Automatica 92 (2018) 225–234

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Robust compensation of a chattering time-varying input delay with jumps ☆

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ARTICLE INFO

Article history:
Received 3 May 2017
Received in revised form 8 November 2017
Accepted 26 February 2018
Available online 5 April 2018

Keywords:
Delay systems
Distributed parameter systems
Reduction method
Prediction-based control
Time-varying delay
Hybrid systems

ABSTRACT

We investigate the design of a prediction-based controller for a linear system subject to a time-varying input delay, not necessarily First-In/First-Out (FIFO). This means that the input signals can be reordered. The feedback law uses the current delay value in the prediction. It does not exactly compensate for the delay in the closed-loop dynamics but does not require to predict future delay values, contrary to the standard prediction technique. Modeling the input delay as a transport Partial Differential Equation, we prove asymptotic stabilization of the system state, that is, robust delay compensation, providing that the average $\mathscr{L}_2$-norm of the delay time-derivative over some time-window is sufficiently small and that the average time between two discontinuities (average dwell time) is sufficiently large.

1. Introduction

Time-delays are ubiquitous in engineering systems. They can take the form of communication lags or physical dead-times and, in all cases, often reveal troublesome in the design and tuning of feedback control laws. Delays are a central concern for numerous systems. When delay stems from transportation of material, as observed in mixing plants for liquid or gaseous fluids (Chèbre, Creff, & Petit, 2010; Petit, Creff, & Rouchon, 1998), automotive engine and exhaust line (Depcik & Assanis, 2005) or heat collector plant (Sbarciog, De Keyser, Cristea, & De Prada, 2008), the dead-time satisfies the First-In/First-Out (FIFO) principle by definition, i.e., the delay D is such that $\dot{D}(t) < 1$ for all time. However, this is not always the case. For example, communication delays can be subject to sudden variations and not satisfy the FIFO principle. This feature, sometimes referred to as fast-varying delay (see Seuret, Gouaisbaut, & Fridman, 2013; Shustin & Fridman, 2007), can also be exhibited for state- or input-dependent input delay systems (Dieulot & Richard, 2001), in which the delay variations can be related to the input in a very intricate manner, like, e.g., for crushing mill devices (Richard, 2003).

We investigate the design of a prediction-based control law (Artstein, 1982; Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Smith, 1959), which is state-of-the-art for constant input delay (Bresch-Pietri, Chauvin, & Petit, 2012; Gu & Niculescu, 2003; Jankovic, 2008; Mazenc & Niculescu, 2011; Michiels & Niculescu, 2007; Moon, Park, & Kwon, 2001) but has only been more recently used for time-varying delays (see Krstic, 2009 or Nihtila, 1991). To compensate for a varying input delay, the prediction has to be calculated over a time window the length of which matches the value of the future delay. In other words, one needs to predict the future variations of the delay to compensate for it. This is the approach followed in Witrant (2005) for a communication time-varying delay, the variations of which are provided by a given known model. It has also been used in Bekiaris-Liberis and Krstic (2012) and Bekiaris-Liberis and Krstic (2013a) for a state-dependent delay or in Bekiaris-Liberis and Krstic (2013b) for a delay depending on delayed state, where variations are anticipated by a careful prediction of the system state. However, in many cases, it is not possible to model the delay and, even if so, to predict the future delay values. For this reason, in this paper, in lieu of seeking exact delay compensation, we consider a prediction horizon equal to the current delay value, which is assumed to be known. This relaxed assumption is realistic. The delay itself can vary to a large extent, can be discontinuous and is not necessarily FIFO. By contrast with previous works accounting explicitly for the delay (that is, without recasting it as a disturbance) and assuming that

☆ The material in this paper was partially presented at the 53rd IEEE Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA. This paper was recommended for publication in revised form by Associate Editor Fouad Giri under the direction of Editor Miroslav Krstic.

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https://doi.org/10.1016/j.automatica.2018.03.058
0005-1098/

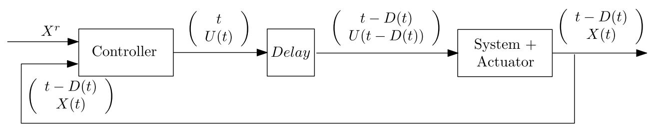

Fig. 1. Example of architecture where the controller knows the current delay value. The communication between the controller and the plant is subject to a delay, while the one between the plant and the controller is not (as they are using different communication paths). The controller is equipped with an internal clock and sends a time stamp with each control input to the block [System + Actuator]. This block then sends back to the controller this (delayed) time stamp, after receiving it. By comparing this delayed time stamp with the time returned by its internal clock, the controller then has access to the current delay affecting the communication path.

$\dot{D}(t) \leq 1$ for $t \geq 0$ (see Bekiaris-Liberis & Krstic, 2013c; Figueredo, Ishihara, Borges, & Bauchspiess, 2011; Yue & Han, 2005), we allow the delay to be such that $\dot{D}(t) > 1$ on some interval of time. A delay of this type, considered for the first time in the preliminary study (Bresch-Pietri & Petit, 2014) in a prediction design context, is also considered in Mazenc, Malisoff, and Niculescu (2017) and Mazenc, Niculescu, and Krstic (2012), but, in these papers the delay is supposed to be equal to a function of class $\mathscr{C}^1$ plus a small discontinuous part, treated as a disturbance. We do not impose such an assumption; in other words, we consider delays with more general types of discontinuities, covering the case where they have large discontinuous jumps.

