Automatica 92 (2018) 1–8

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust stabilization subject to structured uncertainties and mean power constraint[☆]

Yu Feng[a], Xiang Chen[b,*], Guoxiang Gu[c]

[a] College of Information Engineering, Zhejiang University of Technology, Hangzhou, 310032, Zhejiang, PR China
[b] Department of Electrical and Computer Engineering, University of Windsor, Windsor, ON, N9B 3P4, Canada
[c] Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA, 70803-5901, USA

ARTICLE INFO

Article history:
Received 27 July 2017
Received in revised form 30 October 2017
Accepted 8 January 2018

Keywords:
Robust control
Coupled algebraic Riccati equations
Structured uncertainties
Mean power constraint
Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control

ABSTRACT

This paper deals with a robust stabilization problem for discrete-time systems subject to multiple disturbances occurring in controller and actuating channel, where both linear structured uncertainties and white Gaussian noises are included. The desired control law is aimed to robustly stabilize the system and to satisfy some pre-specified mean power constraint, simultaneously. By the philosophy of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control, a solvability condition is first derived for single-input systems that reveals the intrinsic relation between the unstable poles of the plant and the disturbance parameters, together with two cross-coupled algebraic Riccati equations. The result is further generalized to multiple-input systems with a sufficient condition given again by the unstable poles of the plant. An example is included to illustrate the current results.

1. Introduction

In reality, signals are either exchanged by the use of medium with limited capacity or affected by diverse disturbances from internal components and external environment. This fact pushes the classic point-to-point control strategy to its limits. System analysis and design with limited information have thus received increasing attention from many communities over the recent decades (Murray, 2003).

It is well known that analog communication systems are in general subject to power constraints, or so-called signal-to-noise ratio (SNR) constraints. In Braslavsky, Middleton, and Freudenberg (2007), control for single-input-single-output (SISO) systems over an SNR constrained channel is considered, which is closely related to the performance limitation problem in robust control (Chen, 1995; Su, Qiu, & Chen, 2009). The work of Silva, Goodwin, and Quevedo (2010) shows that it is possible to achieve mean square stability at SNRs arbitrarily close to the bound reported in Braslavsky et al. (2007) by using linear time-invariant (LTI) controllers. Moreover, tracking problem over additive white Gaussian noise (AWGN) channels is studied in Li, Tuncel, Chen, and Su (2009) and an explicit expression of the minimal tracking error is given. By the sequential design idea originally reported in Wonham (1967), Qiu et al. introduce the technique of resource allocation and derive a solvability condition for multiple-input systems over AWGN channels in terms of the unstable poles of the plant in Qiu, Gu, and Chen (2013). Stabilization for two-input–two-output systems subject to both input and output SNR constraints is approached in Vargas, Silva, and Chen (2013). Recently, Song, Yang, and Zheng (2016) study the stabilizability and disturbance attenuation of Markov jump linear systems subject to white Gaussian noises through the concept of entropy power. Moreover, systems with multiplicative white Gaussian noises in the process/output channel have also been studied, and relevant control and filtering results have been reported in the literature. For instance, see Bouhtouri, Hinrichsen, and Pritchard (2000), Costa and Oliveira (2012), Feng, Chen, and Gu (2018), Mo and Sinopoli (2012), Su and Checi (2017), Su, Chen, Fu, and Qi (2017) and Xiao, Xie, and Qiu (2012) and the references therein.

In a broader sense, both deterministic and stochastic errors are often involved in system modelling, signal treatment and transmission. Since no mathematical system can precisely model a physical system or procedure, it is of great importance to be aware of how modelling errors might adversely affect the performance of a control system (Doyle, Francis, & Tannenbaum, 2009). The concept of uncertainty and robustness is widely appealed to handle or

☆ The work was supported by the Natural Science Foundation of China under Grant 61573318, Zhejiang Provincial Natural Science Foundation of China for Distinguished Young Scholars under Grant LR17F030003, 111 Project (B12018) in China, and NSERC Discovery Grant in Canada. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

* Corresponding author.
E-mail addresses: yfeng@zjut.edu.cn (Y. Feng), xchen@uwindsor.ca (X. Chen), ggu@lsu.edu (G. Gu).

https://doi.org/10.1016/j.automatica.2018.02.005
0005-1098/

interpret issues with unknown disturbance, unmodelled dynamics, failures (Petersen & Tempo, 2014; Zhou, Doyle, & Glover, 1996), even cellular functions (Steilling, Sauer, Szallasi, Doyle, & Doyle, 2004) and human behaviours (Doyle & Csete, 2011) in certain level. Recently, deterministic uncertainties have also been used as a modelling tool for signal treatment and transmission, and relevant results have been reported in the literature (Fu & Xie, 2005). Stabilization for SISO systems with a norm bounded uncertainty and stochastic multiplicative noise is discussed in Feng, Chen, and Gu (2016) and a set of observer-based stabilizing controllers for such systems is further conducted by solving two algebraic Riccati equations (AREs) and an algebraic Riccati inequality. General dynamic output feedback controller design for such a problem is investigated in Feng, Chen, and Gu (2017) and the solvability condition reveals a trade-off between robust stability and robust performance.

