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Pareto-based guaranteed cost control of the uncertain mean-field stochastic systems in infinite horizon ☆

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ABSTRACT

Pareto-based guaranteed cost control (GCC) problem of uncertain mean-field stochastic systems is investigated in infinite horizon. Firstly, the Pareto game of nominal mean-field stochastic systems is studied. Applying the convexity of the cost functionals, it is shown that all Pareto efficient solutions can be obtained by solving a weighted sum optimal control problem, based on which, Pareto-based GCC problem is solved by the GCC of the weighted sum objective functional. Secondly, applying the Karush–Kuhn–Tucker (KKT) conditions, the necessary conditions for the existence of the Pareto-based guaranteed cost controllers are derived. In particular, it turns out that all controllers are expressed as linear feedback forms involving the state and its mean based on the solutions of the cross-coupled generalized algebraic Riccati equations (CGAREs). Thirdly, this paper presents an LMI-based approach to reduce greatly the computational complexity in the controller design. Finally, two examples are given to show the effectiveness of the proposed results.

1. Introduction

Dynamic games have been extensively researched by many scholars; see, e.g., Basar and Olsder (1999), Bernhard, Gaitsgory, and Pourtallier (2009), Engwerda (2005), Yong (2015) and the references therein. Among various dynamic games, the Pareto game deals with the cooperation between two or more players. In this situation, no player can determine his cost unilaterally. Based on how the players divide their control efforts, each player faces a whole set of possible outcomes. If there are two control strategies v and w such that all players have a lower cost when strategy v is carried out, we say that the solution induced by control strategy v dominates the solution induced by the control strategy w. So, dominance means that the outcome is better for all players. Following this line of thinking, it is reasonable to consider only those cooperative outcomes in which the costs of all players cannot be improved upon simultaneously, the so-called Pareto solutions. Pareto optimality is important in the studies of economic efficiency and income distribution. Also, it plays a central role in the design of investment policies to minimize investment cost and risk, simultaneously (Wu, Chen, & Zhang, 2017). Over the past few decades, the Pareto optimality has been used to analyze some of the most widely used models in economic theory, such as optimal economic growth, environmental economics and engineering (Acemoglu, 2008; Dockner, Jorgensen, Long, & Sorger, 2001; Ramsey, 1928). There have been a great deal of works on Pareto optimality, we refer the reader to Engwerda (2008, 2010), Mukaidani (2013), Mukaidani and Xu (2009) and Reddy and Engwerda (2013, 2014). For instance, Engwerda (2008) determined the set of Pareto efficient equilibria for the regular indefinite LQ control problem of linear affine systems. In Engwerda (2010), Engwerda further presented necessary and sufficient conditions for the existence of a Pareto solution for the finite horizon cooperative differential game of nonlinear systems. Reddy and Engwerda (2013) derived the existence conditions of the Pareto optimal solutions for the LQ infinite horizon cooperative differential games and investigated the relationship between Pareto optimality and weighted sum minimization. Reddy and Engwerda (2014) generalized the results of Engwerda (2010) to the infinite horizon case. However, up to now, most of the work is about deterministic systems, there are

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few results on stochastic systems. Mukaidani and Xu (2009) obtained the decentralized infinite horizon stochastic Pareto-optimal static output feedback strategy for a class of weakly coupled systems with state-dependent noise. In Mukaidani (2013), Mukaidani discussed the Pareto and Nash games for a class of linear stochastic delay systems governed by Itô's stochastic differential equation, respectively.

Mean-field theory was developed to study the collective behaviors resulting from individuals' mutual interactions in various physical and sociological dynamical systems (Huang, Caines, & Malhamé, 2007; Lasry & Lions, 2007; Tembine, Zhu, & Basar, 2014). Huang et al. (2007) investigated large population stochastic dynamical games with mean-field terms. Lasry and Lions (2007) introduced similar problems from the viewpoint of the mean-field theory. Subsequently, Tembine et al. (2014) studied the risk sensitivity of the mean-field game. According to the mean-field theory, the interactions among agents are modeled by mean-field terms. When the number of the individuals goes to infinity, the mean-field term will approach the expected value. Thus, it involves not only the state and the control but also their mathematical expectations in mean-field stochastic differential equations. Mean-field approach has been widely applied to various fields such as engineering, finance and economics; see, e.g., Bensoussan, Frehse, and Yam (2013) and the reference therein. In recent years, there is an increasing interest in mathematics and control theory (Ahdersson & Djehiche, 2011; Bensoussan, Sung, Yam, & Yung, 2016; Elliott, Li, & Ni, 2013; Huang, Li, & Yong, 2015; Ma, Zhang, & Zhang, 2016; Ma, Zhang, Zhang, & Chen, 2015; Ni, Elliott, & Li, 2015; Yong, 2013). Utilizing the mean-field type stochastic maximum principles, Andersson and Djehiche (2011) solved the mean–variance portfolio selection problem. Using the adjoint equation approach, Bensoussan et al. (2016) investigated the unique existence of equilibrium strategies for a class of LQ mean field games. Elliott et al. (2013) presented necessary and sufficient conditions for the solvability of discrete-time mean-field stochastic LQ optimal control problems and derived the optimal control in terms of the solutions to two Riccati difference equations. Ni et al. (2015) extended the results obtained in Elliott et al. (2013) from the finite horizon to the infinite horizon. Ma et al. (2015) investigated stochastic H_2/H_∞ control of the mean-field type continuous-time systems with state- and disturbance-dependent noise. In Ma et al. (2016), Ma et al. considered an H_∞-type control for mean-field stochastic differential equations and gave a sufficient condition for the existence of a stabilizing H_∞ controller in terms of the coupled nonlinear matrix inequalities. Yong (2013) studied the LQ optimal control problem for mean-field stochastic differential equations with deterministic coefficients. Huang et al. (2015) generalized the results of Yong (2013) to the infinite horizon case. However, most of the existing results are about the H_∞ control, the mixed H_2/H_∞ control and the LQ optimal control. To the best of our knowledge, there are few results concerned about the Pareto game of mean-field stochastic systems.

