



Consistent distributed state estimation with global observability over sensor network[☆]

Xingkang He, Wenchao Xue^{*}, Haitao Fang

LSC, NCMIS, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

Article history:

Received 5 January 2017

Received in revised form 19 October 2017

Accepted 25 January 2018

Available online 26 March 2018

Keywords:

Wireless sensor networks

Time-varying systems

State estimation

Covariance intersection

Distributed Kalman filter

Global observability

Semi-definite programming

ABSTRACT

This paper studies the distributed state estimation problem for a class of discrete time-varying systems over sensor networks. Firstly, it is shown that the gain parameter optimization in a networked Kalman filter requires a centralized framework. Then, a sub-optimal distributed Kalman filter (DKF) is proposed by employing the covariance intersection (CI) fusion strategy. It is proven that the proposed DKF is of consistency, that is, an upper bound of error covariance matrix can be provided by the filter in real time. The consistency also enables the design of adaptive CI weights for better filter precision. Furthermore, the boundedness of covariance matrix and the convergence of the proposed filter are proven based on the strong connectivity of directed network topology and the global observability which permits the sub-system with local sensor's measurements to be unobservable. Meanwhile, to keep the covariance of the estimation error bounded, the proposed DKF does not require the system matrix to be nonsingular at each moment, which seems to be a necessary condition in the main DKF designs under global observability. Finally, simulation results of two examples show the effectiveness of the algorithm in the considered scenarios.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Wireless sensor networks (WSNs) usually consist of intelligent sensing devices located at different geographical positions. Since multiple sensors can collaboratively carry out the task by information communication via the wireless channels, WSNs have been widely applied in environmental monitoring (Cao, Chen, Yan, & Sun, 2008), collaborative information processing (Kumar, 2012; Wu, Sun, Lee, & Pan, 2017), data collection (Solis & Obraczka, 2007), distributed signal estimation (Schizas, Ribeiro, & Giannakis, 2008), and etc. In the past decades, state estimation problems of WSNs have drawn more and more attention of researchers. Two approaches are usually considered in existing work. The first one is centralized filtering (Guo, Shi, Johansson, & Shi, 2017; Ren,

Wu, Johansson, Shi, & Shi, 2018), i.e., a data center is set to collect measurements from all sensors at each sampling moment. The centralized Kalman filter (CKF) can be directly designed such that the minimum variance state estimator is achieved for linear systems with Gaussian noises. However, the centralized frame is fragile since it could be easily influenced by link failure, time delay, package loss and so on. The second approach, on the contrary, utilizes distributed strategy, in which no central sensor exists. The implementation of this strategy simply depends on information exchange between neighbors (Boem, Xu, Fischione, & Parisini, 2015; Farina & Carli, 2016; He, Hu, Xue, & Fang, 2017; Hu, Xie, & Zhang, 2012; Speranzon, Fischione, Johansson, & Sangiovanni-Vincentelli, 2008; Sun, Fu, Wang, Zhang, & Marelli, 2016; Xie & Guo, 2015; Yang, Chen, Wang, & Shi, 2014; Yang, Yang, Shi, Shi, & Chen, 2017). Compared with the centralized approach, the distributed frame has stronger ability in robustness and parallel processing.

Information communication between sensors plays an important role in the design of distributed filtering. Generally, communication rate between neighbors could be faster than the rate of measurement sensing. Fast information exchange between neighbors supports the consensus strategy which can achieve the agreement of information variables (e.g. measurements Das & Moura, 2015) of sensors. Actually, Carli, Chiuso, Schenato, and Zampieri (2008), Cattivelli and Sayed (2010), Khan and Moura (2008) and Olfati-Saber (2007) have shown some remarkable results on the

[☆] This work was supported in part by the NSFC61603380, the National Key Research and Development Program of China (2016YFB0901902), the National Basic Research Program of China under Grant No. 2014CB845301. The material in this paper was partially presented at the 14th International Conference on Control, Automation, Robotics and Vision, November 13–15, 2016, Phuket, Thailand. This paper was recommended for publication in revised form by Associate Editor Brett Ninness under the direction of Editor Torsten Söderström.

^{*} Corresponding author at: LSC, NCMIS, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.

E-mail addresses: xkhe@amss.ac.cn (X. He), wenchaoxue@amss.ac.cn (W. Xue), htfang@iss.ac.cn (H. Fang).

convergence and the consensus of local filters with the consensus strategy. However, faster communication rate probably needs larger capability of computation and transmission to conduct the consensus before the updates of filters. In the single-time scale, the neighbor communication and measurement sensing share the same rate, which can not only reduce communication burden but also result in computation cost linearly matching with sensor number over the network (He et al., 2017; Liu, Wang, He, & Zhou, 2015; Matei & Baras, 2012; Zhou, Fang, & Hong, 2013). Additionally, the DKF algorithm with faster communication rate can be designed by combining the filter with single-time scale and the consensus process. Hence, this paper considers distributed state estimation algorithms in the single-time scale.