Motivated from these observations, in the current paper we attempt to explicitly characterize the solution to the robust stabilization problem subject to both LTI structured deterministic norm bounded uncertainty and mean power constraint for multiple-input discrete-time systems. Since norm bounded uncertainties and white noises have distinct feature, deterministic and stochastic models are commonly used to describe them, respectively. Considering these two factors at the same time in general makes the problem significantly complicated, and it yields naturally a multi-objective problem. The contributions of the current paper are threefold. First, we conduct a necessity and sufficient condition to such multi-objective problem by using mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control (Chen & Zhou, 2001, 2002; Feng et al., 2018; Limebeer, Anderson, & Hendel, 1994), and show the mean power constraint is indeed a downgraded $\mathcal{H}_2$ performance due to the presence of the uncertainty. Second, a closed-form solvability condition is derived, which reveals the intrinsic relation between unstable poles of the plant and parameters of transmission disturbance and inaccuracy. Finally, the multiple-input case, which is an essential μ-type synthesis problem, is also analytically solved by adjusting disturbance parameters appropriately. A similar problem is addressed in Feng, Chen, and Gu (2013), with a sufficient condition reported for single-input systems. In the current paper, apart from a sufficient condition for the single-input case, we also conduct two crossed-coupled AREs for characterization of the necessity, and further generalize the sufficient condition to the multiple-input case.

The remainder of the paper is organized as follows. Problem formulation is given in Section 2. In Section 3, a sufficient stabilizability condition is first derived for single-input systems in terms of the unstable poles of the plant, with two cross-coupled AREs characterizing of the necessity. Moreover, we further generalize the result to multiple-input systems and derive a solvability condition again by an inequality involving the unstable poles of the plant. A numerical example is included in Section 4 and concluding remarks are given in Section 5.

The notation in this paper is fairly standard. The superscripts 'T' and '$*$' represent the transpose and complex conjugate transpose, respectively. $\mathbb{R}^m$ represents m-dimension Euclidean space. For a real square matrix P, $P \geq 0$ ($P > 0$) means that P is symmetric positive semidefinite (positive definite). The notations $E\{\cdot\}$, $\|\cdot\|$, $\rho(\cdot)$ and $\lambda(\cdot)$ denote the standard expectation operator, Euclidean norm, spectrum radius and eigenvalue, respectively. Moreover, $j = \sqrt{-1}$ is the imaginary number. In addition, for a matrix $A \in \mathbb{R}^{n\times n}$, its Mahler measure is denoted as $M(A) := \prod_{i=1}^{n} \max\{1, |\lambda_i(A)|\}$. A state-space realization of a rational proper transfer function is denoted by $G(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = D + C(zI - A)^{-1}B$.

2. Problem formulation

Consider a real discrete-time stochastic signal

$$u(k) = \begin{bmatrix} u_1(k), & u_2(k), & \cdots, & u_m(k) \end{bmatrix}^T \in \mathbb{R}^m,$$

where $u_i(k)$, $i = 1, \ldots, m$, are random processes. We define the autocorrelation matrix and power spectral density (PSD) of $u(k)$, if they exist, as follows

$$R_u(\tau) := \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} E\left\{u(k+\tau)u^T(k)\right\},$$

$$\Psi_u(\omega) = \sum_{\tau=-\infty}^{\infty} R_u(\tau)e^{-j\omega\tau},$$

where $\tau \in \mathbb{Z}$.

Definition 1 (*Zhou et al., 1996*)**.** A stochastic signal $u(k)$ is said to have bounded power if both $R_u(\tau)$ and $\Psi_u(\omega)$ exist.

Let $\mathcal{P}$ be the space of all bounded power signals. Then the seminorm (mean power) can be defined on $\mathcal{P}$ as

$$\|u\|_{\mathcal{P}} = \sqrt{\lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} E\left\{\|u(k)\|^2\right\}} = \sqrt{\mathrm{Tr}\{R_u(0)\}}.$$

One has $R_u(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Psi_u(\omega)e^{j\omega\tau}d\omega$. Thus, the power norm of $u(k)$ can also be computed from its PSD by

$$\|u\|_{\mathcal{P}} = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{Tr}\{\Psi_u(\omega)\}d\omega}.$$

Note that the bounded power signals have been widely adopted in mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control (Chen & Zhou, 2001; Zhou et al., 1996) and filtering problem (Chen & Zhou, 2002).

Consider the following discrete-time system

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x_0, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ are the state vector and control input, respectively. The matrices A and B are constant with consistent dimensions. Denote this system by $[A|B]$ for simplicity. For the rest of this paper, we assume that $[A|B]$ is stabilizable and A has no eigenvalue on the unit circle. Note that the latter condition is made to avoid a singular control problem. If it does not hold, we can let $A_\epsilon = (1+\epsilon)A$ with $\epsilon > 0$ such that the eigenvalues on the unit circle move outside the circle after perturbation. Then, by using the same arguments for $[A_\epsilon|B]$ and taking the limit $\epsilon \to 0$, we can obtain the same result.

In our setup, both computation in controller $v(k) = Fx(k)$, $F \in \mathbb{R}^{m\times n}$, and transmission in actuating channels are nonideal, and the individual components $v_i(k)$, $i = 1, \ldots, m$, of $v(k)$ are transmitted independently. The overall model is depicted in Fig. 1, consisting of a received signal-to-error ratio (R-SER) model (Qiu et al., 2013) succeeded by an AWGN model with pre- and post-scaling factors, where $\Gamma_i > 0$, $d_i(k)$ is a white Gaussian noise with variance σ_i^2 and Δ_i is a linear time-invariant norm bounded uncertainty. The induced norm of Δ_i satisfies

$$\|\Delta_i\|_\infty := \sup_{\|q_i\|_{\mathcal{P}}\neq 0} \frac{\|w_i\|_{\mathcal{P}}}{\|q_i\|_{\mathcal{P}}} \leq \delta_i, \tag{2}$$

for some $0 < \delta_i < 1$. The upper bound of δ_i ensures that the feedback interconnection of 1 and Δ_i is causal and bounded. Note that stability of Δ_i does not depend on its initial condition. For simplicity, we assume zero initial condition for Δ_i. Moreover, assume that the noises $d_i(k)$, $i = 1, \ldots, m$, are mutually uncorrelated