It is well known that uncertainty occurs in many engineering systems and is frequently the sources of instability and performance degradation (Dullerud & Paganini, 2000). In recent years, considerable attention has been paid to robust controller designs for uncertain systems, where guaranteed cost control (GCC) is an effective design method. This approach has the advantage of placing an upper bound on the closed-loop value of a given performance index, and it is guaranteed that the resulting closed-loop system is asymptotically stable. Although, there have been a lot of works on GCC (Petersen & McFarlane, 1994; Ugrinovskii, 2000; Ugrinovskii & Petersen, 2000), few of them are about uncertain systems with multiple decision makers. In 2009, the reference (Mukaidani, 2009) first investigated the GCC problem for a class of uncertain stochastic systems with N decision makers, where the Pareto efficient strategies were introduced to describe the mutual cooperation among the decision makers. Reference (Mukaidani, 2009) provided us a useful idea to deal with various control problems, such as controllability, stabilizability and adaptation, of the game-based controlled systems.

Motivated by the above discussion, in this paper, we consider the Pareto-based GCC problem of the uncertain mean-field stochastic Itô systems. Our work extends the results in Mukaidani (2009) to the mean-field stochastic systems. The appearance of the mean-field terms makes this study more challenging. In addition, compared with Mukaidani (2009), the new contributions of this paper are as follows: (i) We discuss the relationship between the Pareto optimality and the weighted sum minimization, and point out that these two concepts are equivalent under the existing conditions. (ii) On account of the equivalence, we study the Pareto game of nominal mean-field stochastic systems and derive all Pareto efficient strategies based on the solutions of two generalized algebraic Riccati equations (GAREs).

The rest of this paper is organized as follows: Section 2 presents some useful definitions and lemmas. Section 3.1 is devoted to investigating the Pareto game of nominal mean-field stochastic systems. By using the relationship between the Pareto optimality and the weighted sum minimization, it is shown that all Pareto efficient strategies can be obtained via the weighted sum method. Section 3.2 proceeds with the discussion of the Pareto-based GCC problem for the uncertain mean-field stochastic systems. Based on the equivalence between the Pareto optimality and the weighted sum minimization, the Pareto-based GCC problem is transformed into the GCC of the weighted sum objective functional. Employing the KKT theorem, the necessary conditions are obtained via the solvability of the CGAREs. Section 4 employs the LMI approach to simplify the computational complexity. Section 5 provides two examples to show the effectiveness of the proposed results. Finally, we end this paper in Section 6 with a concluding remark.

Notation: $\mathcal{R}^n$: the set of all real n-dimensional vectors. $\mathcal{R}^{m\times n}$: the set of all $m \times n$ real matrices. $A > 0$ (resp. $A \geq 0$): A is a real positive definite (resp. positive semi-definite) symmetric matrix. A^T: the transpose of a matrix A. $\mathrm{Tr}(A)$: the trace of a square matrix A. I_n: a $n\times n$ identity matrix. $\mathbb{E}(x)$: the mathematical expectation of a random variable x. $\|x\|$: the Euclidean norm of a vector x. $\|A\|_F$: the Frobenius norm of a matrix A. $\mathcal{A} := \{\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N) | 0 \leq \alpha_i \leq 1 \text{ and } \sum_{i=1}^{N}\alpha_i = 1\}$.

2. Preliminaries

In this section, we consider the following uncertain mean-field stochastic system

$$\begin{cases} dx(t) = \big[(A + \Delta A(t))x(t) + (\widehat{A} + \Delta\widehat{A}(t))\mathbb{E}x(t) \\ \qquad + (B + \Delta B(t))u(t) + (\widehat{B} + \Delta\widehat{B}(t)) \\ \qquad \times \mathbb{E}u(t)\big]dt + \big[q_1 Cx(t) + q_2\widehat{C}\mathbb{E}x(t) \\ \qquad + q_3 Du(t) + q_4\widehat{D}\mathbb{E}u(t)\big]dw(t), \\ x(0) = \xi, \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the control input, $w(t)$ is a one-dimensional standard Wiener process that is defined on the filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}; \mathcal{F}_t)$ with $\mathcal{F}_t = \sigma(w(s) : 0 \leq s \leq t)$. ξ is an $\mathcal{F}_0$-measurable random vector. $A, \widehat{A}, B, \widehat{B}, C, \widehat{C}, D$ and $\widehat{D}$ are known matrices of appropriate dimensions. $\Delta A(t), \Delta\widehat{A}(t)$, $\Delta B(t)$, $\Delta\widehat{B}(t)$ represent the uncertainties of system (1), which are