Parameter design of algorithms is one of the most essential parts in the study of distributed state estimation problems. In He, Xue, and Fang (2016), it is shown that a networked Kalman filter with optimal gain parameter requires a centralized framework since the calculation of time-varying gain parameter is dependent on information of non-neighbors. Then a modified sub-optimal distributed filter under undirected graph is proposed. Distributed filters with constant filtering gains are well studied in Khan and Jadbabaie (2014) and Khan, Kar, Jadbabaie, and Moura (2010), which evaluate the relationship between the instability of system and the boundedness of estimation error. In Das and Moura (2015), measurement consensus based DKF is presented and design methods of the consensus weights as well as the filtering gains are rigorously studied. In Cattivelli and Sayed (2010), a general diffusion DKF based on time-invariant weights is proposed and performance of the distributed algorithm is analyzed in detail. To achieve better estimation precision, time-varying parameters are considered in Speranzon et al. (2008), which provides a distributed minimum variance estimator for a scalar time-varying signal. In Boem et al. (2015), a distributed prediction method for dynamic systems is proposed to minimize bias and variance. The method can effectively compute time-varying weights of the distributed algorithm. A scalable partition-based distributed Kalman filter is investigated in Farina and Carli (2016) to deal with coupling terms and uncertainty among sub-systems. Furthermore, stability of this algorithm is guaranteed through designing proper parameters. Nevertheless, the work mentioned above have not considered the distributed filter problem with global observability condition which allows the sub-system with local sensor's measurements to be unobservable.

Research of distributed filter for time-varying systems based on global observability is an important but difficult problem. Since sensors of WSNs are sparsely located in different positions, the observability condition assumed for the sub-system with respect to one sensor is much stronger than that assumed for the overall system based on global network. However, the work mentioned above pay little attention to boundedness analysis of covariance matrix and convergence analysis of the algorithm under global observability. Regarding time-invariant systems, conditions on global observability are usually determined by the system matrix, the network topology and the global observation matrix which collects model information of all sensors (Khan & Jadbabaie, 2014; Khan et al., 2010). This means that distributed filters with constant filtering gain can be designed to guarantee stability of the algorithm. However, most of the methods fail for time-varying systems. Battistelli and Chisci (2014) and Battistelli, Chisci, Mugnai, Farina, and Graziano (2015) give some pioneer work on building consensus DKF algorithms under the global observability for time-invariant systems. Nevertheless, they require the assumption that the system matrix is nonsingular, which seems to be severe for time-varying systems at every moment. In this paper, we aim to develop a scalable and totally distributed algorithm for a class of discrete linear time-varying systems in the WSNs. The main contributions are summarized as follows.

- (1) The proposed consistent distributed Kalman filter (CDKF) guarantees the error covariance matrix can be upper bounded by a parameter matrix, which is timely calculated by each sensor using local information. This property is quite of importance since it supports an effective error evaluation principle in real time.
- (2) A set of adaptive weights based on CI fusion is determined through a Semi-definite Programming (SDP) convex optimization method. It is proven that the proposed adaptive CI weights ensure lower error covariance bound than that with constant CI weights which are mainly used in existing work (Battistelli & Chisci, 2014, 2016; Battistelli et al., 2015). Therefore, adaptive CI weights can achieve improvement of estimation performance.
- (3) Global observability instead of local observability is assumed for the system over networks. This allows the sub-system with local sensor's measurements to be unobservable. Additionally, the assumption of system matrix being nonsingular at each moment is loosened (Battistelli & Chisci, 2014, 2016; Battistelli et al., 2015; He et al., 2016; Wang & Ren, 2017). Since the nonsingularity of system matrix at each moment is difficult to be satisfied for time-varying systems, the proposed filter can greatly enlarge application range of the distributed state estimation algorithms.

The remainder of this paper is organized as follows. Section 2 presents some necessary preliminaries and notations of this paper. Section 3 is on problem formulation and distributed filtering algorithms. Section 4 considers performance of the proposed algorithm. Section 5 is on simulation studies. The conclusion of this paper is given in Section 6.

2. Preliminaries and notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed graph, which consists of the set of nodes $\mathcal{V} = \{1, 2, \dots, N\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the weighted adjacent matrix $\mathcal{A} = [a_{ij}]$. In the weighted adjacent matrix \mathcal{A} , all elements are nonnegative, row stochastic and the diagonal elements are all positive, i.e., $a_{i,i} > 0$, $a_{i,j} \geq 0$, $\sum_{j \in \mathcal{V}} a_{i,j} = 1$. If $a_{i,j} > 0$, $j \neq i$, then there is an edge $(i, j) \in \mathcal{E}$, which means node i can directly receive the information of node j . In this situation, node j is called the neighbor of node i . All neighbors of node i including itself can be represented by the set $\{j \in \mathcal{V} | (i, j) \in \mathcal{E}\} \cup \{i\} \triangleq \mathcal{N}_i$, whose size is denoted as $|\mathcal{N}_i|$. \mathcal{G} is called strongly connected if for any pair nodes (i_1, i_l) , there exists a directed path from i_1 to i_l consisting of edges $(i_1, i_2), (i_2, i_3), \dots, (i_{l-1}, i_l)$. According to Horn and Johnson (2012) and Varga (2009), the following lemma is obtained.

Lemma 1. *If the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is strongly connected with $\mathcal{V} = \{1, 2, \dots, N\}$, then all elements of \mathcal{A}^s , $s \geq N - 1$, are positive.*

Throughout this paper, the notations used are fairly standard. The superscript "T" represents transpose. The notation $A \geq B$ (or $A > B$), where A and B are both symmetric matrices, means that $A - B$ is a positive semidefinite (or positive definite) matrix. I_n stands for the identity matrix with n rows and n columns. $E\{x\}$ denotes the mathematical expectation of the stochastic variable x , and $blockcol\{\cdot\}$ means the block elements are arranged in columns. $blockdiag\{\cdot\}$ and $diag\{\cdot\}$ represent the diagonalizations of block elements and scalar elements, respectively. $tr(P)$ is the trace of matrix P . The notation \otimes stands for tensor product. The integer set from a to b is denoted as $[a : b]$.

Download English Version:

<https://daneshyari.com/en/article/7108771>

Download Persian Version:

<https://daneshyari.com/article/7108771>

[Daneshyari.com](https://daneshyari.com)