponential distribution or geometric distribution,

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which indicates that the transition probabilities in S-MJSs are time-varying and have the so-called "memory" property. Therefore, S-MJSs are capable of describing much broader scope of stochastic switching systems, and MJSs can be regarded as a special class of S-MJSs. However, due to the generality of semi-Markov chain in the capability of modeling stochastic switchings, investigations on S-MJSs are more challenging than MJSs. Up to date, developed theories on S-MJSs are still far away from maturity, even for the basic semi-Markov jump linear systems (S-MJLSs).

In some earlier works on S-MJLSs, several special types of probability distributions of sojourn time are utilized to address the stability problem, see, e.g., Hou, Luo, and Shi (2005), Hou, Luo, Shi, and Nguang (2006), Huang and Shi (2013), Huang, Shi, and Zhang (2014) and Schwartz and Haddad (2003), in which the sojourn time of each system mode possesses a single distribution with certain parameters. In order to overcome such a limitation, very recently, the so-called semi-Markov kernel (SMK) approach is exploited in Zhang, Leng, and Colaneri (2016). With the introduction of SMK, the statistic characteristics of sojourn time in S-MJLSs can be dependent on both the current and the next system modes, which is more general than those in the previous studies. In addition, as the transition probabilities rely on all the history information of past switching sequences in a semi-Markov chain, it is rather difficult to obtain numerically testable stability or stabilization condition if to completely utilize the probability density function (PDF) information of sojourn time (Zhang, Yang, & Colaneri, 2017). To tackle this problem, as well as considering the fact that the practical sojourn time is generally finite, a recent attempt in Zhang et al. (2016) is to make a truncation on the sojourn time of each system mode to be upper-bounded, such that a finite number of stability conditions can be obtained, which can be numerically tested.

Note that in reality, there may be a nonzero possibility for some stochastic switching systems that mode switchings cannot occur immediately after the previous switching, and consequently the existence of lower bound of sojourn time is highly possible. One typical example can be found in a machine with three modes, i.e., "good", "fair" and "broken" (Ross, 2009), as it usually takes time to recover to "good" or "fair" mode from "broken" mode. Another typical example is an ecological system that can be modeled as an S-MJLS (Zhang et al., 2017), in which evident characteristics are often displayed by different seasons that cannot change rapidly. Recent studies on sojourn time with lower bound can be found in stochastic timed automata (Bertrand, 2015) and quantum resonances (Asch, Bourget, Cortés, & Fernandeza, 2016). However, in the control community, no insightful investigations have been reported on the issue of S-MJLSs with the consideration of lower bounds of sojourn time so far, and the derived stability or stabilization condition may be quite conservative if ignoring the existence of such lower bounds. Whereas the concept of mean square stability (MSS) considers the random sojourn time to be any length (even infinity) and there exists an error to approximate MSS by allowing for finite sojourn time, the concept of σ-error mean square stability (σ-EMSS) is introduced in Zhang et al. (2016) where σ characterizes the degree of approximation error of σ-EMSS to MSS. Unfortunately, such defined σ-EMSS is not applicable if there exist lower bounds of sojourn time in any system modes. Another drawback of the σ-EMSS lies in that σ becomes larger if more system modes are considered, which leads to an incomparability of the degrees of approximation errors among systems with different numbers of modes.

Based on the aforementioned discussions, this paper investigates the stability and stabilization problems for a class of discrete-time S-MJLSs with finite sojourn time that has both upper and lower bounds for each system mode. The main contributions are summarized as follows: (i) the lower bound of sojourn time for each system mode is taken into account as a first attempt, which generalizes the previous results only considering the upper bound; (ii) a novel framework of σ-EMSS is proposed by allowing for both upper and lower bounds of sojourn time for each system mode, which describes a broader scope of stability and is better scaled than the traditional σ-EMSS; (iii) the approximation error σ of σ-EMSS to MSS is defined in the form of average value of components of all the system modes, such that the σ among different S-MJLSs with different system modes can be comparable. The remainder of the paper is organized as follows. Section 2 formulates the problem and gives two examples to show the reasonability of considering lower bound of sojourn time. Section 3 is devoted to the development of stability and stabilization criteria which can be tested numerically. To evaluate the theoretical results, two illustrative examples are provided in Section 4. Finally, Section 5 draws the conclusion and proposes the perspectives for future works.