We follow our preliminary study (Bresch-Pietri & Petit, 2014) which, as a first step, considered the delay function to be continuously differentiable (a demanding assumption from a practical point of view) and apply the novel time-varying version of Halanay inequality proposed in Mazenc et al. (2017) to address delay jumps. In this paper, as a result, the delay is only assumed to be piecewise continuously differentiable, encompassing potential sudden delay jumps and discontinuities, which are quite common, e.g., in the context of networks and communication protocols. Recasting the problem as an Ordinary Differential Equation (ODE) cascaded with a transport Partial Differential Equation (PDE), we use a backstepping transformation recently introduced in Krstic and Smyshlyaev (2008) to analyze the closed-loop stability. We prove asymptotic convergence of the system state provided that the delay time-derivative is sufficiently small in average, in the sense of an average $\mathscr{L}_2$-norm, and that the delay non-differentiability times are sufficiently sparse in time, in the sense of the average dwell time (Hespanha & Morse, 1999).

The paper is organized as follows. In Section 2, we introduce the problem at stake, before designing our control strategy and stating our main result. The latter is proven in Section 3. Section 4 presents an illustrative simulation example.

Notations. In the following, a function f is said to be piecewise continuous on an interval $[a, b] \subset \mathbb{R}$ if the interval can be partitioned by a finite number of points $a = t_0 < t_1 < \cdots < t_n = b$ so that f is continuous on each subinterval (t_{i-1}, t_i) and f admits finite right-hand and left-hand limits at t_i, $i \in \{0, \ldots, n\}$. A function f is said to be piecewise continuous on $\mathbb{R}$ if the restriction of f to any interval is piecewise continuous. A function f is said to be piecewise continuously differentiable on $\mathbb{R}$ if both f and f' are piecewise continuous on $\mathbb{R}$. Standardly, we denote $\mathscr{C}_{pw}(I, \mathbb{R})$ (resp. $\mathscr{C}_{pw}(\mathbb{R}, \mathbb{R})$) the set of real-valued piecewise continuous function on an interval $I \subset \mathbb{R}$ (resp. on $\mathbb{R}$) and $f(t^+)$ (resp. $f(t^-)$) the right-hand (resp. left-hand) limit of f at point t, if it exists.

$|\cdot|$ is the usual Euclidean norm and, for a signal $u(x, \cdot)$ for $x \in [0, 1]$, $\|u(\cdot)\|$ denotes its spatial $\mathscr{L}_2$-norm, i.e.,

$$\|u(t)\| = \sqrt{\int_0^1 u(x, t)^2 dx}. \tag{1}$$

In the sequel, integrals should be understood in the Riemann integrability sense, that is, when the signal $x \mapsto u(x, \cdot)$ is not defined on a set $S \subset [0, 1]$ of measure zero, we write

$$\|u(t)\| = \sqrt{\int_0^1 u(x, t)^2 dx} = \sqrt{\int_{[0,1]\setminus S} u(x, t)^2 dx} \tag{2}$$

and similarly for time signals. Finally, for a matrix M the eigenvalues of which are all real numbers, $\underline{\lambda}(M)$ and $\overline{\lambda}(M)$ refer to the minimal and maximal eigenvalues of M.

2. Problem statement and control design

We consider the following (potentially) unstable linear dynamics

$$\dot{X}(t) = AX(t) + BU(t - D(t)) \tag{3}$$

in which $X \in \mathbb{R}^n$, U is scalar and the delay D satisfies the following assumption.

Assumption 1. The delay D is a **piecewise** continuously differentiable function with set of time instants of non-differentiability

$$\mathscr{T} = \{t_i\,,\ i \in \mathbb{N}\} \tag{4}$$

and which satisfies

(i) $D(t) \in [\underline{D}, \overline{D}]$ for $t \geq 0$, with $0 < \underline{D} \leq \overline{D}$
(ii) there exists $\underline{\Delta} > 0$ such that $t_i - t_j \geq \underline{\Delta}$, $(t_j, t_i) \in \mathscr{T}^2$, $i > j$
(iii) there exist $T > 0$ and $\delta > 0$ such that, for all $i \in \mathbb{N}$,

$$\frac{1}{T}\int_t^{t+T} \dot{D}(s)^2 ds \leq \delta\,, \quad t \in (t_i, t_{i+1} - T), \quad t_i \in \mathscr{T}. \tag{5}$$

Note that no assumption is made a priori on the time-derivative of D. In particular, it is possible that $\dot{D}(t) > 1$ for certain intervals of time. Also, it is worth observing that D is not necessarily continuous at time $t_i \in \mathscr{T}$.

In the sequel, we consider that the current value of the delay is known. For instance, this is the case of the architecture presented in Fig. 1.

Our control objective is to design a prediction-based controller stabilizing the plant (3), using the knowledge of the current value of the delay $D(t)$ at time $t \geq 0$. With this aim in view, consider the following control law

$$U(t) = K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^t e^{A(t-s)}BU(s)ds\right] \tag{6}$$

in which the feedback gain K is such that $A + BK$ is Hurwitz.

This controller approximately forecasts value of the state over a time window of varying length $D(t)$. Indeed, this prediction is only an approximation in the sense that it does not correspond to the future value $X(t + D(t))$ as

$$X(t + D(t)) = e^{AD(t)}X(t) + \int_{t-D(t)}^t e^{A(t-s)}BU(s + D(t) - D(s))ds. \tag{7}$$

However, this last expression is not implementable as it is not always causal.[1] However, it can be approximated by the one used in (6) if $D(t) - D(s) \approx 0$ for "most" instants t, i.e., under the assumption that the variations of the delay are sufficiently small

[1] In details, if there exists $s \in [t - D(t), t]$ such that $s - D(s) \geq t - D(t)$, i.e., if the delay $D(t)$ is suddenly high and the information received at time t older than some previously received), this expression is not causal while the one employed in (6) always is.