Notations: The notation used throughout this note is standard except where otherwise stated. The superscripts "T" and "−1" indicate vector or matrix transposition and inverse, respectively. $\mathbb{R}^m$ denotes the m-dimensional Euclidean space, and $\|\cdot\|$ is the Euclidean vector norm in $\mathbb{R}^m$. $\mathbb{R}_{\geq 0}$, $\mathbb{R}_{>0}$ and $\mathbb{N}$ represent the sets of non-negative real numbers, positive real numbers and non-negative integers, respectively; $\mathbb{R}_{[s_1,s_2]}$, $\mathbb{N}_{\geq s_1}$ and $\mathbb{N}_{[s_1,s_2]}$ denote the sets $\left\{i \in \mathbb{R}_{\geq 0} | s_1 \leq i \leq s_2\right\}$, $\{i \in \mathbb{N} | i \geq s_1\}$ and $\{i \in \mathbb{N} | s_1 \leq i \leq s_2\}$, respectively. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x\}|_y$ denote, respectively, the mathematical expectation of x and the mathematical expectation of x conditional on y. The notation $P \succ \mathbf{0}$ ($P \prec \mathbf{0}$) means that P is positive (negative) definite. In addition, $\left[\omega_{ij}\right]_{i,j\in\{1,2,\ldots,N\}}$ refers to an $N \times N$ matrix with ω_{ij} representing the ith row, jth column element. $\otimes$ refers to the Kronecker product of matrices, and diag$\{\cdots\}$ means a block-diagonal matrix. In symmetric block matrices, the symbol "$*$" is used as an ellipsis for the terms that are introduced by symmetry. I_m and $\mathbf{0}$ are identity matrix of order m and zero matrix with appropriate dimensions, respectively. Matrices, if their dimensions are not explicitly mentioned, are assumed to be compatible for algebraic operations.

2. Preliminaries and problem formulation

Let us consider the following discrete-time stochastic switching system on the complete probability space $(\Psi, \mathcal{F}, \Pr)$:

$$x(k+1) = A_{r(k)}x(k) + B_{r(k)}u(k) \tag{1}$$

where Ψ is the sample space, $\mathcal{F}$ the σ-algebra of subsets of the sample space, and Pr the probability measure on $\mathcal{F}$; $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ denote the system state and control input, respectively. The jump process $\{r(k)\}_{k\in\mathbb{N}}$ is considered to be a semi-Markov chain, which takes values in the finite set $\mathbb{M} \triangleq \{1, 2, \ldots, M\}$, and governs the switchings among the M system modes. Then, system (1) is regarded as a semi-Markov jump linear system (S-MJLS).

To recall the formal definition of semi-Markov chain, we first present the concept of Markov renewal chain.

Definition 1 (*Barbu & Limnios, 2006*). Let us consider a stochastic process $\{R_n, k_n\}_{n\in\mathbb{N}}$, where R_n is the index of system mode at the nth jump and takes values in $\mathbb{M}$; k_n denotes the time instant at the nth jump with $k_0 = 0$. The stochastic process $\{(R_n, k_n)\}_{n\in\mathbb{N}}$ is a discrete-time homogeneous Markov renewal chain (MRC) if $\forall a \neq b \in \mathbb{M}, \forall \tau \in \mathbb{N}_{\geq 1}$ and $\forall n \in \mathbb{N}$, $\Pr(R_{n+1} = b, S_n = \tau | R_0, k_0; R_1, k_1; \ldots; R_n = a, k_n) = \Pr(R_{n+1} = b, S_n = \tau | R_n = a) = \Pr(R_1 = b, S_0 = \tau | R_0 = a)$, where $S_n \triangleq k_{n+1} - k_n \in \mathbb{N}_{\geq 1}, \forall n \in \mathbb{N}$ denotes the sojourn time of the system mode between the nth jump and $(n+1)$th jump.